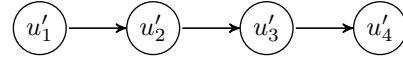
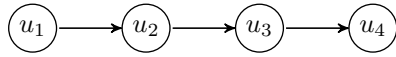


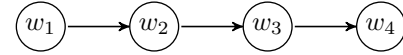
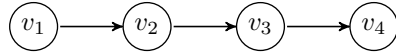
Tutorial 3: Ehrenfeucht-Fraïssé games

In this tutorial the vocabulary consists of only one relation symbol E of arity 2. In other words, we will consider only graphs as structures.

- (1) Consider the following graph G :

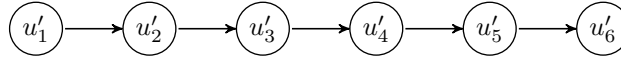
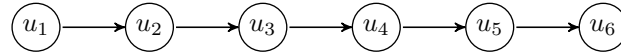


and the following graph H :

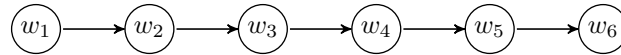
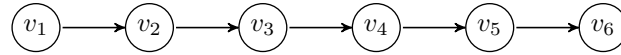


Describe a winning strategy for Duplicator in 1-round EF game on (G, u_1, u_4) and (H, v_1, w_4) .

- (2) Is there a winning strategy for Duplicator for 2-round EF game on (G, u_1, u_4) and (H, v_1, w_4) ?
 (3) Consider the following graph G' :



and the following graph H' :



Describe a winning strategy for Duplicator in 2-round EF game on (G', u_1, u_6) and (H', v_1, w_6) .

- (4) For every $n \geq 1$, construct two graphs (G, x_1, x_2) and (H, y_1, y_2) , each with two designated vertices such that:
- Duplicator has a winning strategy in n -round EF game on (G, x_1, x_2) and (H, y_1, y_2) ,
 - there is a path from x_1 to x_2 in G ,
 - there is *no* path from y_1 to y_2 in H .

What can you conclude about reachability query, i.e., a query that on input graph I , output all pairs (u, v) such that there is a path from u to v in I ?

- (5) Show that there is no FO sentence that describe “*there is an even length cycle in the input graph.*”