If Content is King,
Context is God!

Word Embeddings

Oct 16th & 19th, 2017

ADL x MLDS

YUN-NUNG (VIVIAN) CHEN

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National Taiwan University

Slides credited from Dr. Richard Socher

Announcement

Guest Lecture Report Submission (5% for participation)

- Report content requirement
  - Length: A4 1 page
  - Content:
    1. What did you learn? 我學到了甚麼？
    2. What do I want to know? 我還想知道甚麼？
    3. How can I leverage my expertise and the learned knowledge to benefit the company’s product? 如果我是公司員工，我想要如何利用我的expertise及本課程所學來benefit公司的product？
    4. Can you draft a project proposal based on the available company data and the learned skills from ADLxMLDS? 若根據課程所學以及公司的資源，我想要propose一個新的project，可能的內容為何？
- Deadline: midnight of 10/21 (Sat)
- Submitted via Ceiba

Review

Meaning Representations in Computers

Knowledge-based representation

Corpus-based representation

✓Atomic symbol

✓Neighbors

- High-dimensional sparse word vector
- Low-dimensional dense word vector
  - Method 1 – dimension reduction
  - Method 2 – direct learning

Meaning Representations in Computers

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Corpus-based representation

Atomic symbols: ***one-hot*** representation

car $[0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ \ldots\ 0]$

car

Issues: difficult to compute the similarity
(i.e. comparing "car" and "motorcycle")

$[0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ \ldots\ 0]$ AND $[0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ \ldots\ 0] = 0$

car motorcycle

Idea: words with similar meanings often have similar neighbors

Meaning Representations in Computers

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Window-based Co-occurrence Matrix

Example

- Window length=1
- Left or right context
- Corpus:

I love NTU.
I love deep learning.
I enjoy learning.

similarity > 0

Counts	I	love	enjoy	NTU	deep	learning
I	0	2	1	0	0	0
love	2	0	0	1	1	0
enjoy	1	0	0	0	0	1
NTU	0	1	0	0	0	0
deep	0	1	0	0	0	1
learning	0	0	1	0	1	0

Issues:

- matrix size increases with vocabulary
- high dimensional
- sparsity → poor robustness

Idea: low dimensional word vector

Meaning Representations in Computers

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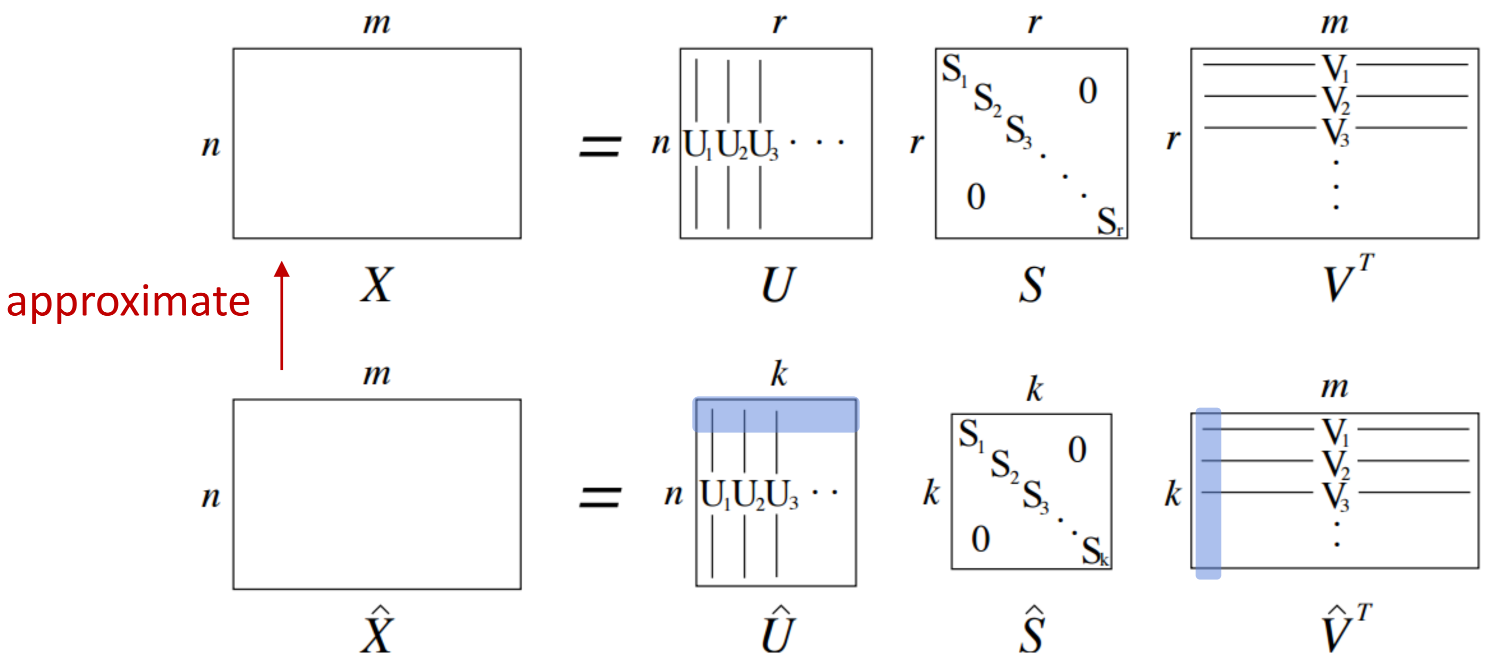
Low-Dimensional Dense Word Vector

Method 1: dimension reduction on the matrix

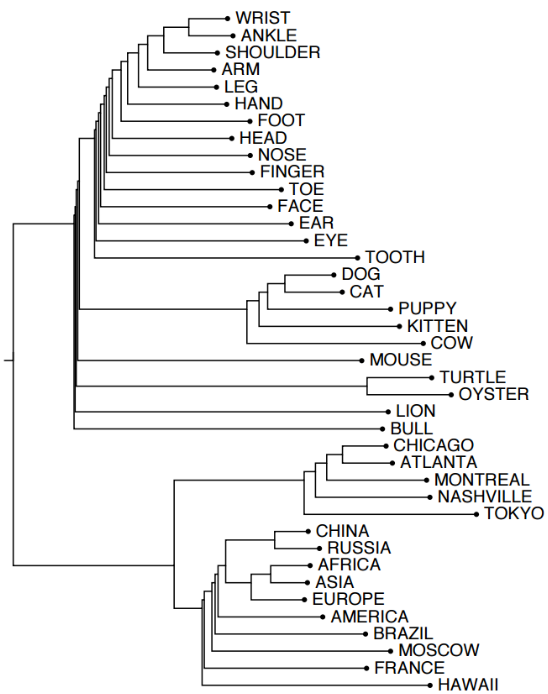
Singular Value Decomposition (SVD) of co-occurrence matrix X



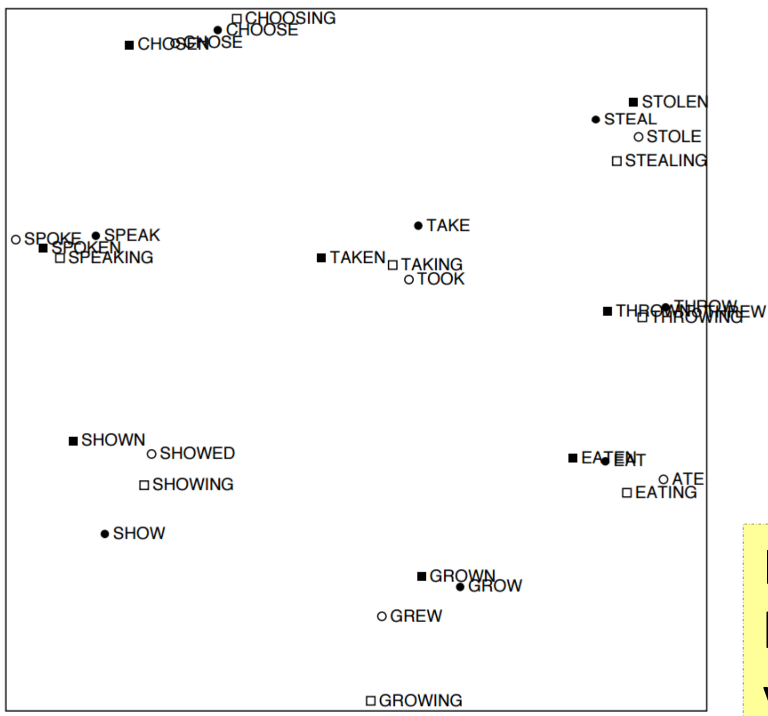
Low-Dimensional Dense Word Vector

Method 1: dimension reduction on the matrix

Singular Value Decomposition (SVD) of co-occurrence matrix X



semantic relations



syntactic relations

Issues:

- computationally expensive: $O(mn^2)$ when n<m for n x m matrix
- difficult to add new words

Idea: directly learn low-dimensional word vectors

Word Representation

Knowledge-based representation

Corpus-based representation

- ✓ Atomic symbol
- ✓ Neighbors
  - High-dimensional sparse word vector
  - Low-dimensional dense word vector
    - Method 1 – dimension reduction
    - Method 2 – direct learning → word embedding

Word Embedding

Method 2: directly learn low-dimensional word vectors

- Learning representations by back-propagation. (Rumelhart et al., 1986)
- A neural probabilistic language model (Bengio et al., 2003)
- NLP (almost) from Scratch (Collobert & Weston, 2008)
- Recent and most popular models: word2vec (Mikolov et al. 2013) and Glove (Pennington et al., 2014)

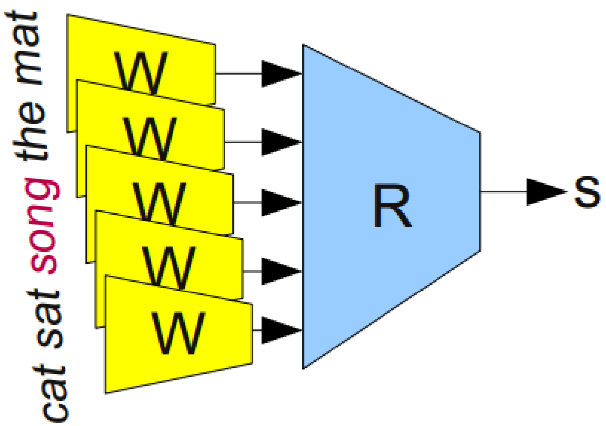
Word Embedding Benefit

Given an unlabeled training corpus, produce a vector for each word that encodes its semantic information. These vectors are useful because:

① semantic similarity between two words can be calculated as the cosine similarity between their corresponding word vectors

② word vectors as powerful features for various supervised NLP tasks since the vectors contain semantic information

③ propagate any information into them via neural networks and update during training



Word2Vec Skip-Gram

Mikolov et al., “Distributed representations of words and phrases and their compositionality,” in *NIPS*, 2013.

Mikolov et al., “Efficient estimation of word representations in vector space,” in *ICLR Workshop*, 2013.

Word2Vec – Skip-Gram Model

Goal: predict surrounding words within a window of each word

Objective function: maximize the probability of any context word given the current center word

$$w_1, w_2, \cdots, \underbrace{w_{t-m}, \cdots, w_{t-1}, w_t, w_{t+1}, \cdots, w_{t+m}}_{\text{context window}}, \cdots, w_{T-1}, w_T$$

w_I $\quad$ w_O

$$p(w_{O,1}, w_{O,2}, \cdots, w_{O,C} \mid w_I) = \prod_{c=1}^{C} p(w_{O,c} \mid w_I)$$

target word vector

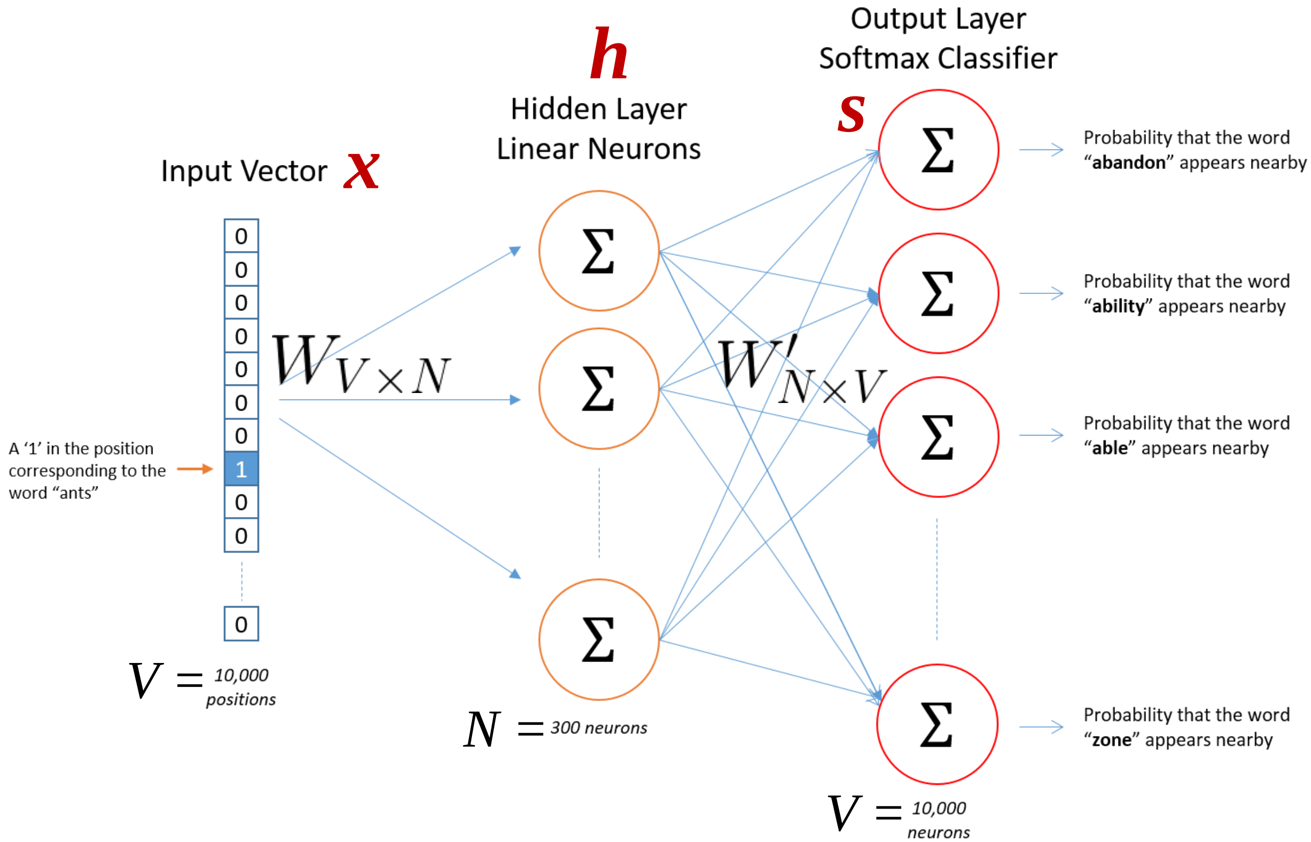
$$C(\theta) = -\sum_{w_I} \sum_{c=1}^{C} \log p(w_{O,c} \mid w_I) \quad p(w_O \mid w_I) = \frac{\exp(v'^T_{w_O} v_{w_I})}{\sum_j \exp(v'^T_{w_j} v_{w_I})}$$

outside $\quad$ target word

Benefit: faster, easily incorporate a new sentence/document or add a word to vocab

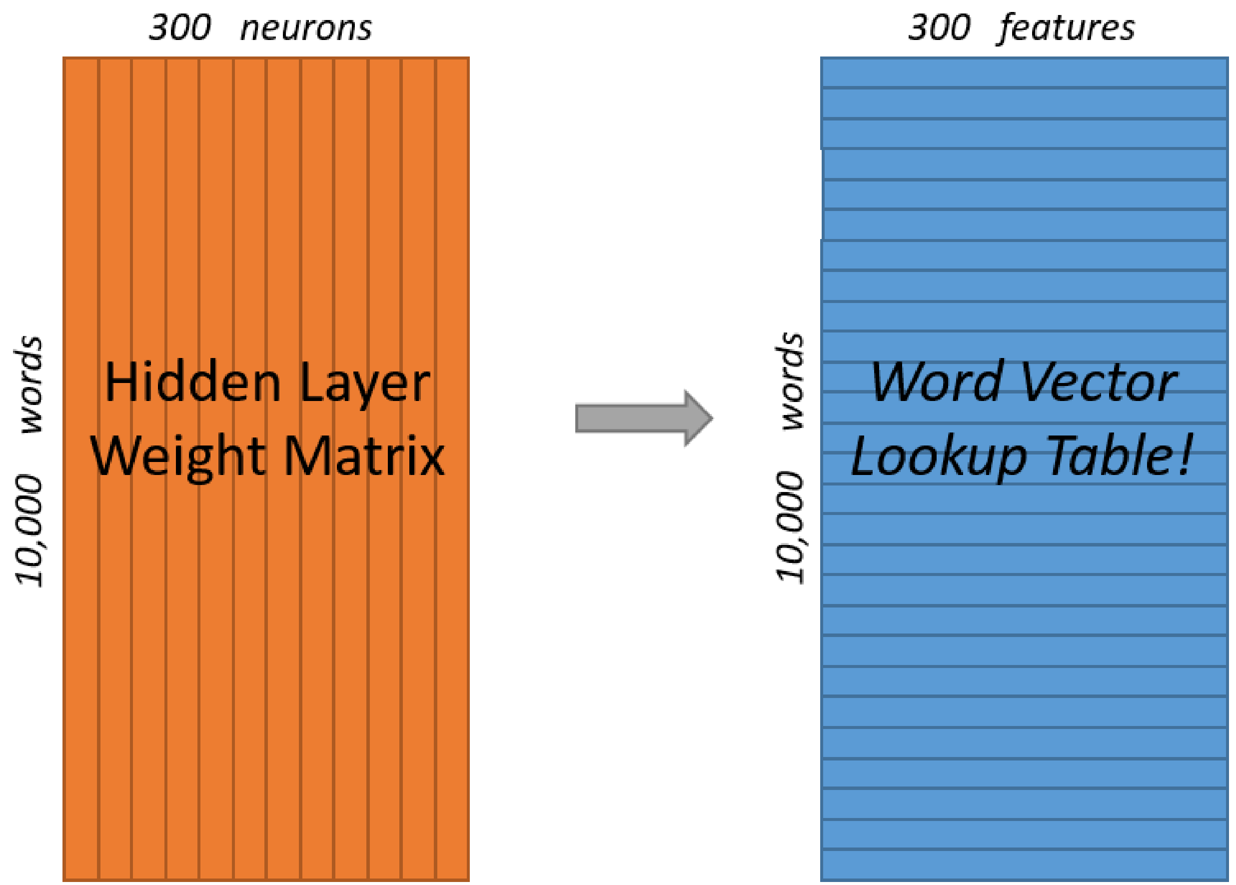
Word2Vec Skip-Gram Illustration

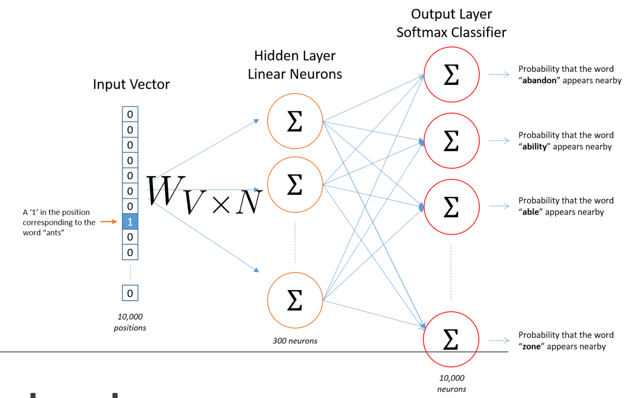
Goal: predict surrounding words within a window of each word



Hidden Layer Weight Matrix → Word Embedding Matrix

$$W_{V \times N}$$



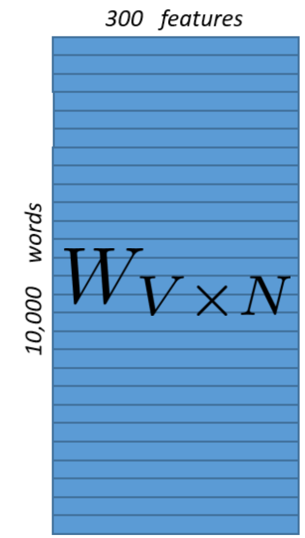


Weight Matrix Relation

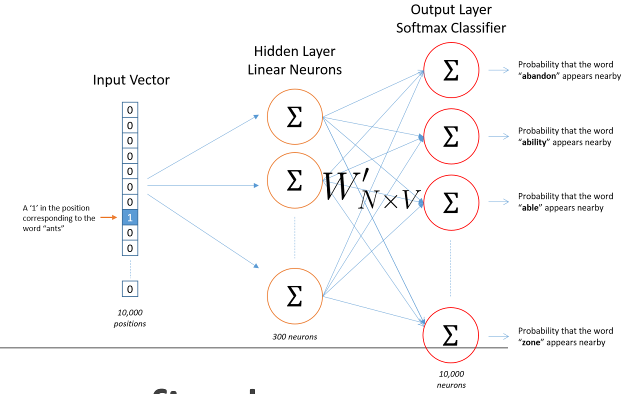
Hidden layer weight matrix = word vector lookup

$$h = x^T W = W_{(k,.)} := v_{w_I}$$

$$[0 \quad 0 \quad 0 \quad 1 \quad 0] \times \begin{bmatrix} 17 & 24 & 1 \\ 23 & 5 & 7 \\ 4 & 6 & 13 \\ 10 & 12 & 19 \\ 11 & 18 & 25 \end{bmatrix} = [10 \quad 12 \quad 19]$$



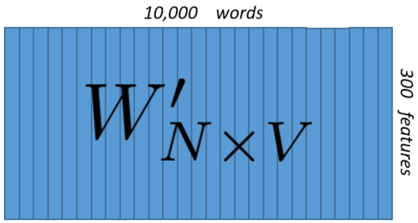
Each vocabulary entry has two vectors: as a **target** word and as a **context** word



Weight Matrix Relation

Output layer weight matrix = weighted sum as final score

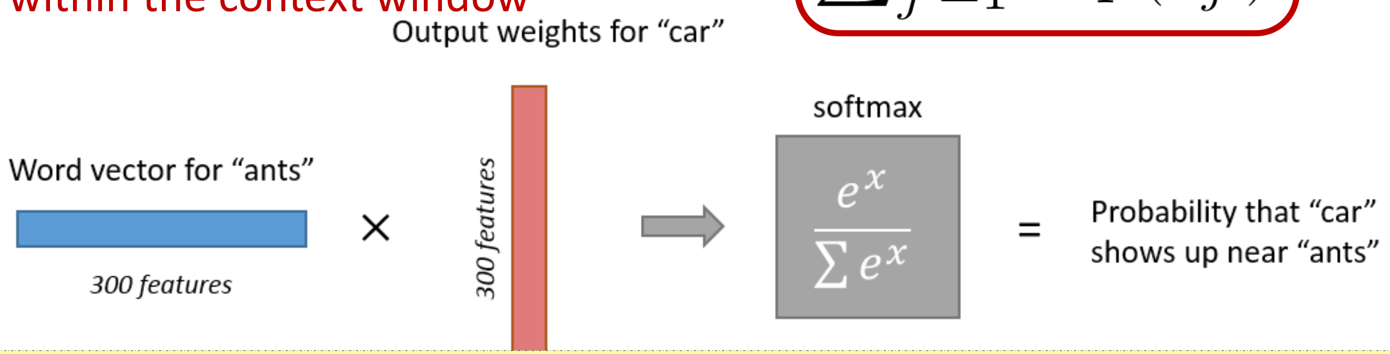
$$s_j = h v'_{w_j}$$



$$p(w_j = w_{O,c} \mid w_I) = y_{jc} = \boxed{\frac{\exp(s_{jc})}{\sum_{j'=1}^{V} \exp(s_{j'})}}$$

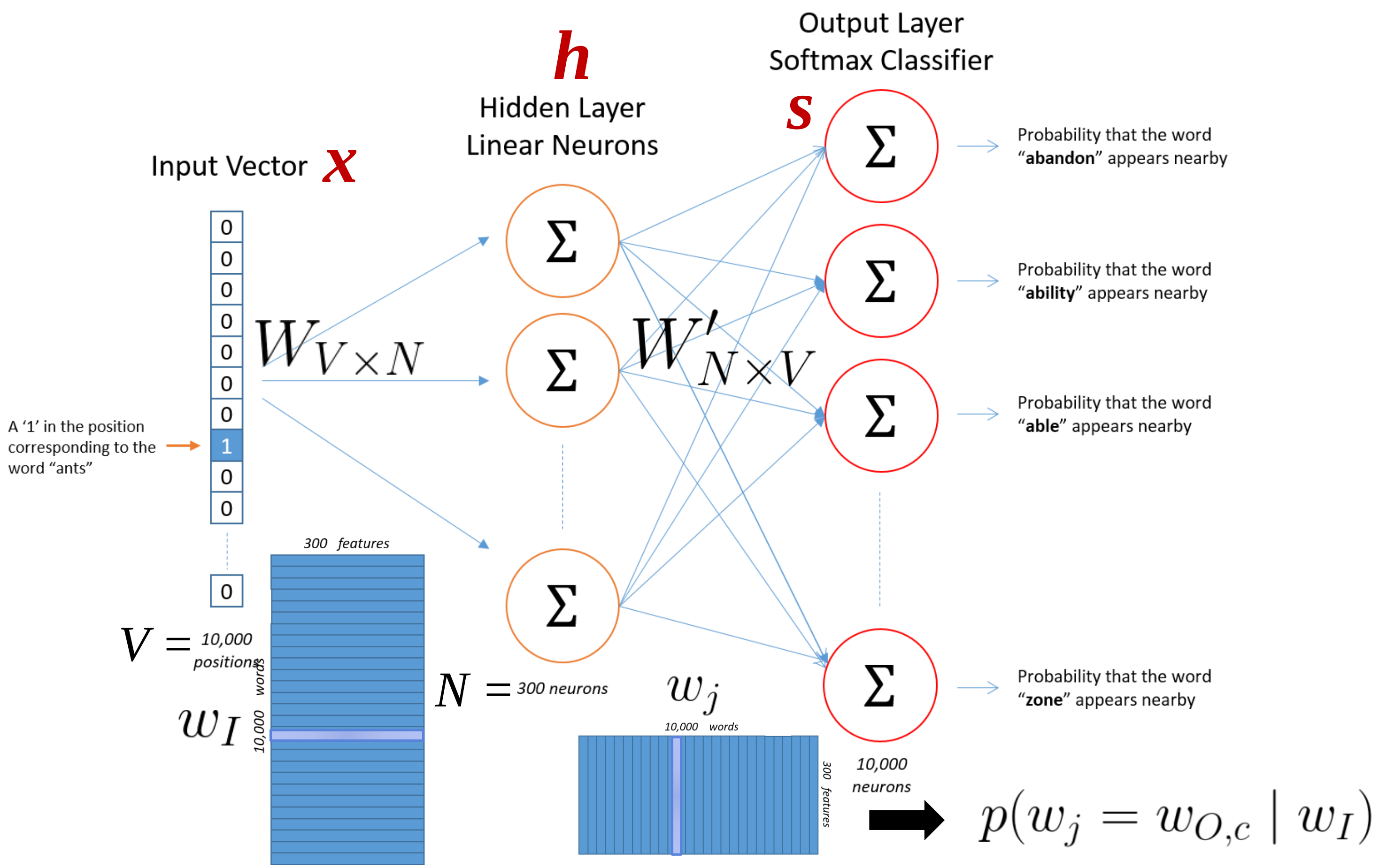
softmax

within the context window



Each vocabulary entry has two vectors: as a **target** word and as a **context** word

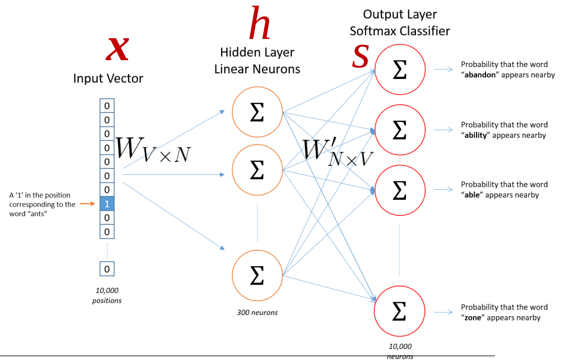
Word2Vec Skip-Gram Illustration



Loss Function

Given a target word (w_I)

$$C(\theta) = -\log p(w_{O,1}, w_{O,2}, \cdots, w_{O,C} \mid w_I)$$
$$= -\log \prod_{c=1}^{C} \frac{\exp(s_{j_c})}{\sum_{j'=1}^{V} \exp(s_{j'})}$$
$$= -\sum_{c=1}^{C} s_{j_c} + C \log \sum_{j'=1}^{V} \exp(s_{j'})$$



SGD Update for W'

Given a target word (w_I)

$$\frac{\partial C(\theta)}{\partial w'_{ij}} = \sum_{c=1}^{C} \frac{\partial C(\theta)}{\partial s_{j_c}} \frac{\partial s_{j_c}}{\partial w'_{ij}} = \sum_{c=1}^{C} (y_{j_c} - t_{j_c}) \cdot h_i$$

$$\boxed{s_j = {v'_{w_j}}^T \cdot h}$$

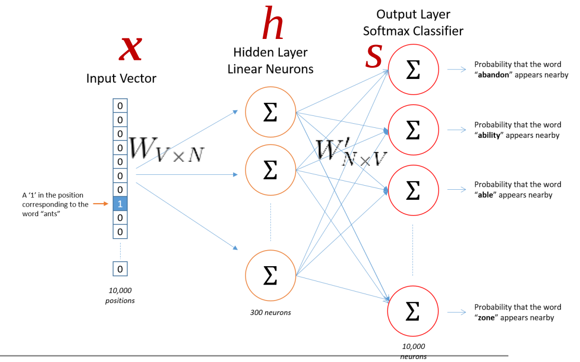
$$\frac{\partial C(\theta)}{\partial s_{j_c}} = y_{j_c} - t_{j_c} := e_{j_c}$$

error term

=1, when w_{jc} is within the context window

=0, otherwise

$$\boxed{{w'_{ij}}^{(t+1)} = {w'_{ij}}^{(t)} - \eta \cdot \sum_{c=1}^{C} (y_{j_c} - t_{j_c}) \cdot h_i}$$



SGD Update for W

$$\frac{\partial C(\theta)}{\partial w_{ki}} = \frac{\partial C(\theta)}{\partial h_i}\frac{\partial h_i}{\partial w_{ki}} = \sum_{j=1}^{V}\sum_{c=1}^{C}(y_{j_c} - t_{j_c}) \cdot w'_{ij} \cdot x_k$$

$$\boxed{h = x^T W}$$

$$\frac{\partial C(\theta)}{\partial h_i} = \sum_{j=1}^{V}\frac{\partial C(\theta)}{\partial s_j}\frac{\partial s_j}{\partial h_i} = \sum_{j=1}^{V}\sum_{c=1}^{C}(y_{j_c} - t_{j_c}) \cdot w'_{ij}$$

$$\boxed{s_j = {v'_{w_j}}^T \cdot h}$$

$$w_{ij}^{(t+1)} = w_{ij}^{(t)} - \eta \cdot \sum_{j=1}^{V}\sum_{c=1}^{C}(y_{j_c} - t_{j_c}) \cdot w'_{ij} \cdot x_j$$

SGD Update

$$w'^{(t+1)}_{ij} = w'^{(t)}_{ij} - \eta \cdot \sum_{c=1}^{C} (y_{j_c} - t_{j_c}) \cdot h_i$$

$$EI_j = \sum_{c=1}^{C} (y_{j_c} - t_{j_c})$$

$$v'^{(t+1)}_{w_j} = v'^{(t)}_{w_j} - \eta \cdot EI_j \cdot h$$

$$w^{(t+1)}_{ij} = w^{(t)}_{ij} - \eta \cdot \sum_{j=1}^{V} \sum_{c=1}^{C} (y_{j_c} - t_{j_c}) \cdot w'_{ij} \cdot x_j$$

$$v^{(t+1)}_{w_I} = v^{(t)}_{w_I} - \eta \cdot EH^T$$

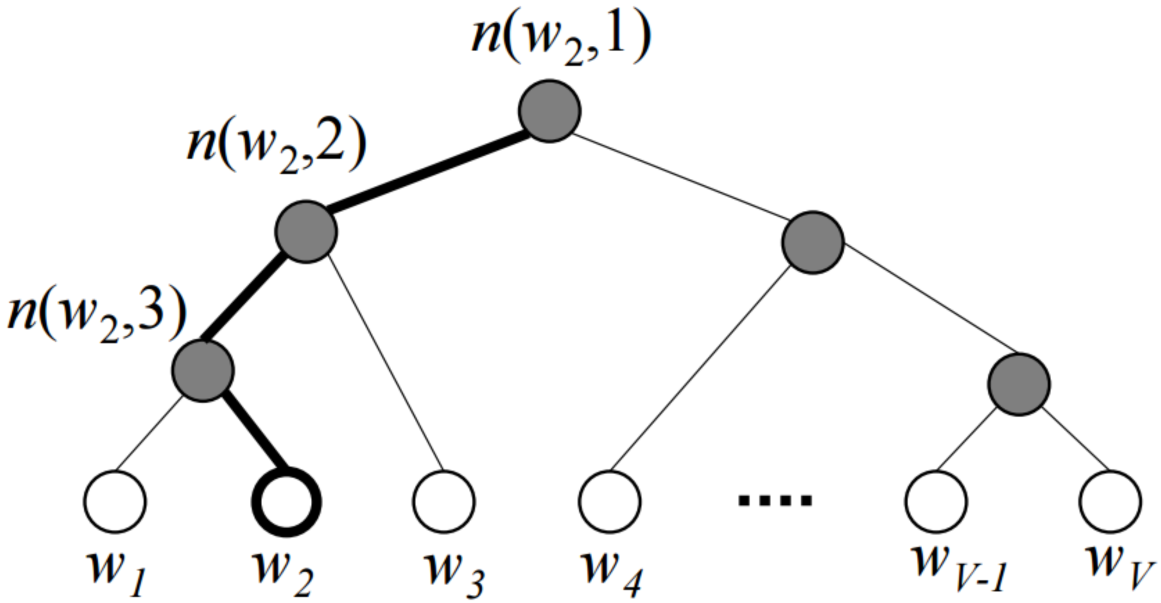
$$EH_i = \sum_{j=1}^{V} EI_j \cdot w'_{ij} \cdot x_j$$

large vocabularies or large training corpora → expensive computations

limit the number of output vectors that must be updated per training instance
→ hierarchical softmax, sampling

Hierarchical Softmax

Idea: compute the probability of leaf nodes using the paths



$$O(N) \rightarrow O(\log N)$$

Negative Sampling

Idea: only update a sample of output vectors

$$C(\theta) = -\log \sigma({v'_{w_O}}^T v_{w_I}) + \sum_{w_j \in \mathcal{W}_{\text{neg}}} \log \sigma({v'_{w_j}}^T v_{w_I})$$

$${v'_{w_j}}^{(t+1)} = {v'_{w_j}}^{(t)} - \eta \cdot EI_j \cdot h$$

$$EI_j = \sigma({v'_{w_j}}^T v_{w_I}) - t_j$$

$$v_{w_I}^{(t+1)} = v_{w_I}^{(t)} - \eta \cdot EH^T$$

$$EH = \sum_{w_j \in \{w_O\} \cup \mathcal{W}_{\text{neg}}} EI_j \cdot v'_{w_j}$$

$$w_j \in \{w_O\} \cup \mathcal{W}_{\text{neg}}$$

Negative Sampling

Sampling methods $w_j \in \{w_O\} \cup \mathcal{W}_{\text{neg}}$

- Random sampling
- Distribution sampling: w_j is sampled from $P(w)$

What is a good $P(w)$?

Idea: less frequent words sampled more often

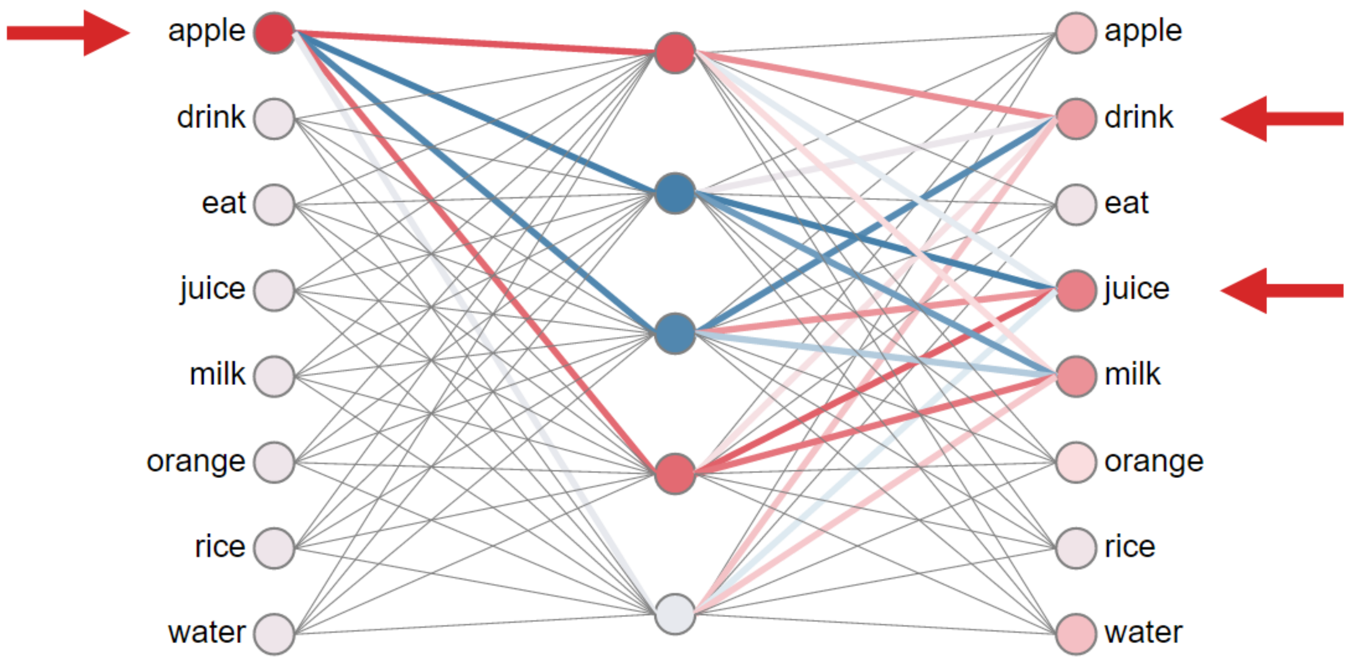
Empirical setting: unigram model raised to the power of 3/4

Word	Probability to be sampled for "neg"
is	$0.9^{3/4} = 0.92$
constitution	$0.09^{3/4} = 0.16$
bombastic	$0.01^{3/4} = 0.032$

Word2Vec Skip-Gram Visualization

https://ronxin.github.io/wevi/

Skip-gram training data:
apple|drink^juice,orange|eat^apple,rice|drink^juice,juice|drink^milk,
milk|drink^rice,water|drink^milk,juice|orange^apple,juice|apple^drink
,milk|rice^drink,drink|milk^water,drink|water^juice,drink|juice^water



Word2Vec Variants

Skip-gram: predicting surrounding words given the target word (Mikolov+, 2013)

better

$$p(w_{t-m}, \cdots w_{t-1}, w_{t+1}, \cdots, w_{t+m} \mid w_t)$$

CBOW (continuous bag-of-words): predicting the target word given the surrounding words (Mikolov+, 2013)

$$p(w_t \mid w_{t-m}, \cdots w_{t-1}, w_{t+1}, \cdots, w_{t+m})$$

LM (Language modeling): predicting the next words given the proceeding contexts (Mikolov+, 2013)

first

$$p(w_{t+1} \mid w_t)$$

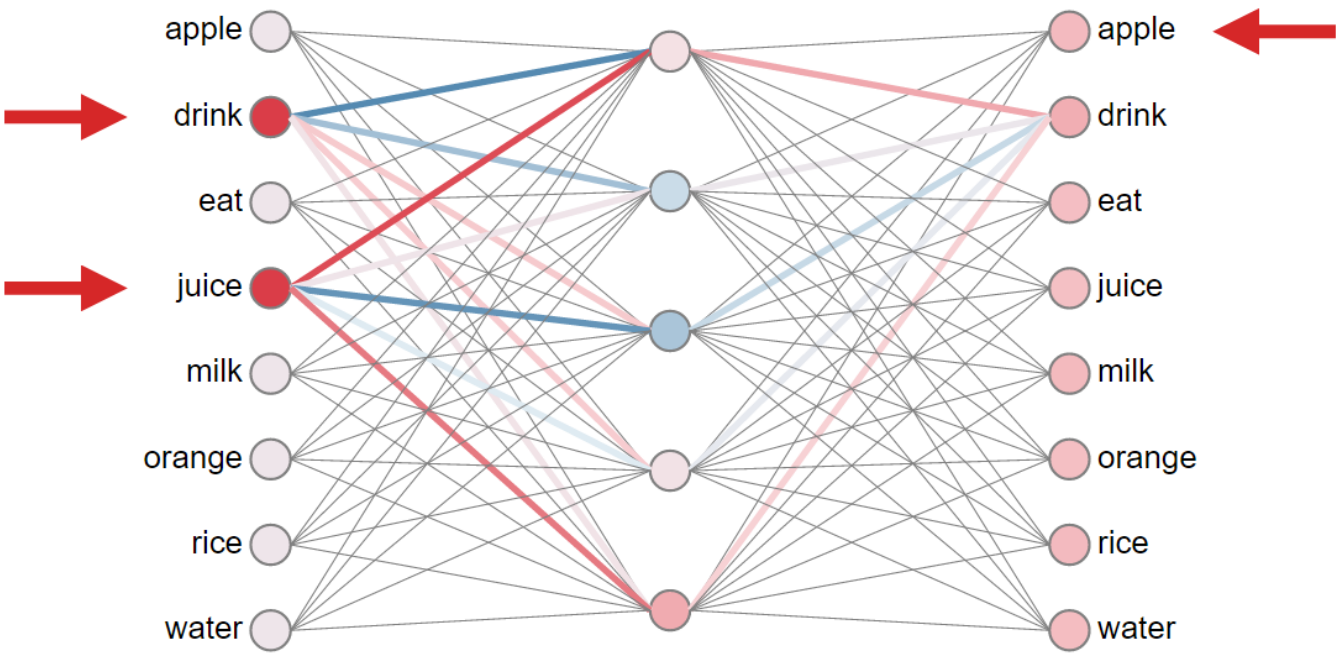
Practice the derivation by yourself!!

Mikolov et al., "Efficient estimation of word representations in vector space," in *ICLR Workshop*, 2013.
Mikolov et al., "Linguistic regularities in continuous space word representations," in *NAACL HLT*, 2013.

Word2Vec CBOW

Goal: predicting the target word given the surrounding words

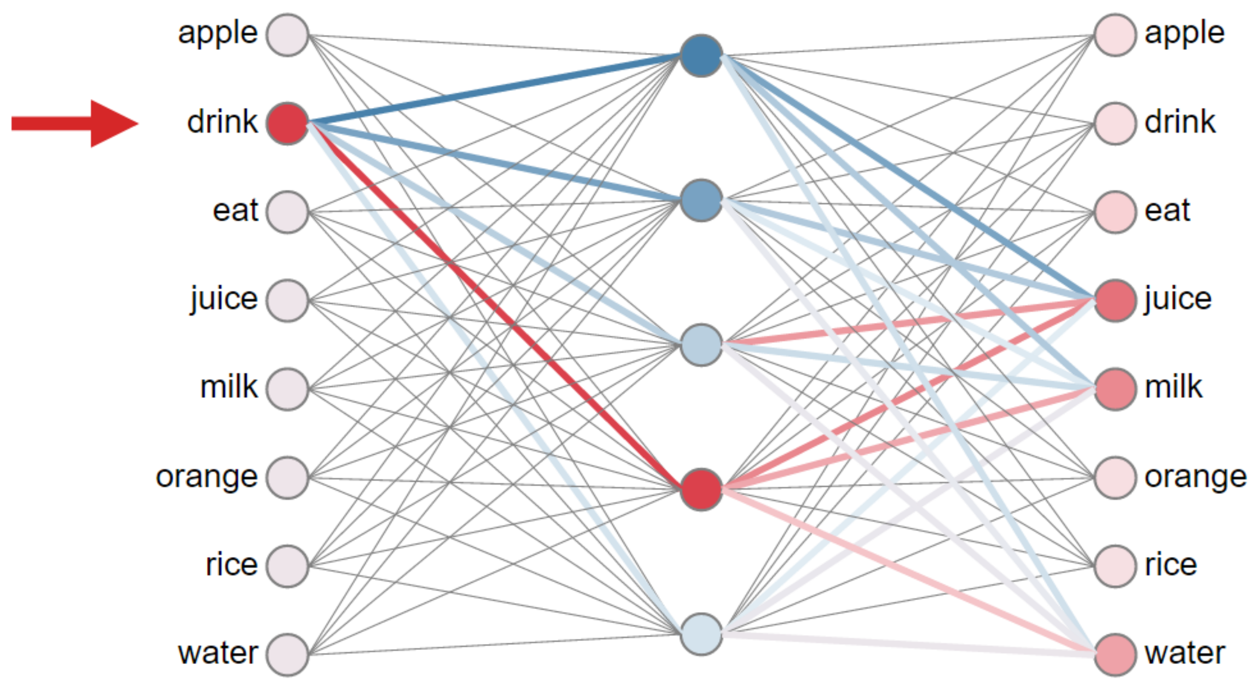
$$p(w_t \mid w_{t-m}, \cdots w_{t-1}, w_{t+1}, \cdots , w_{t+m})$$



Word2Vec LM

Goal: predicting the next words given the proceeding contexts

$$p(w_{t+1} \mid w_t)$$



Comparison

Count-based

- Example
  - LSA, HAL (Lund & Burgess), COALS (Rohde et al), Hellinger-PCA (Lebret & Collobert)
- Pros
  - ✓ Fast training
  - ✓ Efficient usage of statistics
- Cons
  - ✓ Primarily used to capture word similarity
  - ✓ Disproportionate importance given to large counts

Direct prediction

- Example
  - NNLM, HLBL, RNN, Skipgram/CBOW, (Bengio et al; Collobert & Weston; Huang et al; Mnih & Hinton; Mikolov et al; Mnih & Kavukcuoglu)
- Pros
  - ✓ Generate improved performance on other tasks
  - ✓ Capture complex patterns beyond word similarity
- Cons
  - ✓ Benefits mainly from large corpus
  - ✓ Inefficient usage of statistics

Combining the benefits from both worlds → GloVe

GloVe

Pennington et al., "GloVe: Global Vectors for Word Representation," in EMNLP, 2014.

GloVe

Idea: ratio of co-occurrence probability can encode meaning

P_{ij} is the probability that word w_j appears in the context of word w_i

$$P_{ij} = P(w_j \mid w_i) = X_{ij}/X_i$$

Relationship between the words w_i and w_j

	x = solid	x = gas	x = water	x = random
$P(x \mid \text{ice})$	large	small	large	small
$P(x \mid \text{stream})$	small	large	large	small
$\dfrac{P(x \mid \text{ice})}{P(x \mid \text{stream})}$	large	small	~ 1	~ 1

GloVe

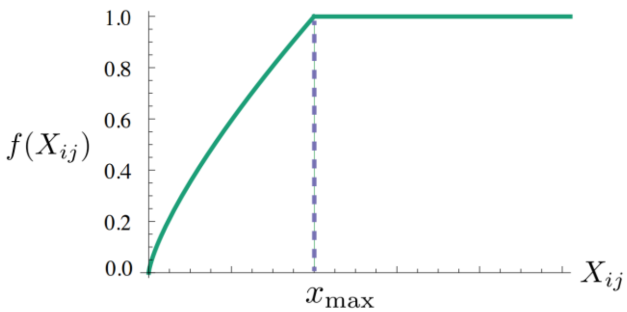
The relationship of w_i and w_j approximates the ratio of their co-occurrence probabilities with various w_k

$$F(w_i, w_j, \tilde{w}_k) = \frac{P_{ik}}{P_{jk}}$$

$$F(w_i - w_j, \tilde{w}_k) = \frac{P_{ik}}{P_{jk}}$$

$$F((v_{w_i} - v_{w_j})^T v'_{\tilde{w}_k}) = \frac{P_{ik}}{P_{jk}} \qquad F(\cdot) = \exp(\cdot)$$

$$v_{w_i} \cdot v'_{\tilde{w}_k} = v_{w_i}^T v'_{\tilde{w}_k} = \log P(w_k \mid w_i)$$



GloVe

$$P_{ij} = X_{ij}/X_i$$

$$\begin{aligned} v_{w_i} \cdot v'_{\tilde{w}_j} &= v_{w_i}^T v'_{\tilde{w}_j} = \log P(w_j \mid w_i) \\ &= \log P_{ij} = \log(X_{ij}) - \log(X_i) \end{aligned}$$

$$v_{w_i}^T v'_{\tilde{w}_j} + b_i + \tilde{b}_j = \log(X_{ij})$$

$$C(\theta) = \sum_{i,j=1}^{V} f(P_{ij})(v_{w_i} \cdot v'_{\tilde{w}_j} - \log P_{ij})^2$$

$$C(\theta) = \sum_{i,j=1}^{V} f(X_{ij})(v_{w_i}^T v'_{\tilde{w}_j} + b_i + \tilde{b}_j - \log X_{ij})^2$$

fast training, scalable, good performance even with small corpus, and small vectors

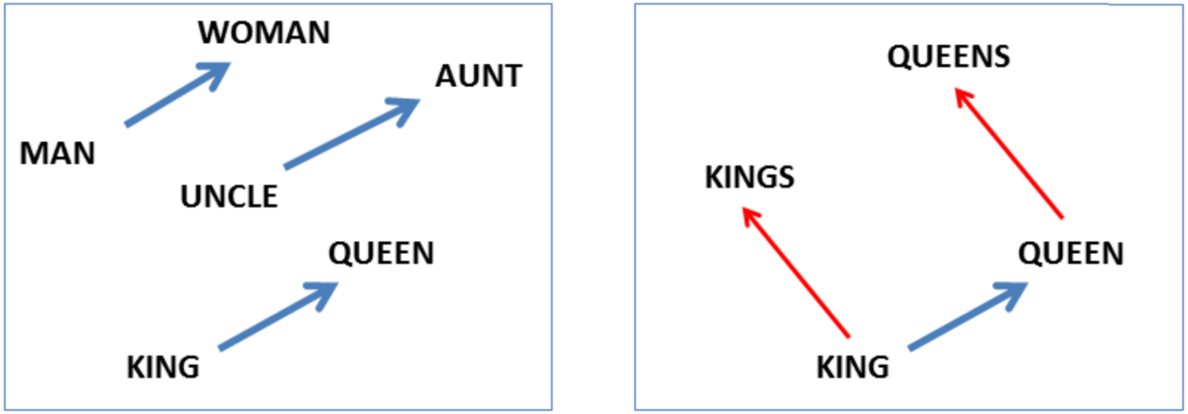
Word Vector Evaluation

Intrinsic Evaluation – Word Analogies

Word linear relationship $w_A : w_B = w_C : w_x$

$$x = \arg\max_x \frac{(v_{w_B} - v_{w_A} + v_{w_C})^T v_{w_x}}{\left\| v_{w_B} - v_{w_A} + v_{w_C} \right\|}$$

Syntactic and **Semantic** example questions [link]



Issue: what if the information is there but not linear

Intrinsic Evaluation – Word Analogies

Word linear relationship $w_A : w_B = w_C : w_x$

Syntactic and **Semantic** example questions [link]

city---in---state

Chicago : Illinois = Houston : Texas
Chicago : Illinois = Philadelphia : Pennsylvania
Chicago : Illinois = Phoenix : Arizona
Chicago : Illinois = Dallas : Texas
Chicago : Illinois = Jacksonville : Florida
Chicago : Illinois = Indianapolis : Indiana
Chicago : Illinois = Aus8n : Texas
Chicago : Illinois = Detroit : Michigan
Chicago : Illinois = Memphis : Tennessee
Chicago : Illinois = Boston : Massachusetts

Issue: different cities may have same name

capital---country

Abuja : Nigeria = Accra : Ghana
Abuja : Nigeria = Algiers : Algeria
Abuja : Nigeria = Amman : Jordan
Abuja : Nigeria = Ankara : Turkey
Abuja : Nigeria = Antananarivo : Madagascar
Abuja : Nigeria = Apia : Samoa
Abuja : Nigeria = Ashgabat : Turkmenistan
Abuja : Nigeria = Asmara : Eritrea
Abuja : Nigeria = Astana : Kazakhstan

Issue: can change with time

Intrinsic Evaluation – Word Analogies

Word linear relationship $w_A : w_B = w_C : w_x$

Syntactic and Semantic example questions [link]

superlative

bad : worst = big : biggest
bad : worst = bright : brightest
bad : worst = cold : coldest
bad : worst = cool : coolest
bad : worst = dark : darkest
bad : worst = easy : easiest
bad : worst = fast : fastest
bad : worst = good : best
bad : worst = great : greatest

past tense

dancing : danced = decreasing : decreased
dancing : danced = describing : described
dancing : danced = enhancing : enhanced
dancing : danced = falling : fell
dancing : danced = feeding : fed
dancing : danced = flying : flew
dancing : danced = generating : generated
dancing : danced = going : went
dancing : danced = hiding : hid
dancing : danced = hiding : hit

Intrinsic Evaluation – Word Correlation

Comparing word correlation with human-judged scores

Human-judged word correlation [link]

Word 1	Word 2	Human-Judged Score
tiger	cat	7.35
tiger	tiger	10.00
book	paper	7.46
computer	internet	7.58
plane	car	5.77
professor	doctor	6.62
stock	phone	1.62

Ambiguity: synonym or same word with different POSs

Extrinsic Evaluation – Subsequent Task

Goal: use word vectors in neural net models built for subsequent tasks

Benefit

- Ability to also classify words accurately
  - Ex. countries cluster together a classifying location words should be possible with word vectors
- Incorporate any information into them other tasks
  - Ex. project sentiment into words to find most positive/negative words in corpus

Softmax & Cross-Entropy

Revisit Word Embedding Training

Goal: estimating vector representations s.t.

$$p(w_j = w_{O,c} \mid w_I) = y_{j_c} = \frac{\exp(s_{j_c})}{\sum_{j'=1}^{V} \exp(s_{j'})}$$

Softmax classification on *x* to obtain the probability for class *y*

- Definition

$$p(y \mid x) = \frac{\exp(W_y x)}{\sum_{c=1}^{C} \exp(W_c x)}$$

Softmax Classification

Softmax classification on x to obtain the probability for class y

- Definition

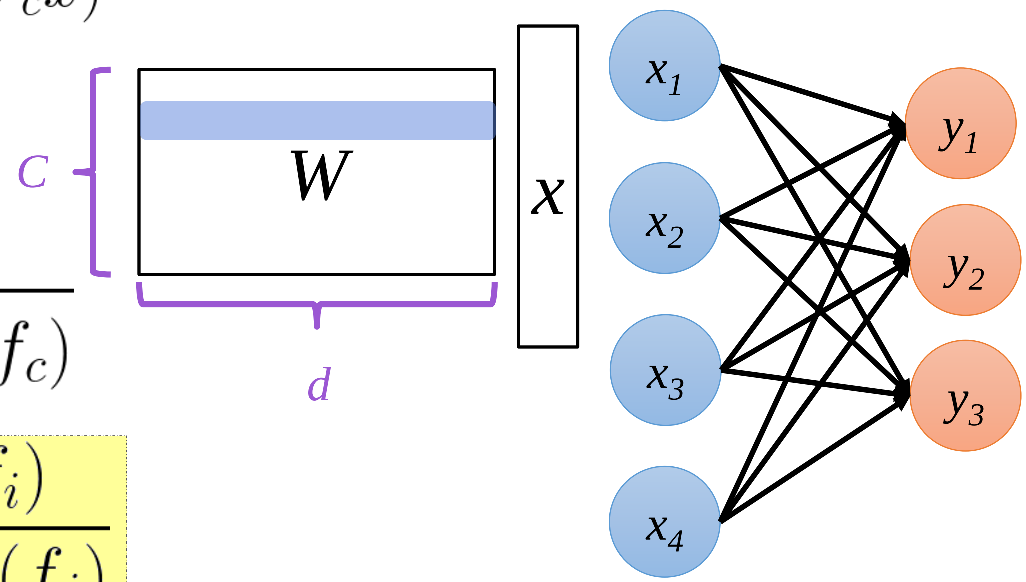
$$p(y \mid x) = \frac{\exp(W_y x)}{\sum_{c=1}^{C} \exp(W_c x)}$$

$W \in \mathbb{R}^{C \times d}$

usually C > 2
(multi-class classification)

$$W_y x = \sum_{i=1}^{d} W_{y_i} x_i = f_y$$

$$p(y \mid x) = \frac{\exp(f_y)}{\sum_{c=1}^{C} \exp(f_c)}$$



$$\boxed{\mathrm{softmax}(f)_i = \frac{\exp(f_i)}{\sum_j \exp(f_j)}}$$

Loss of Softmax

Objective function

$$O(\theta) = \mathrm{softmax}(f)_i = \frac{\exp(f_i)}{\sum_j \exp(f_j)}$$

Loss function

$$C(\theta) = -\log \mathrm{softmax}(f)_i = -f_i + \underline{\log \sum_j \exp(f_j)}$$

$$\approx \max_j f_j$$

- If the correct answer already has the largest input to the softmax, then the first term and the second term will roughly cancel
- the correct sample contributes little to the overall cost, which will be dominated by other examples not yet correctly classified

Softmax function always strongly penalizes the most active incorrect prediction

Cross Entropy Loss

Cross entropy of target and predicted probability distribution

- Definition

$$H(p,q) = -\sum_i p_i \log q_i$$

p: target one-hot vector
q: predicted probability distribution

- Re-written as the entropy and Kullback-Leibler divergence

$$H(p,q) = H(p) + D_{KL}(p \parallel q) \qquad D_{KL}(p \parallel q) = \sum_i p_i \log \frac{p_i}{q_i}$$

- KL divergence is not a distance but a non-symmetric measure of the difference between p and q

p: target *one-hot* vector

cross entropy loss

$$D_{KL}(p \parallel q) = \log \frac{1}{q_i} = -\log q_i$$

loss for softmax

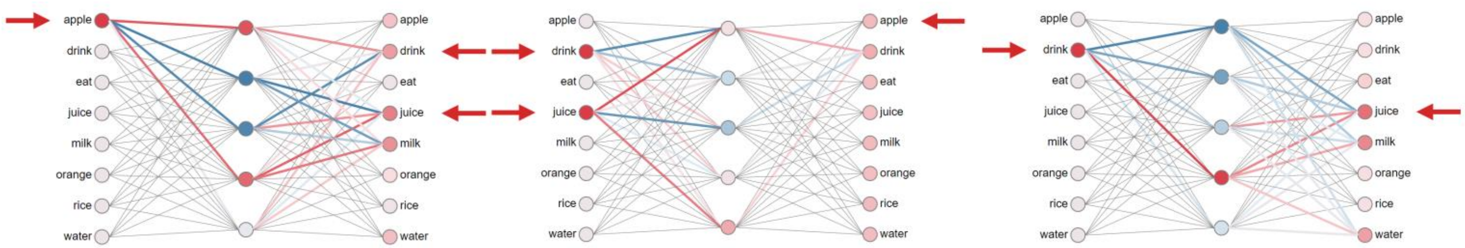
$$-\log \mathrm{softmax}(f)_i = -\log \frac{\exp(f_i)}{\sum_j \exp(f_j)} = -\log q_i$$

cross entropy loss = loss for softmax

Concluding Remarks

Low dimensional word vector

- word2vec



Skip-gram CBOW LM

- GloVe: combining count-based and direct learning

Word vector evaluation

- Intrinsic: word analogy, word correlation
- Extrinsic: subsequent task

Softmax loss = cross-entropy loss