Lesson 10: Width of conjunctive queries

Theme: The notion of width of conjunctive queries.

1 Strong generalized hypertree decompositions (SGHD)

Let $\mathcal{H} = (V, \mathcal{E})$ be a hypergraph with $V = \bigcup \mathcal{E}$. A strong generalized hypertree decomposition (SGHD) $T = (I, F)$ of $\mathcal{H}$ is a tree, where

(H1) the set I of vertices in T is $I = \{(X_1, Y_1), \ldots, (X_m, Y_m)\}$, where each $X_i \subseteq V$ and $Y_i \subseteq \mathcal{E}$;

(H2) for every vertex $u \in V$, there is a node (X_i, Y_i) in T such that $u \in X_i$;

(H3) for every edge $e \in \mathcal{E}$, there is a node (X_i, Y_i) in T such that $e \subseteq X_i$ and $e \in Y_i$;

(H4) $X_i \subseteq \bigcup Y_i$, for each $(X_i, Y_i) \in I$;

(H5) for every vertex $u \in V$, the set of vertices $\{(X_i, Y_i) \mid X_i \text{ contains } u\}$ is connected in T.

The tree T is called an SGHD of $\mathcal{H}$. The width of T is $\max_i |Y_i|$. The *strong generalized hypertreewidth* of a hypergraph $\mathcal{H}$, denoted by $\mathrm{sghw}(\mathcal{H})$, is the minimum width among all possible SGHDs of $\mathcal{H}$.

2 Evaluations of conjunctive queries with bounded width

The sghw of a CQ φ, denoted by $\mathrm{sghw}(\varphi)$, is the sghw of the hypergraph associated with φ. The following problem can be solved in polynomial time for every fixed $k \geqslant 1$.

- CQ-EVALUATION$_k$.

  Input: A database DB and an SGHD of a CQ φ with width at most k.
  Task: Determine whether $\mathsf{DB} \models \varphi$.

- CQ-EVALUATION$_{k,\mathsf{DB}}$.

  Input: An SGHD of a Boolean CQ φ with width at most k.
  Task: Determine whether $\mathsf{DB} \models \varphi$.

The algorithm is a direct modification of Yannakakis' algorithm for acyclic queries.

Appendix

There are a few notions of hypertree decompositions and their associated widths of hypergraphs, such as tree decompositions, generalized hypertree decompositions and hypertree decompositions. All of them are quite similar, but subtly different. For more details, see, for example, a survey paper [2]. There is another notion of width of CQs that are more related to treewidth of standard graphs in [1].

References

[1] C. Chekuri and A. Rajaraman. Conjunctive query containment revisited. *Theoretical Computer Science*, 239(2):211–229, 2000.

[2] G. Gottlob, M. Grohe, N. Musliu, M. Samer, and F. Scarcello. Hypertree decompositions: Structure, algorithms, and applications. In *Proceedings of 31st International Workshop on Graph-Theoretic Concepts in Computer Science (WG)*, pages 1–15, 2005.