Semantically-Aligned Equation Generation for Solving and Reasoning Math Word Problems

Ting-Rui Chiang and Yun-Nung (Vivian) Chen

https://github.com/MiuLab/E2EMathSolver

NTU MIULAB

Math Word Problem

*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*

Reasoning & Solving

$$x = 10 - 1 \times 5 \div 0.5$$

Prior Work

Non-neural approaches

- Template-based
  (Kushman et al., Upadhyay and Chang)

$$x = (? + ?) \times ? - ?$$

fill

$$x = (1 + 2) \times 3 - 4$$

Rely on hand-crafted features!

Deep learning

- Seq2Seq
  (Wang et al., Ling et al.)

$$\textit{Problem}$$

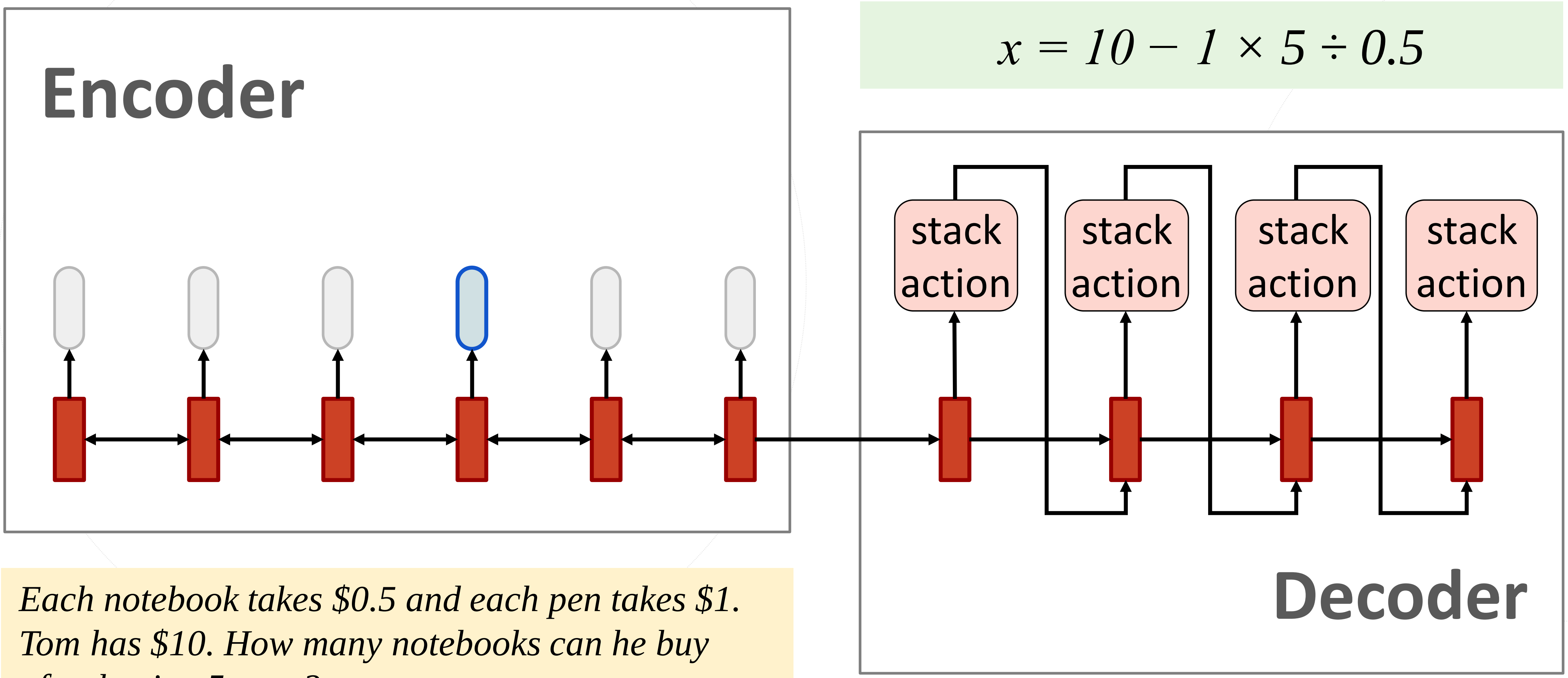
generate

$$x = (1 + 2) \times 3 - 4$$

Does not use the structure of math expression.

Our model is **end-to-end** and **structural**!

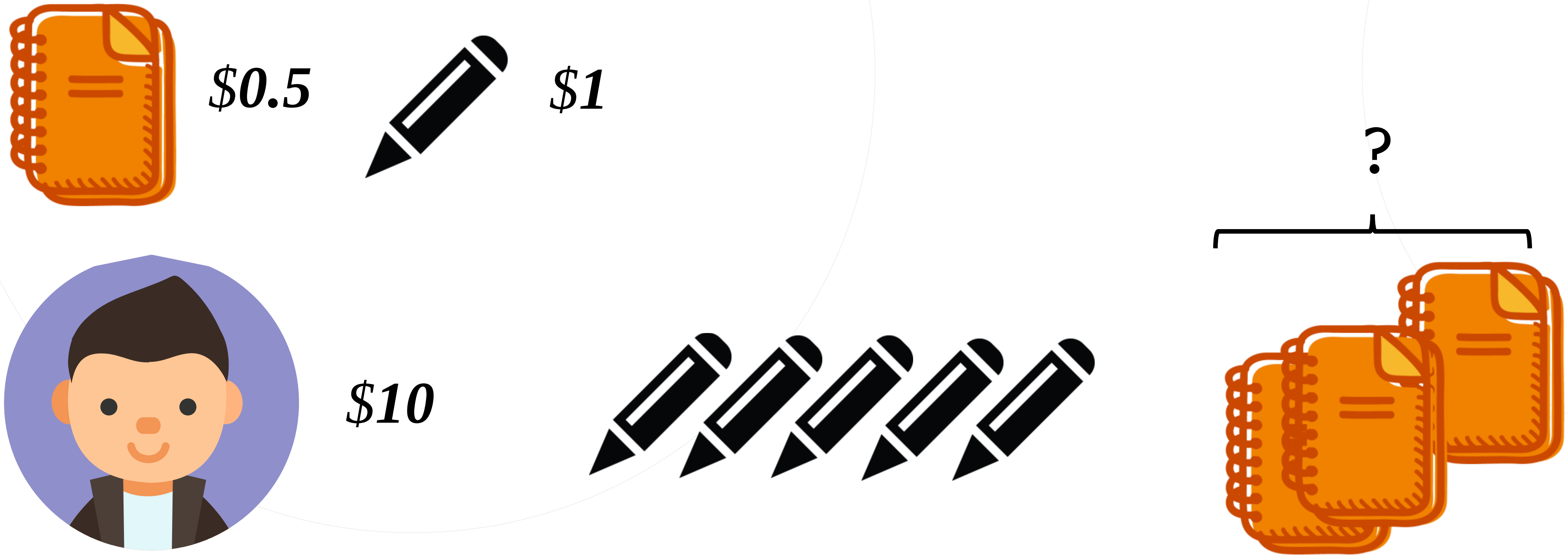
Overview of the Proposed Model



Each notebook takes \$0.5 and each pen takes \$1. Tom has \$10. How many notebooks can he buy after buying 5 pens?

Look Again at the Problem

*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*

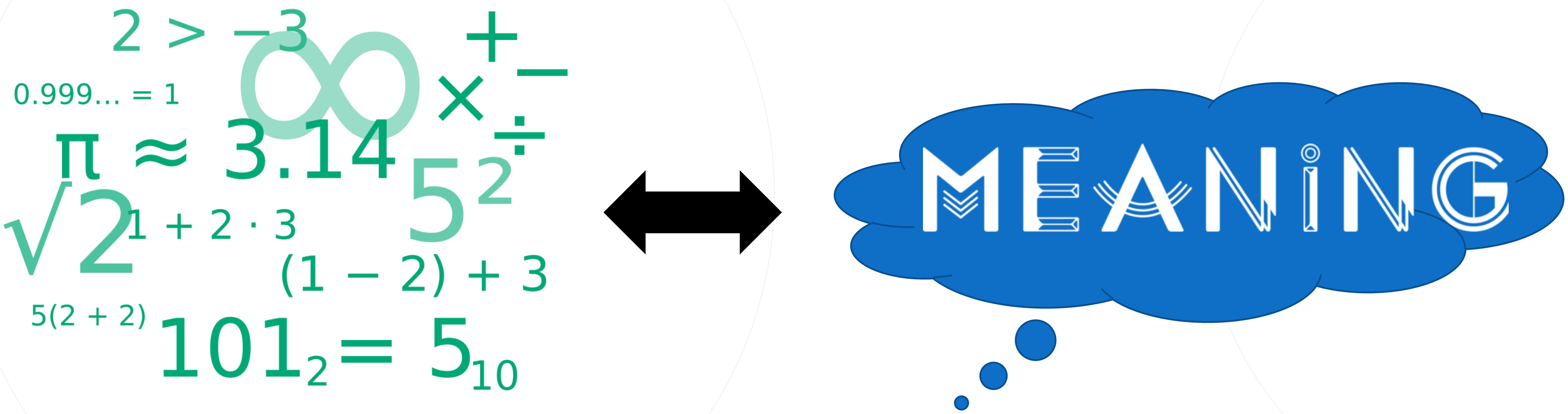


Semantic Meaning of the Operands

*Each notebook takes \$**0.5** and each pen takes \$**1**. Tom has \$**10**. How many notebooks can he buy after buying **5** pens?*

$$x = (\overbrace{\boxed{10}}^{\text{The amount of money Tom has}} - \underbrace{\boxed{1}}_{\text{Price of a pen}} \times \underbrace{\boxed{5}}_{\text{Number of pens bought}}) \div \overbrace{\boxed{0.5}}^{\text{Price of a notebook}}$$

Idea: Bridging Symbolic and Semantic Worlds



Symbolic World

Semantic World

NTU MIULAB

Preprocess

*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*

Preprocess

Symbolic Part

0.5

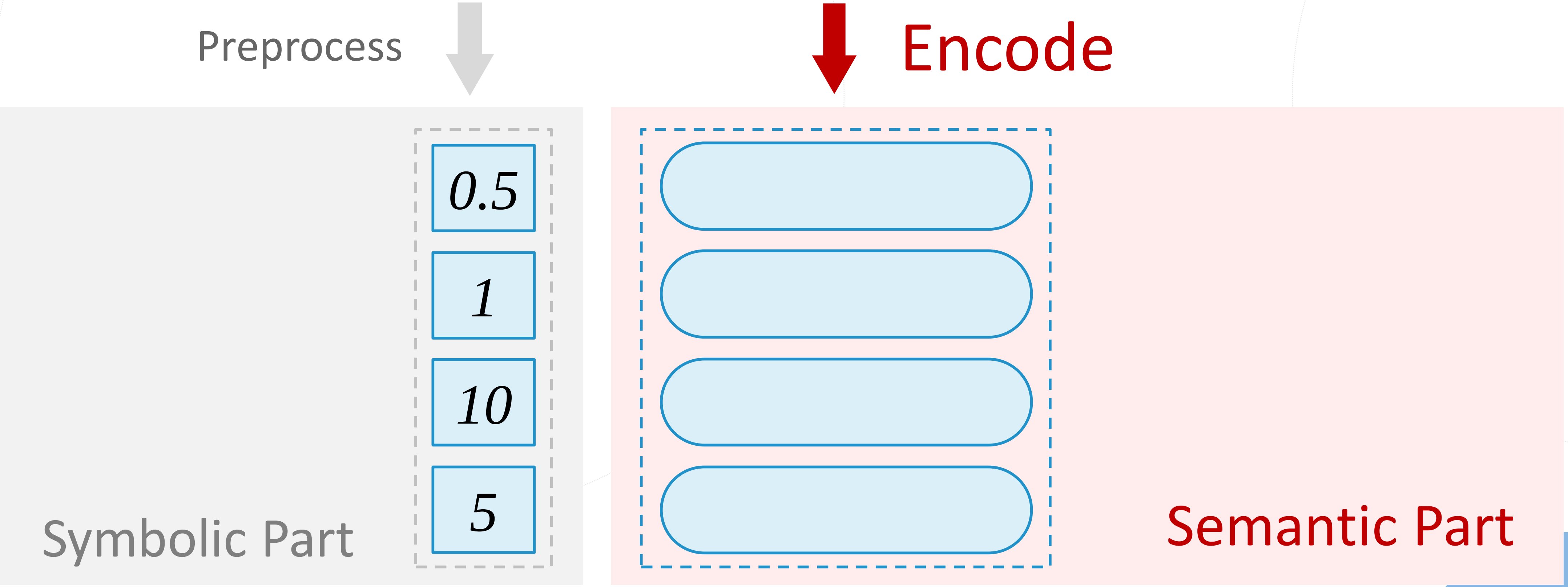
1

10

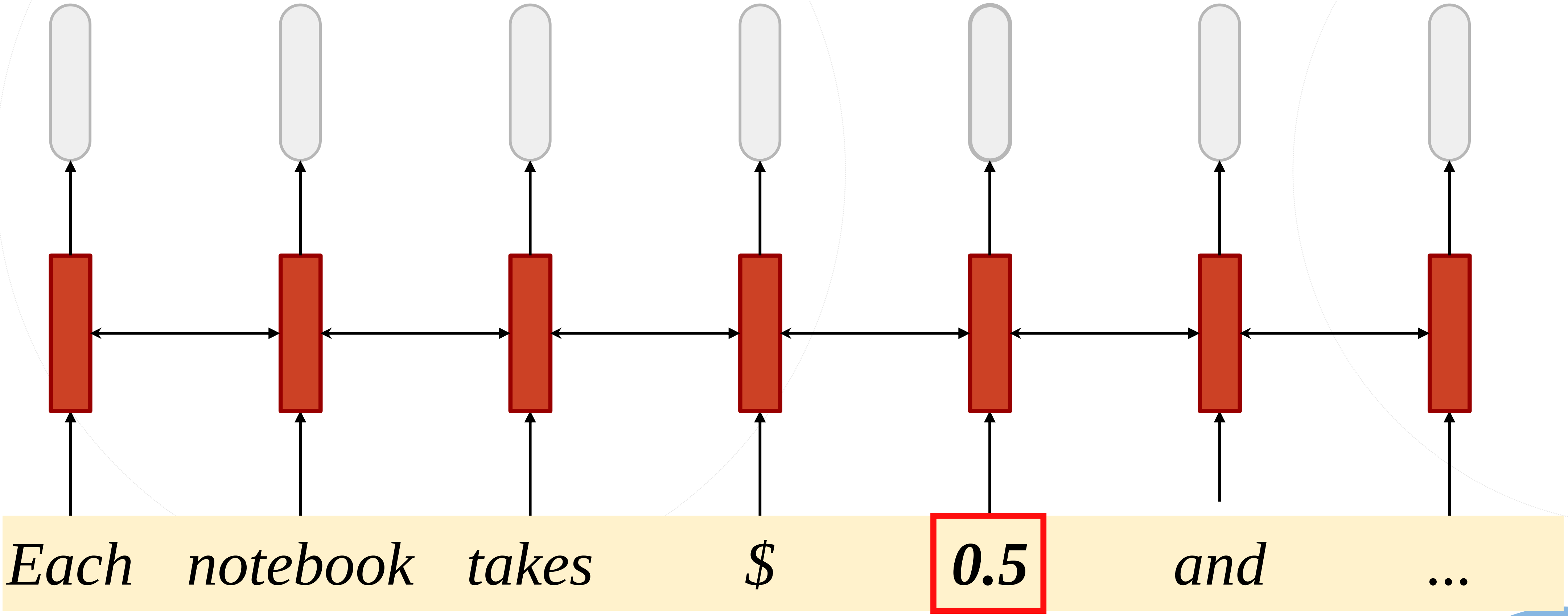
5

Symbol Encoding

*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*

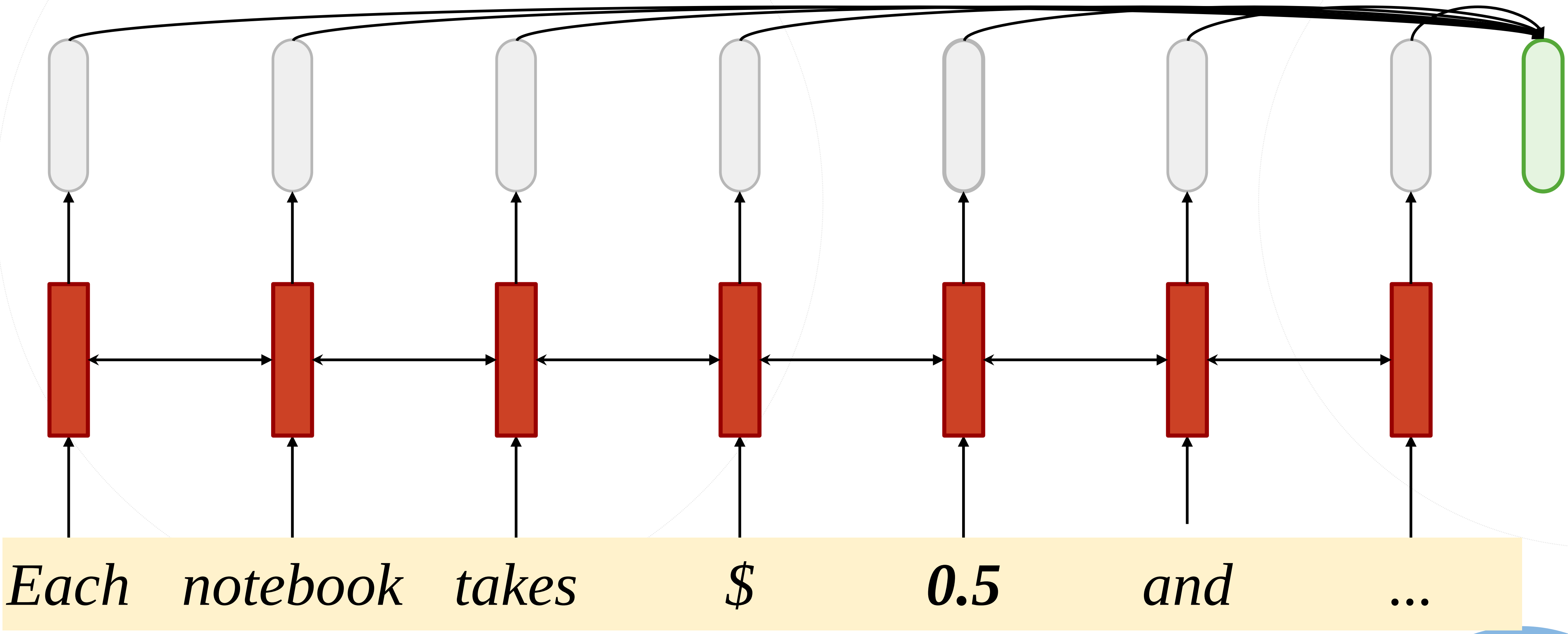


Inside Encoder



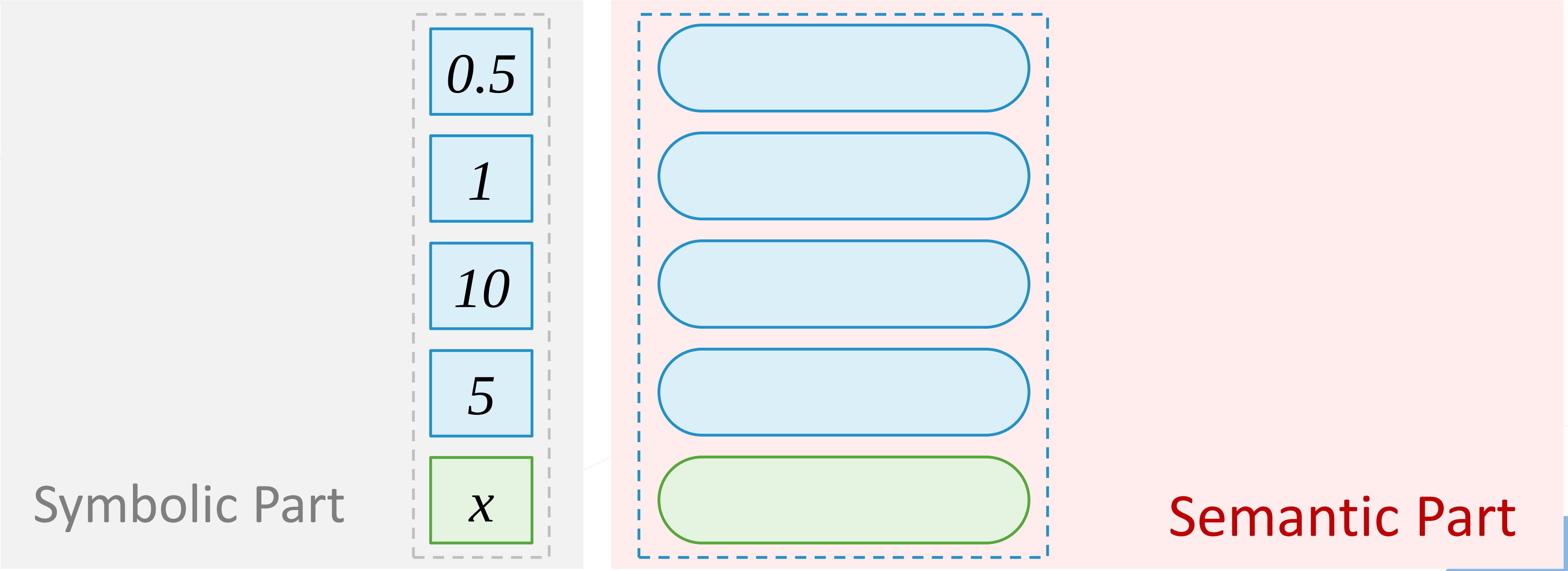
NTU MIULAB

Semantic Generation for Unknown *x*



Operands & Their Semantics

*Each notebook takes \$**0.5** and each pen takes \$**1**. Tom has \$**10**. How many notebooks can he buy after buying **5** pens?*



Intuition of Using Semantics

*Each notebook takes $**0.5** and each pen takes $**1**. Tom has $**10**. How many notebooks can he buy after buying **5** pens?*

Number of pens bought.

$$x = (10 - \boxed{1} \; {\color{red}?} \; \boxed{5})$$

Price of a pen.

Equation Generation in Postfix

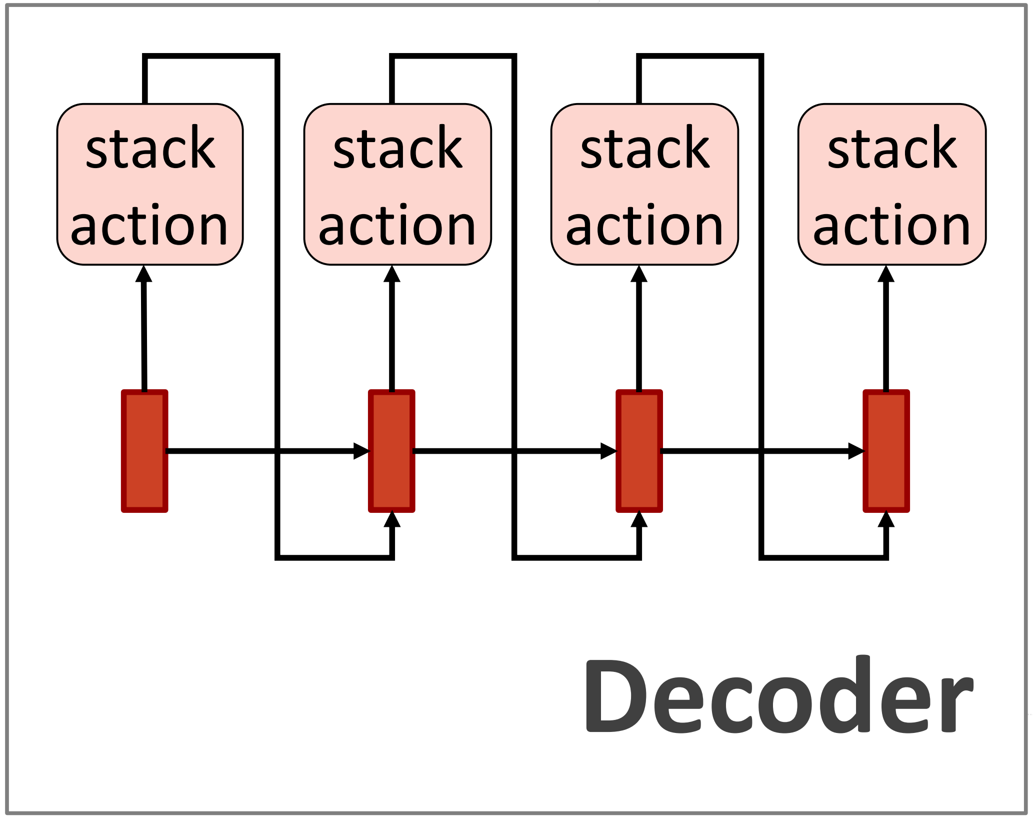
*Each notebook takes \$**0.5** and each pen takes \$**1**. Tom has \$**10**. How many notebooks can he buy after buying **5** pens?*

$$x \;\; 10 \;\; 1 \;\; 5 \;\; \times \;\; - \;\; 0.5 \;\; \div \;\; =$$

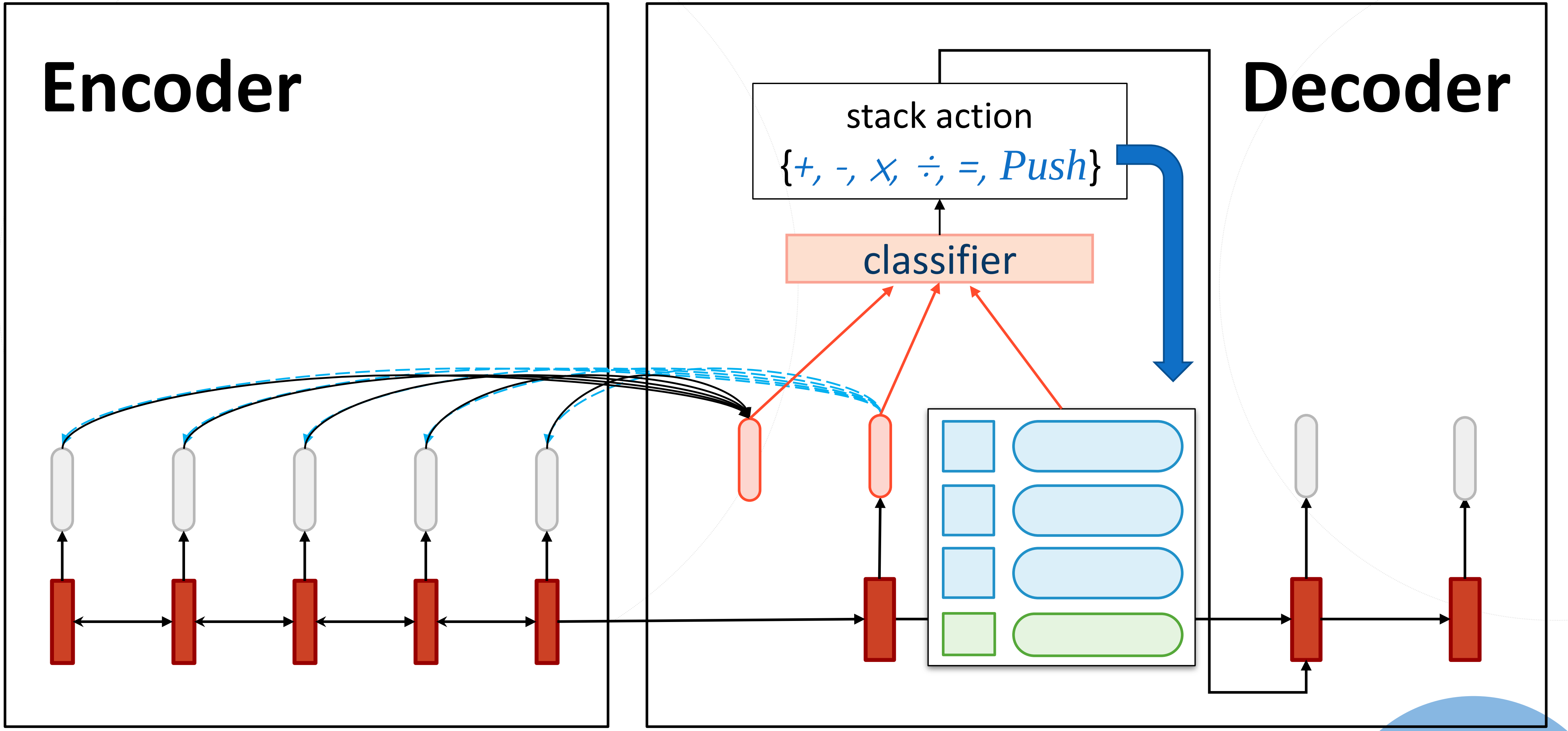
Equation Generation by Stack Actions

- Stack is used
- The decoder generates stack actions.
- An equation is generated with actions on stack.

$$x = 10 - 1 \times 5 \div 0.5$$



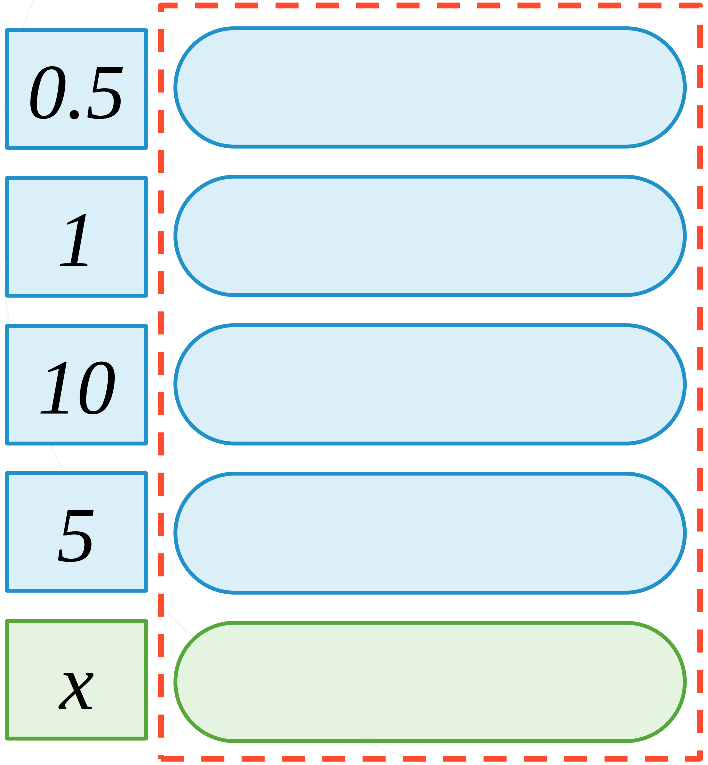
Action Selection in Each Step



Equation Generation by Stack Actions

Target Equation: $x = 10 - 1 \times 5 \div 0.5$

Generated Actions:

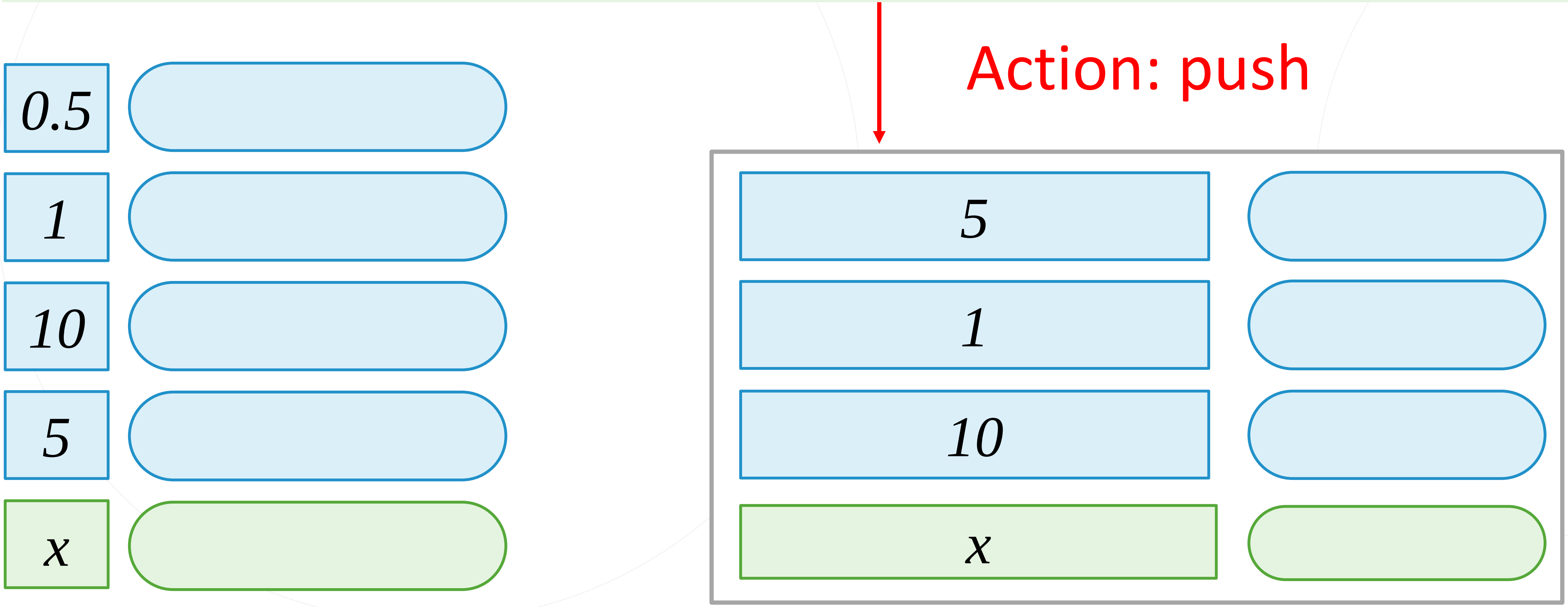


Action: push

Equation Generation by Stack Actions

Target Equation: $x = 10 - 1 \times 5 \div 0.5$

Generated Actions: *x 10 1* ***5***

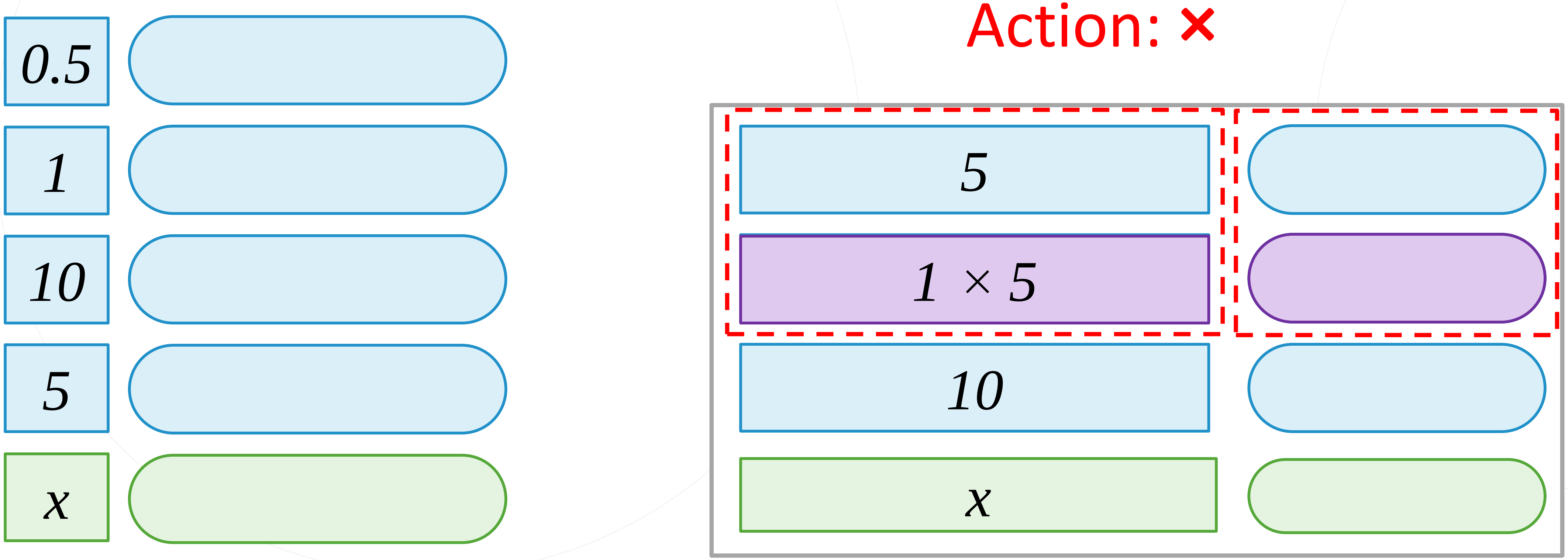


Equation Generation by Stack Actions

Target Equation: $x = 10 - 1 \times 5 \div 0.5$

Generated Actions: *x 10 1* ***5***

Action: ×



Equation Generation by Stack Actions

Target Equation: $x = 10 - 1 \times 5 \div 0.5$

Generated Actions: $x\ 10\ 1\ 5\ \times\ 0.5\ \div\ =$

After many steps…

0.5

1

10

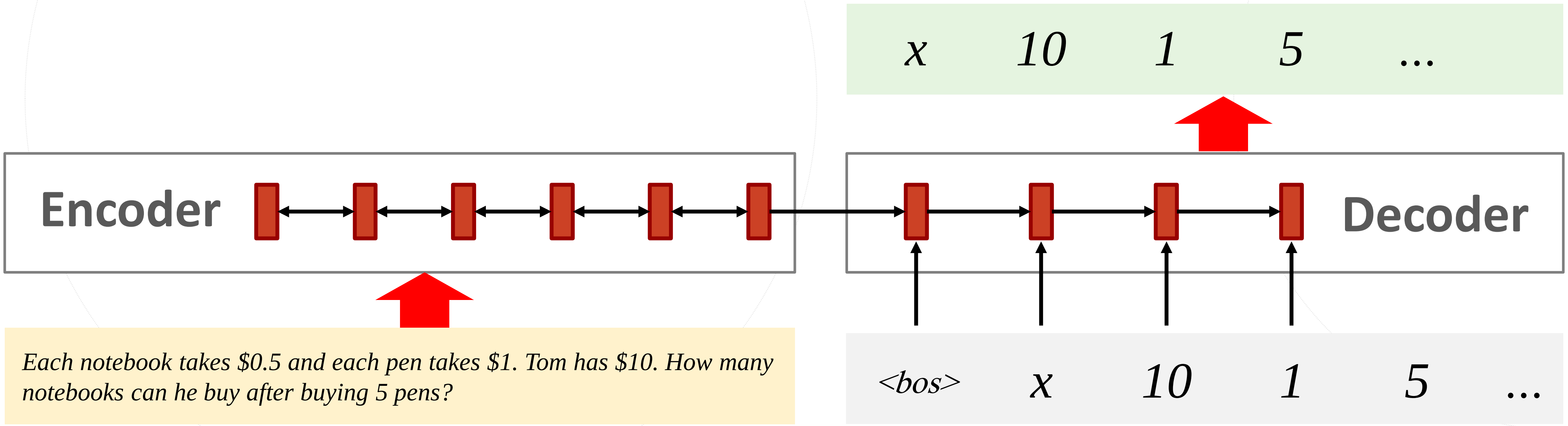
5

x

$x=(10-1\times 5)\div 0.5$

Training Process

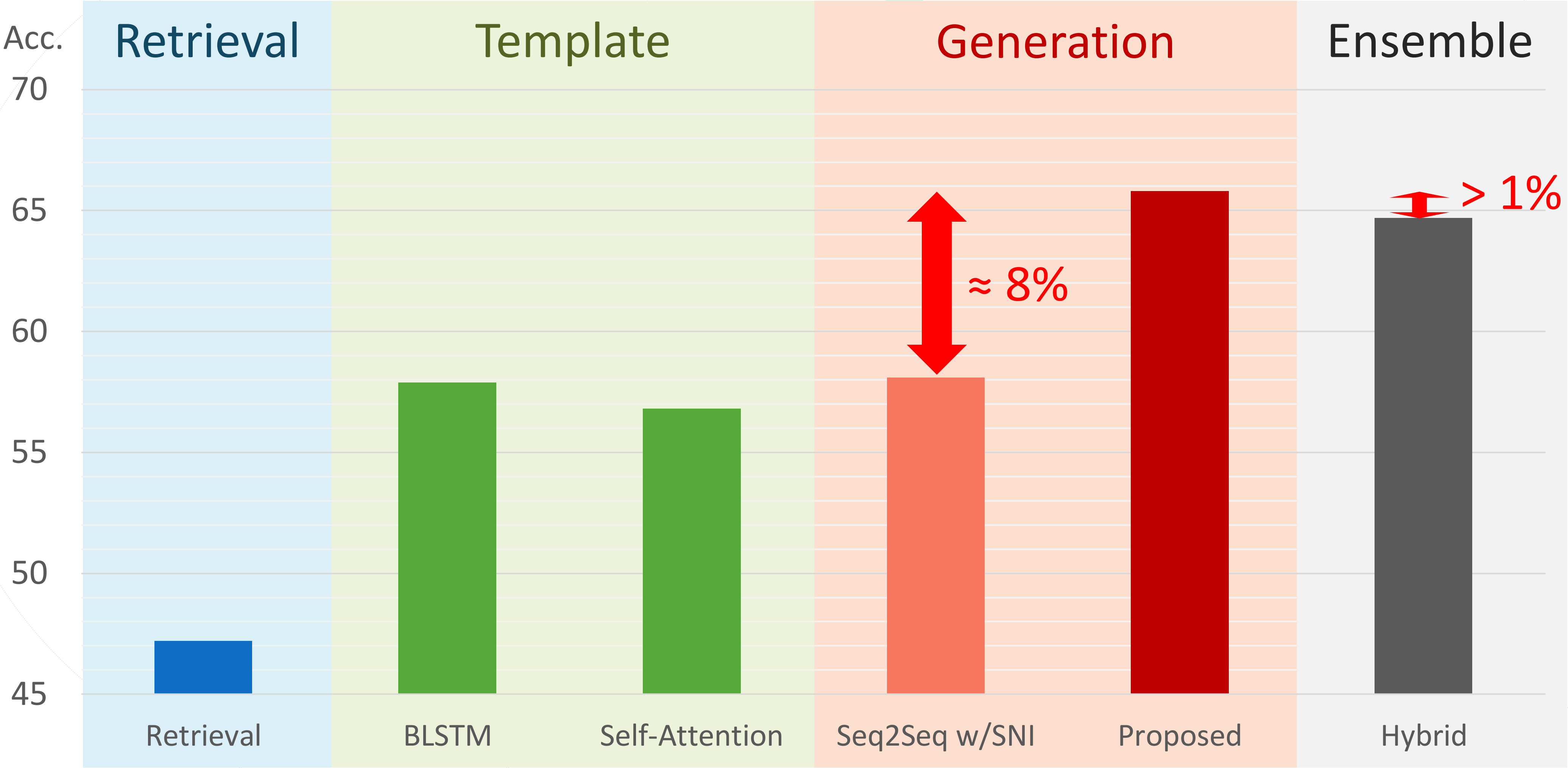
- Target equation is given.
- Trained as Seq2Seq.



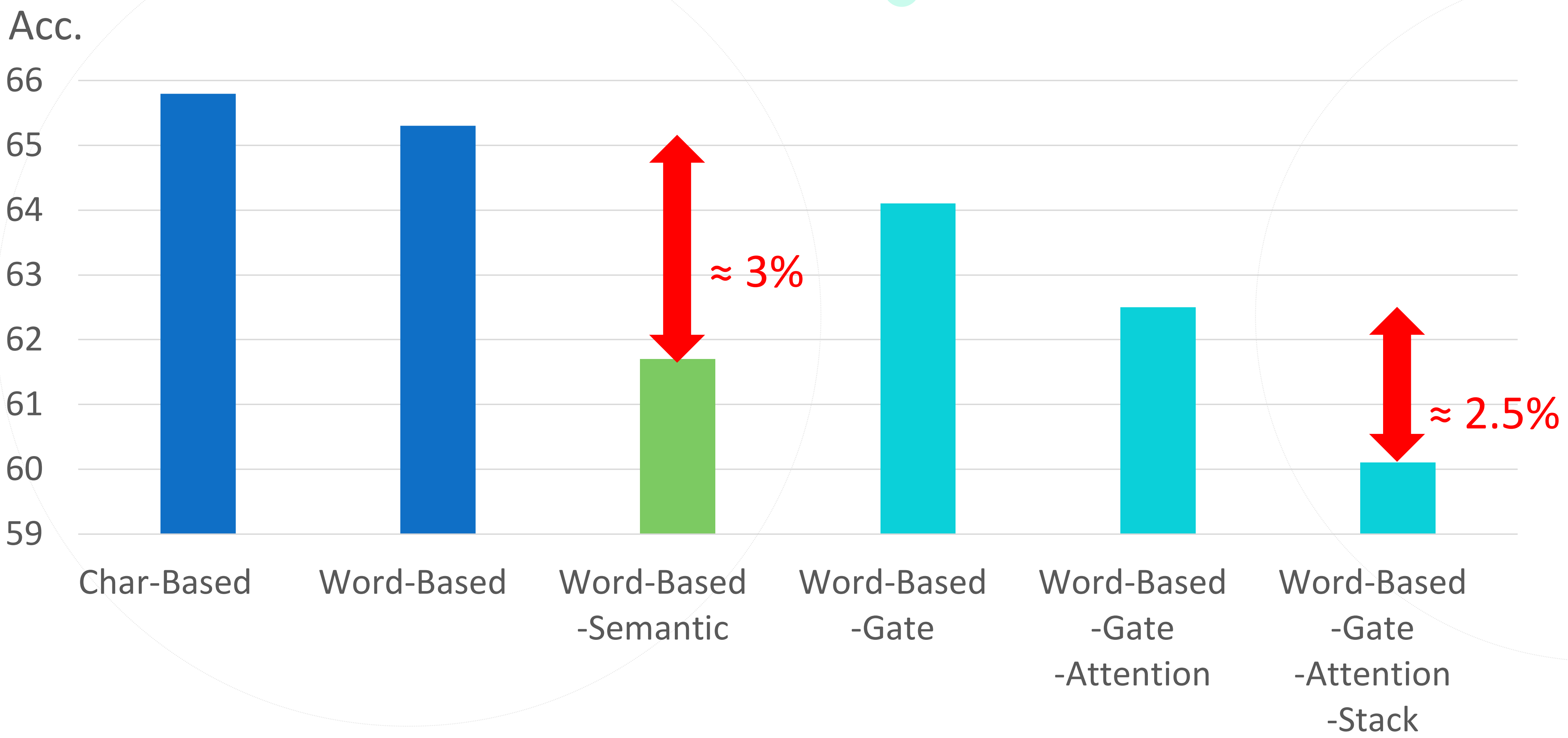
Experiments

- Dataset: Math23k
  - In Chinese
  - 23000 math word problems.
  - Operators: +, -, ×, ÷

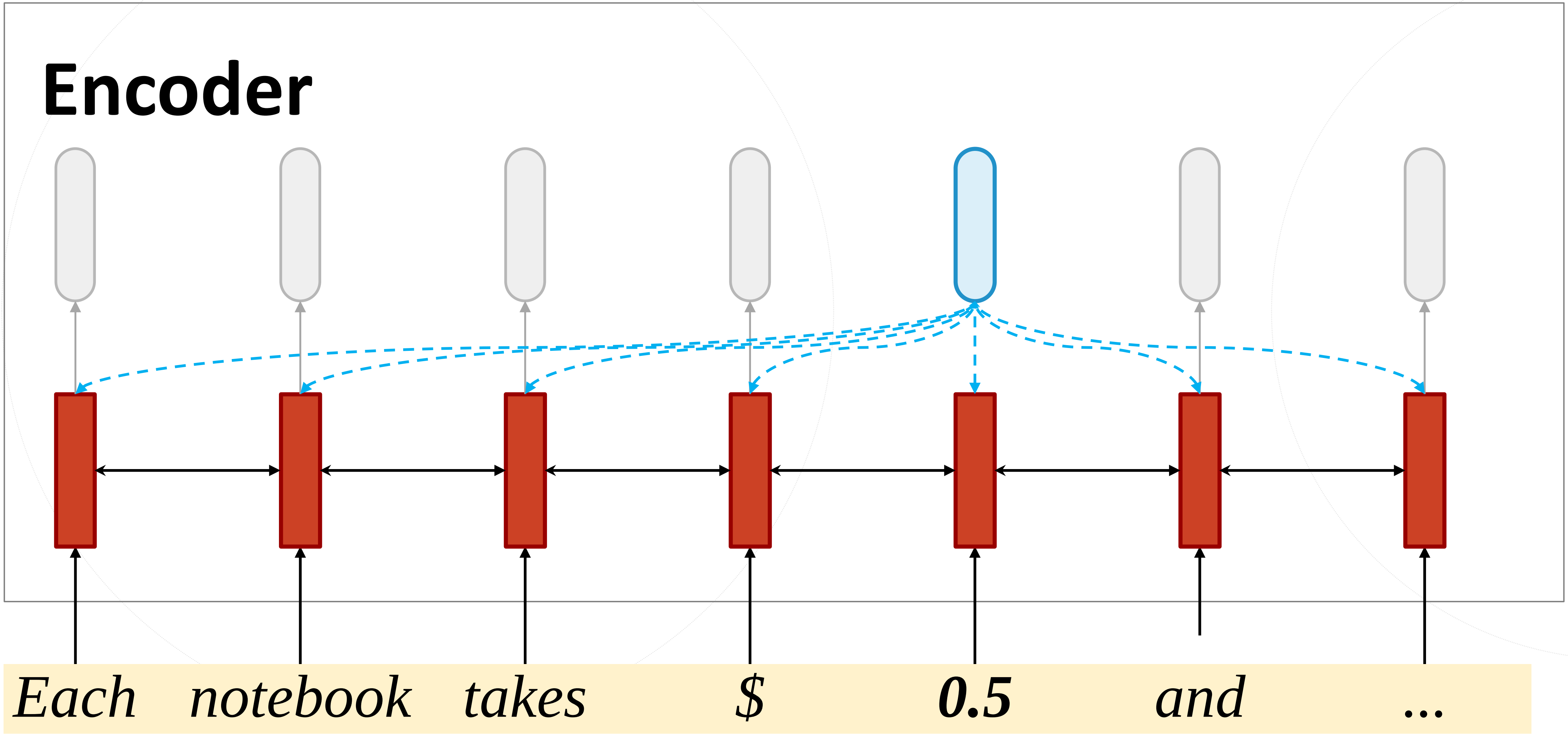
Results


Acc.
Retrieval
Template
Generation
Ensemble
70
65
60
55
50
45
≈ 8%
> 1%
Retrieval
BLSTM
Self-Attention
Seq2Seq w/SNI
Proposed
Hybrid

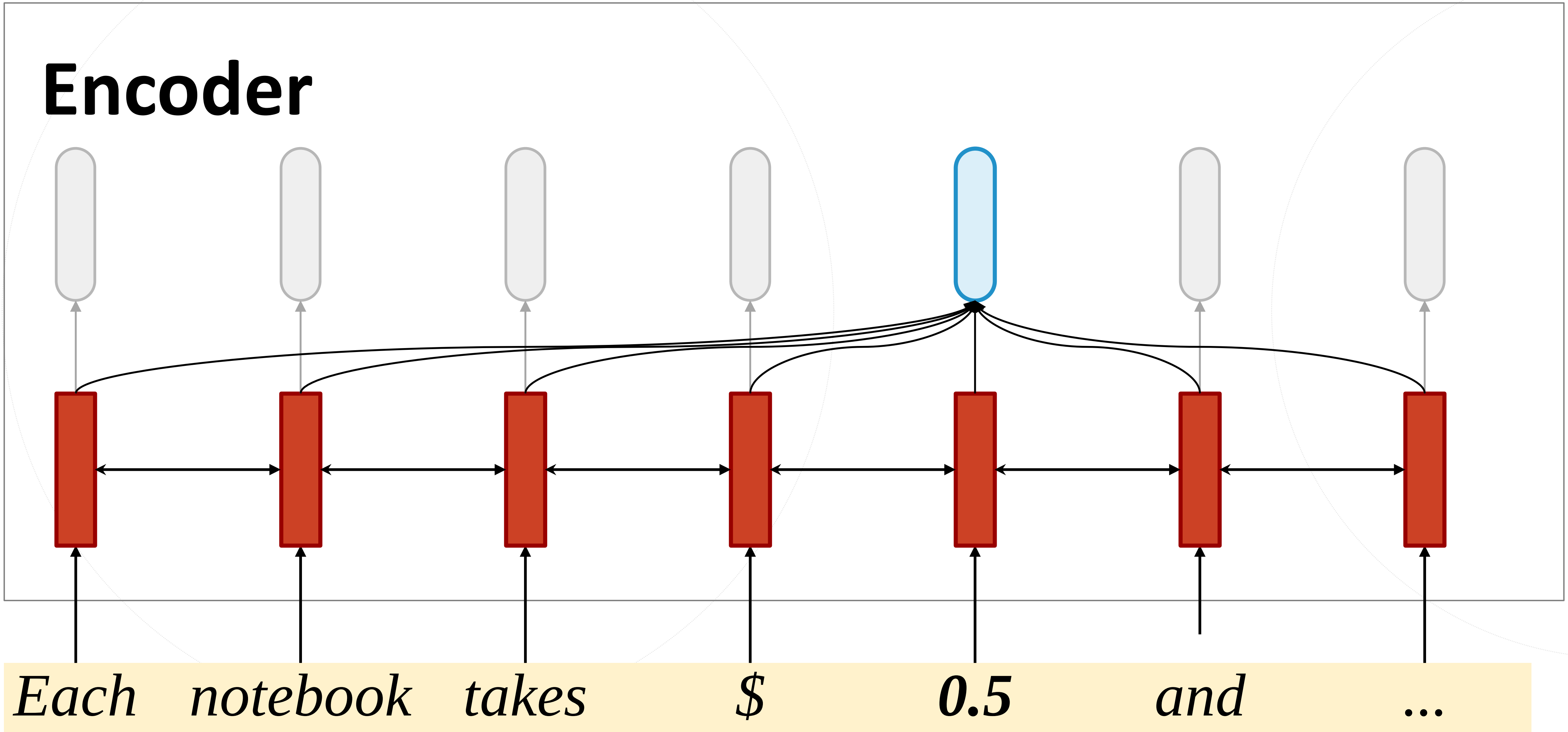
Ablation Test



Self-Attention for Qualitative Analysis



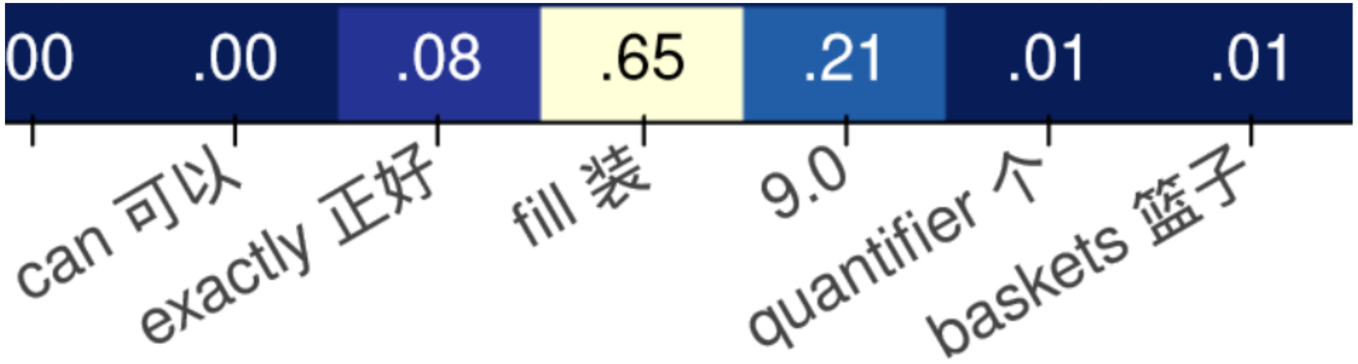
Self-Attention for Qualitative Analysis



Attention for Operand Semantics

The attention focuses on:

- Informative verbs
  - *"gain"*, *"get"*, *"fill"*, etc.
- Quantifier-related words
  - *"every"*, *"how many"*, etc.



Conclusion

Three main contributions

- **Approach:** equation generation with stack
- **Originality:** automatic extraction of operand semantics
- **Performance:** a SOTA end-to-end neural model on Math23k

Code Available @
https://github.com/MiuLab/E2EMathSolver