Lesson 5. Turing machines

CSIE 3110 – Formal Languages and Automata Theory

Tony Tan
Department of Computer Science and Information Engineering
College of Electrical Engineering and Computer Science
National Taiwan University

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Turing machines and algorithms

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For such problem, we may write a piece of program in some well-known programming languages such as C++ or psuedo-codes in some acceptable format.

We can rightly call such codes/pseudo-codes "algorithms."

Turing machines and algorithms – continued

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Turing machines and algorithms – continued

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- The definition should not depend on any programming languages.
  Such languages are ever changing and depend on their "design."
- It must not be ambiguous.
  Pseudo-codes are often "ambiguous" on what constitutes "basic instructions." For example, "sorting" may/may not be regarded as a basic instruction.

Church-Turing thesis

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Every "algorithm" is equivalent to a Turing machine.

(Note) Turing machine is not the only mathematical definition of algorithm. There are other models such as λ-calculus, recursive functions, etc, but they are all equivalent.

The formal definition of Turing machines

We reserve a special symbol $\sqcup$ called *the blank symbol* and $\triangleleft$ called *the left-end symbol*.

(Def.) A *Turing machine* (TM) is a system $\mathcal{M} = \langle \Sigma, \Gamma, Q, q_0, q_{\mathrm{acc}}, q_{\mathrm{rej}}, \delta \rangle$:

- Σ is a finite alphabet, called *the input alphabet*, where $\sqcup, \triangleleft \notin \Sigma$.
- Γ is a finite alphabet, called *the tape alphabet*, where $\Sigma \subseteq \Gamma$ and $\sqcup, \triangleleft \in \Gamma$.
- Q is a finite set of states and $q_0 \in Q$ is the initial state.
- $q_{\mathrm{acc}}, q_{\mathrm{rej}} \in Q$ are two special states called the *accept* and *reject* states, respectively.
- $\delta : Q - \{q_{\mathrm{acc}}, q_{\mathrm{rej}}\} \times \Gamma \to Q \times \Gamma \times \{\mathtt{Left}, \mathtt{Right}\}$ is the transition function, whose elements are written in the form:

$$(p, a) \;\to\; (q, b, \alpha)$$

where $p, q \in Q$, $a, b \in \Gamma$ and $\alpha \in \{\mathtt{Left}, \mathtt{Right}\}$.

What does a Turing machine $\mathcal{M} = \langle \Sigma, \Gamma, Q, q_0, q_{\mathbf{acc}}, q_{\mathbf{rej}}, \delta \rangle$ have?

Assume $\Sigma = \{0, 1\}$ and $\Gamma = \{\triangleleft, 0, 1, x, \#, \sqcup\}$.

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- The input word is a word over Σ.
- It has a finite number of states.
- It has one tape, which is divided into infinitely many cells.
  Each cell contains exactly one symbol from Γ.

$\triangleleft$	0	x	1	$\#$	1	$\#$	0	x	x	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\cdots$

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$\triangleleft$	0	x	1	$\#$	1	$\#$	0	x	x	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\cdots$

- It has one head reading one cell at a time.

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- The tape initially contains the input word w:
  For example, if the input word $w = 10010$:

$\triangleleft$	1	0	0	1	0	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\cdots$

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- It moves from state to state according to the transitions.

  The transitions also specify where the head moves to and what symbol it writes onto the tape.

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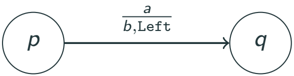
- The head initially reads the first symbol of the input word.
- It moves from state to state according to the transitions.

  The transitions also specify where the head moves to and what symbol it writes onto the tape.
- It stops when it is in q_{acc} or q_{rej}.

Illustration of a transition $(p, a) \to (q, b, \texttt{Left})$



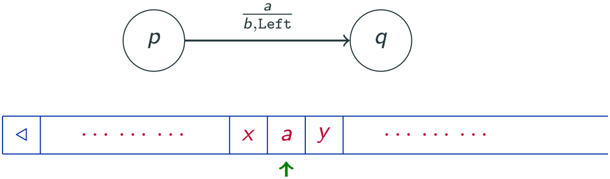
(The intuitive meaning) If:

- the TM is in state p,
- the head is reading symbol a,

then:

- it writes symbol b (on top of a),
- the head moves left,
- the TM enters state q,

Illustration of a transition $(p, a) \rightarrow (q, b, \texttt{Left})$



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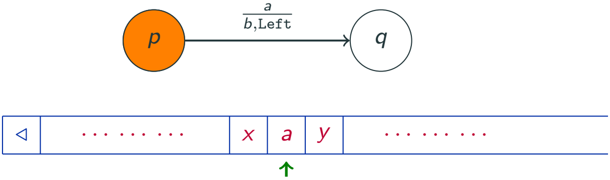

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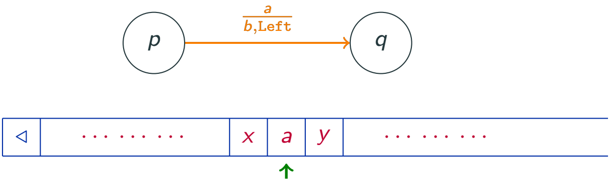

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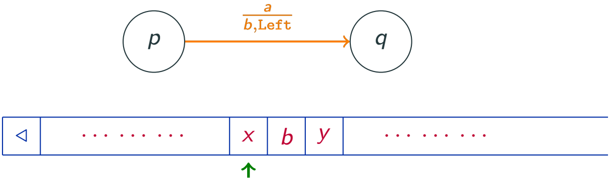

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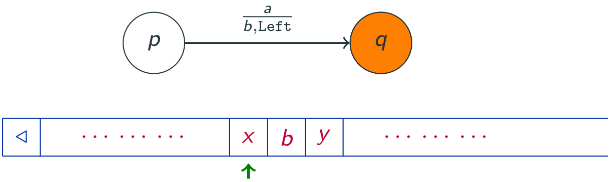

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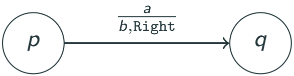
Illustration of a transition $(p, a) \to (q, b, \texttt{Right})$



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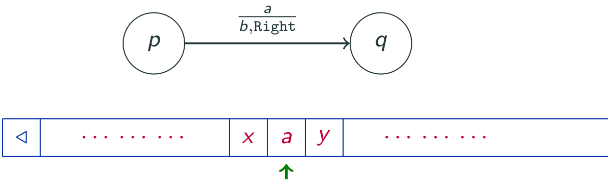

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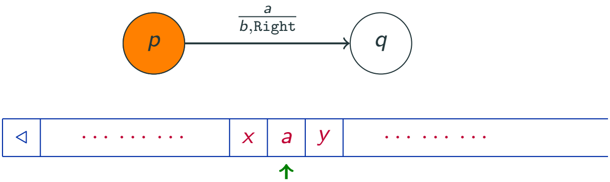

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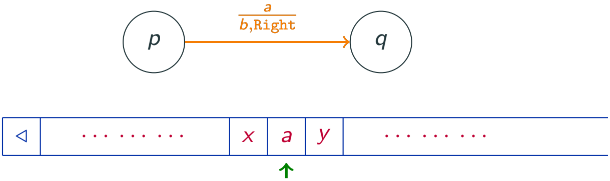

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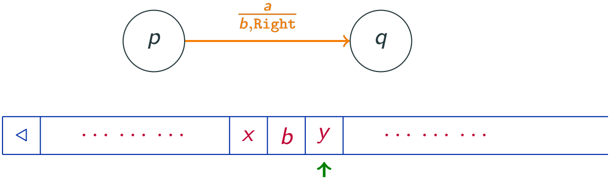

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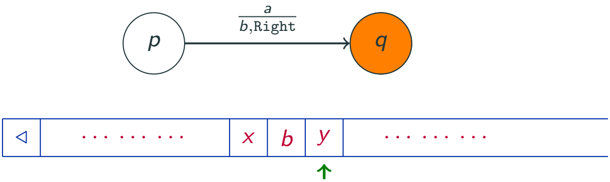

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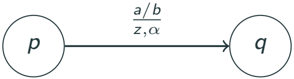
Some conventions in depicting transitions – part 1



This means if:

- the TM is in state p,
- the head reads a or b,

then:

- the head writes z,
- moves according to $\alpha \in \{\texttt{Left}, \texttt{Right}\}$,
- the TM enters state q.

Some conventions in depicting transitions – part 2

$$\begin{tikzcd} p \arrow[r, "\frac{a_1/a_2/\cdots/a_k}{z,\alpha}"] & q \end{tikzcd}$$

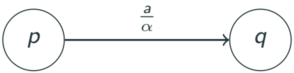
This means if:

- the TM is in state p,
- the head reads a_1 or a_2 or $\ldots$ or a_k,

then:

- the head writes z,
- moves according to $\alpha \in \{\mathtt{Left}, \mathtt{Right}\}$,
- the TM enters state q.

Some conventions in depicting transitions – part 3



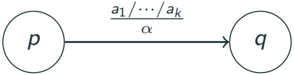
We don't specify the symbol it writes. This means if:

- the TM is in state p,
- the head reads a,

then:

- the head writes a (i.e., the same symbol it reads),
- moves according to $\alpha \in \{\texttt{Left}, \texttt{Right}\}$,
- the TM enters state q.

Some conventions in depicting transitions – part 4



This means if:

- the TM is in state p,
- the head reads a_1 or a_2 or ... or a_k,

then:

- the head writes the same symbol it reads,
- moves according to $\alpha \in \{\texttt{Left}, \texttt{Right}\}$,
- the TM enters state q.

Some conventions in depicting transitions – part 5

q_{rej} is not depicted explicitly in a Turing machine.

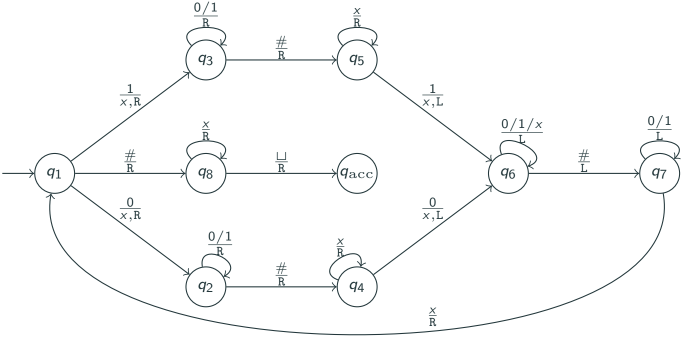
Any transition not depicted is assumed to enter q_{rej}.

We assume that the head always moves right when it reads $\triangleleft$ (the left-end marker).

The moves `Left` and `Right` are often abbreviated as `L` and `R`.

An example of a Turing machine where $\Gamma = \{\triangleleft, 0, 1, x, \#, \sqcup\}$

q_1 is the initial state.



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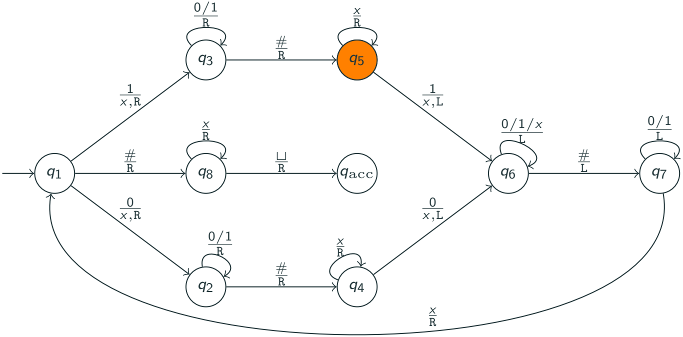
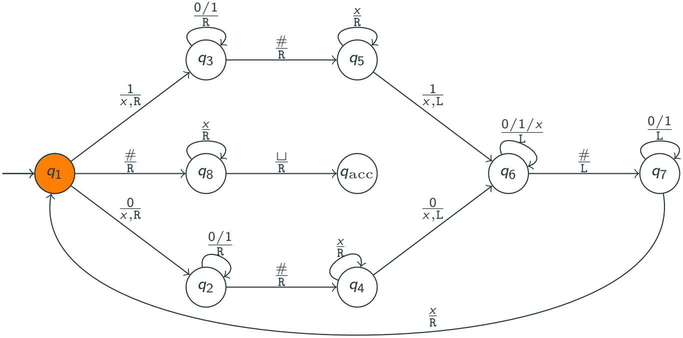


Note, for example, we do not depict the transition when it is in state q_5 and the head reads symbol 0.

This means the TM enters q_{rej}.

An example of a Turing machine on input 01101#01101



$\triangleleft$	0	1	1	0	1	#	0	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$
	↑													

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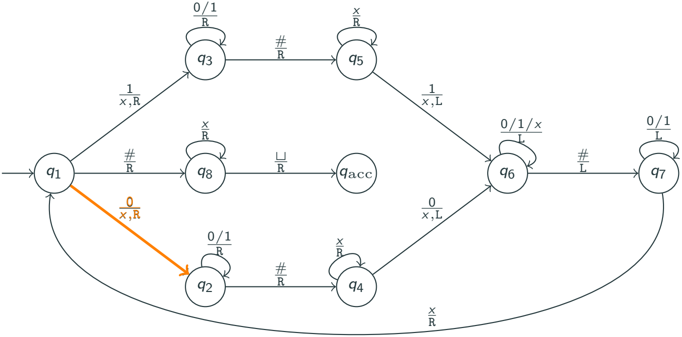

◁	0	1	1	0	1	#	0	1	1	0	1	⊔	⊔	···
	↑													

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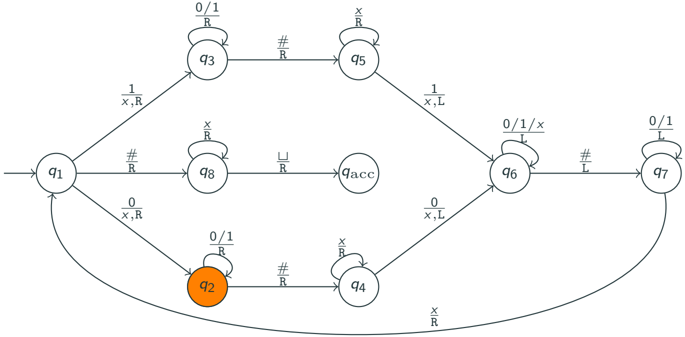

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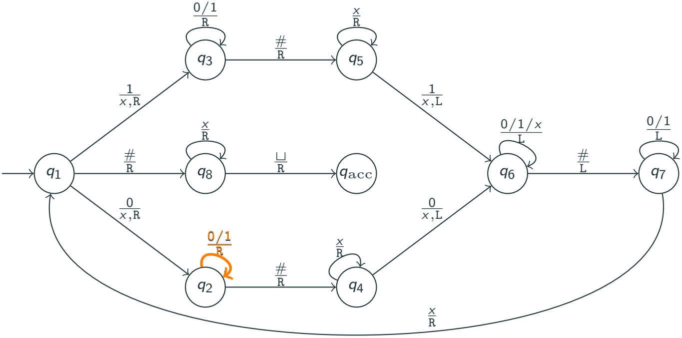

$\triangleleft$	x	1	1	0	1	#	0	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$
		$\uparrow$												

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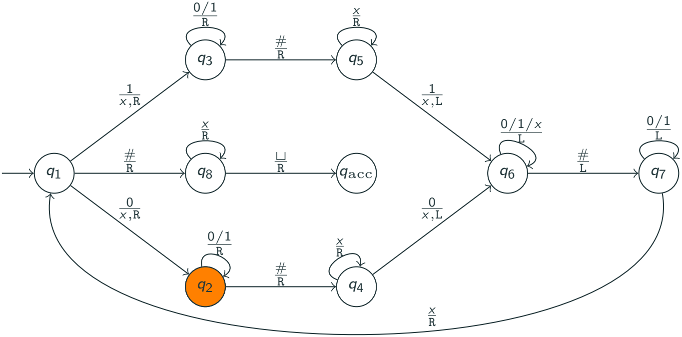

$\triangleleft$	x	1	1	0	1	#	0	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$
			↑											

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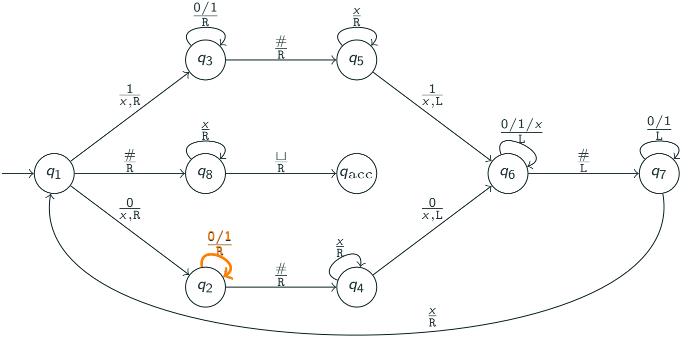
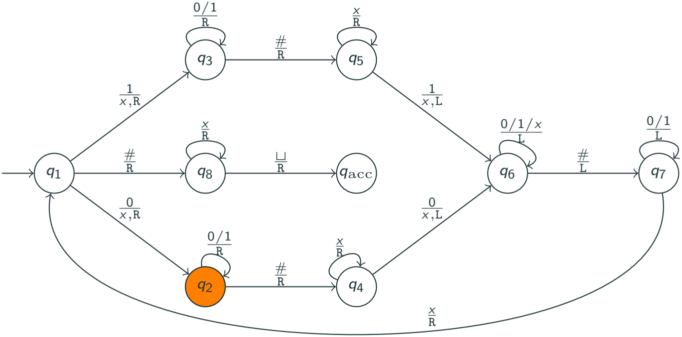


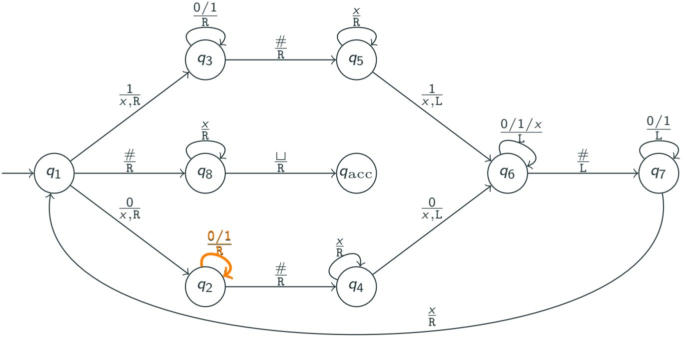
$\triangleleft$	x	1	1	0	1	#	0	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input $01101\#01101$



$\triangleleft$	x	1	1	0	1	$\#$	0	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



◁	x	1	1	0	1	#	0	1	1	0	1	⊔	⊔	⋯

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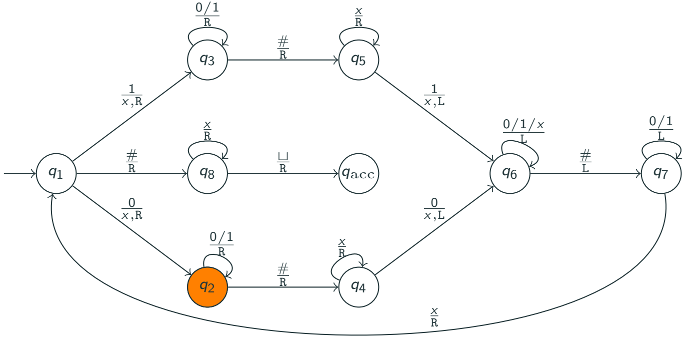

$\triangleleft$	x	1	1	0	1	#	0	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$
					$\uparrow$									

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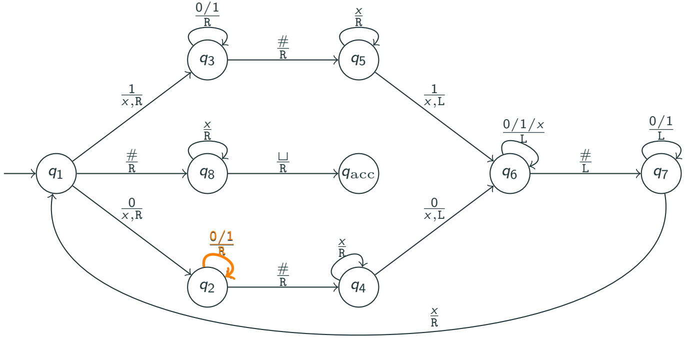

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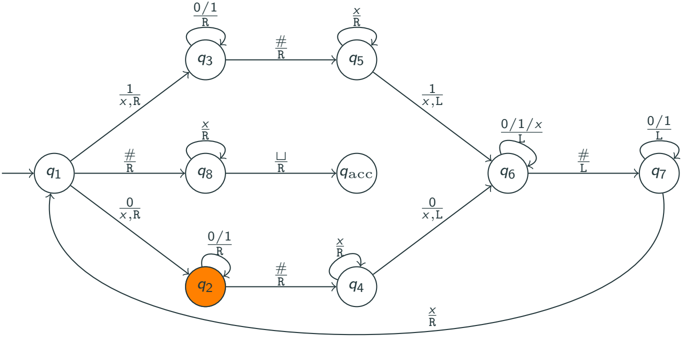

$\triangleleft$	x	1	1	0	1	#	0	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

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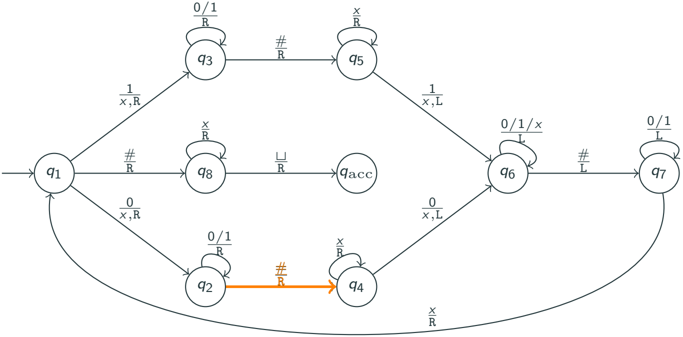

$\triangleleft$	x	1	1	0	1	#	0	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

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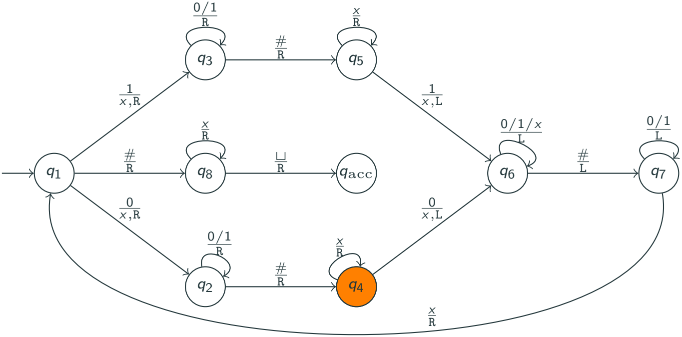

$\triangleleft$	x	1	1	0	1	$\#$	0	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

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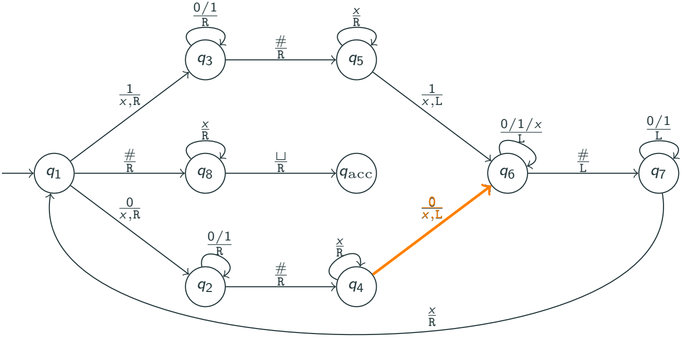
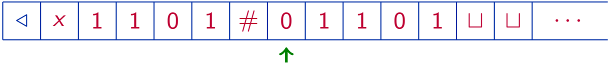
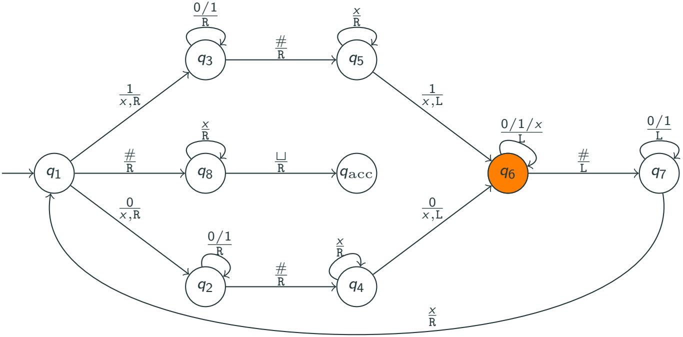
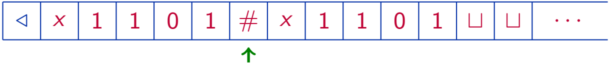


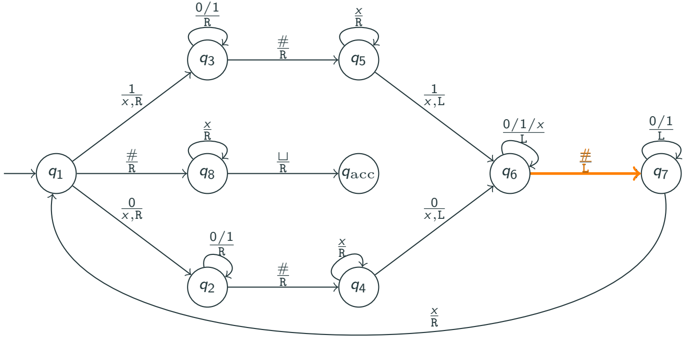


An example of a Turing machine on input 01101#01101



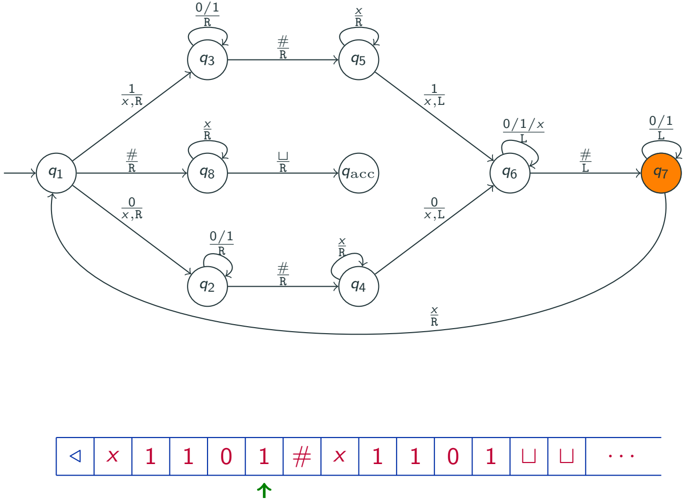


An example of a Turing machine on input 01101#01101

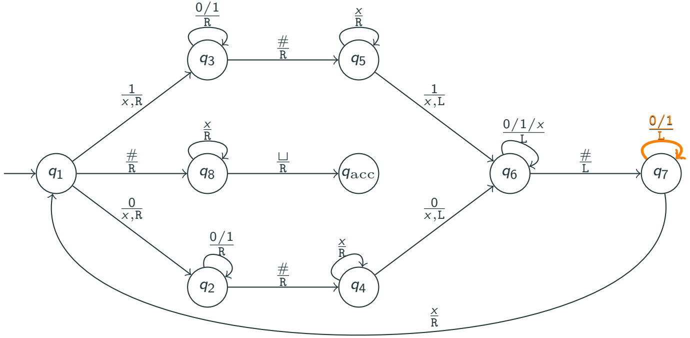


$\triangleleft$	x	1	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

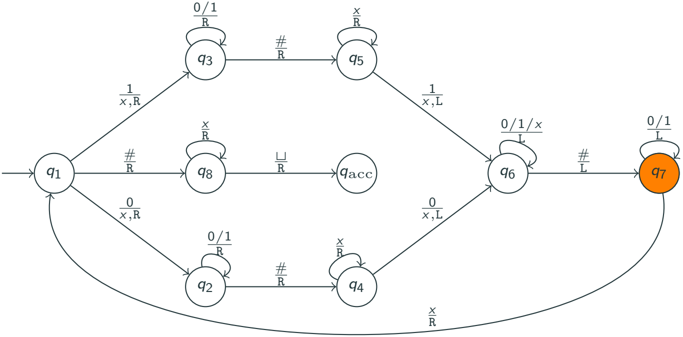


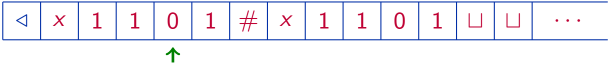
An example of a Turing machine on input 01101#01101



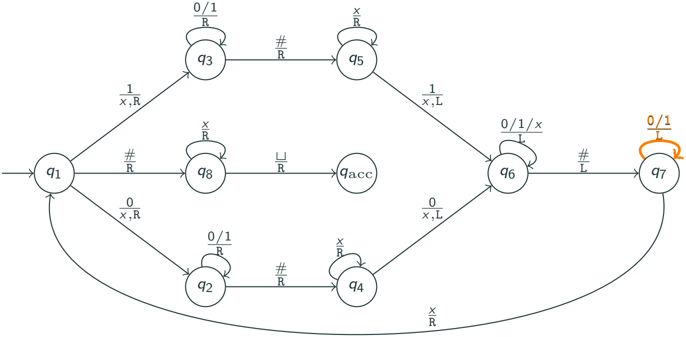
$\triangleleft$	x	1	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



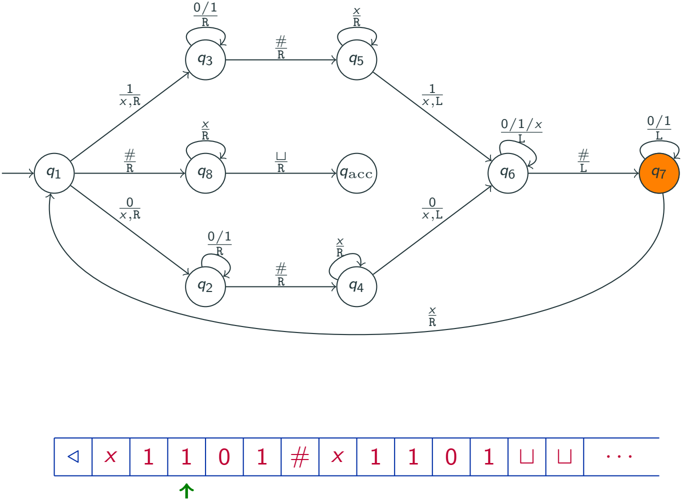


An example of a Turing machine on input 01101#01101

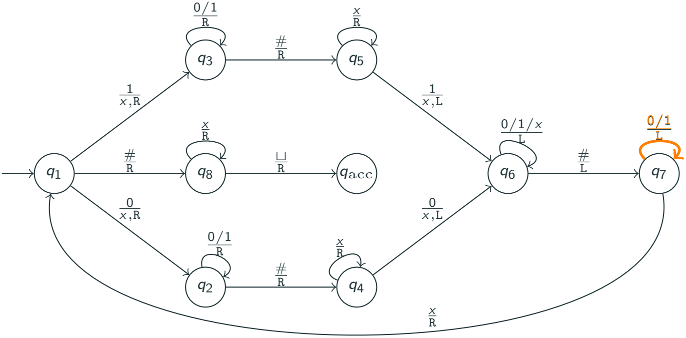


◁	x	1	1	0	1	#	x	1	1	0	1	⊔	⊔	⋯

An example of a Turing machine on input 01101#01101

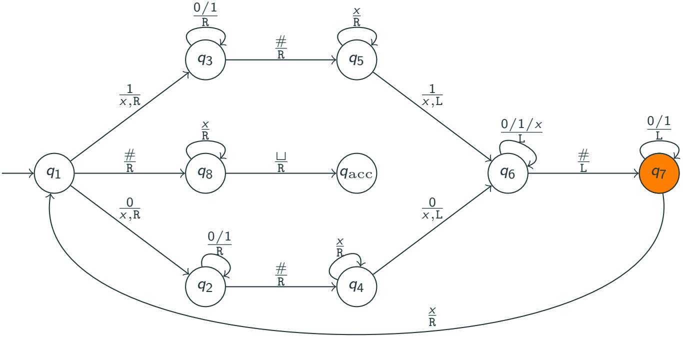


An example of a Turing machine on input 01101#01101



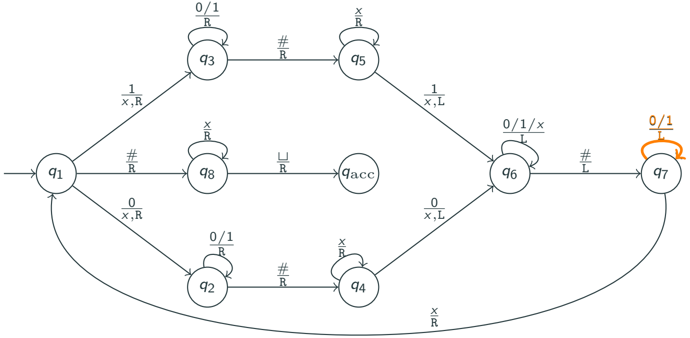
$\triangleleft$	x	1	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



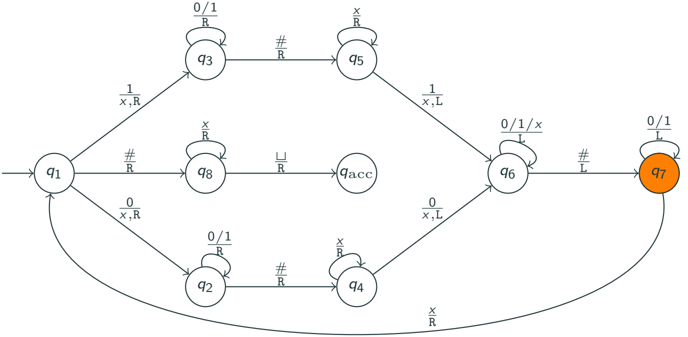
$\triangleleft$	x	1	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



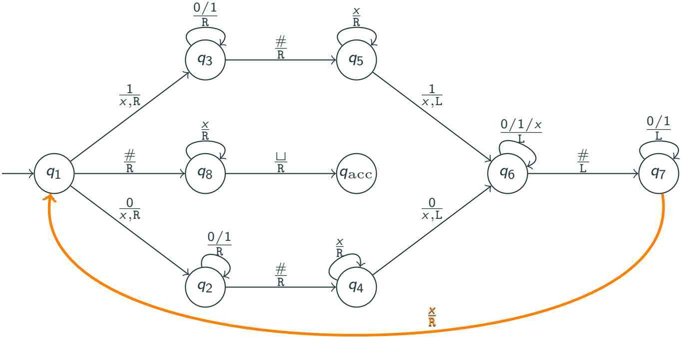
$\triangleleft$	x	1	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



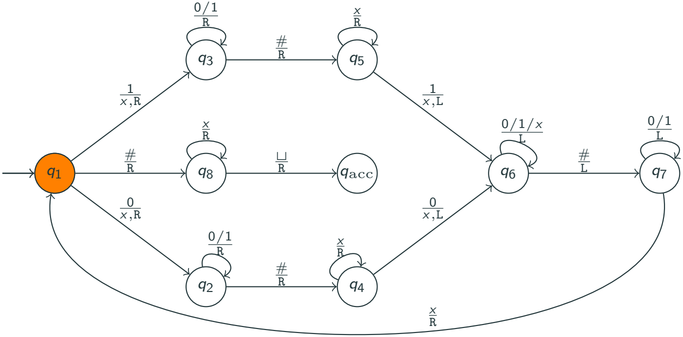
$\triangleleft$	x	1	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



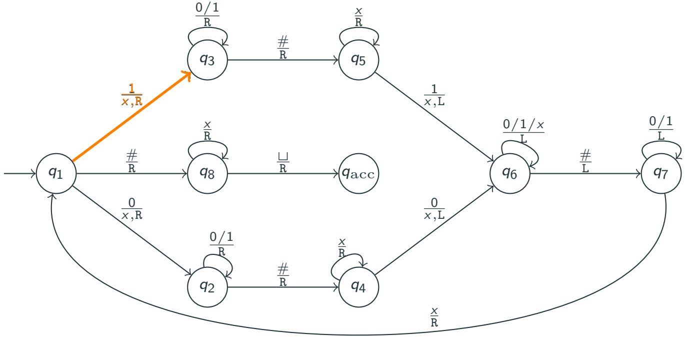
$\triangleleft$	x	1	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



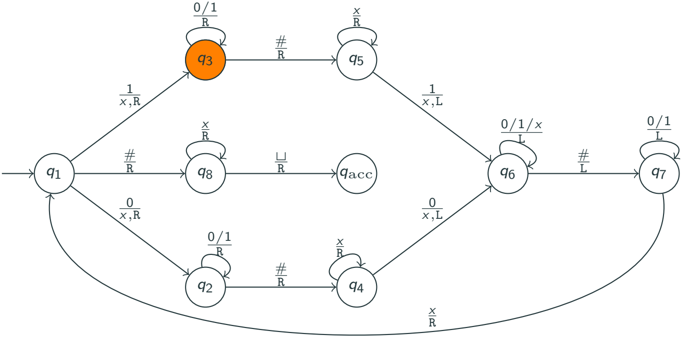
$\triangleleft$	x	1	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$
		$\uparrow$												

An example of a Turing machine on input 01101#01101



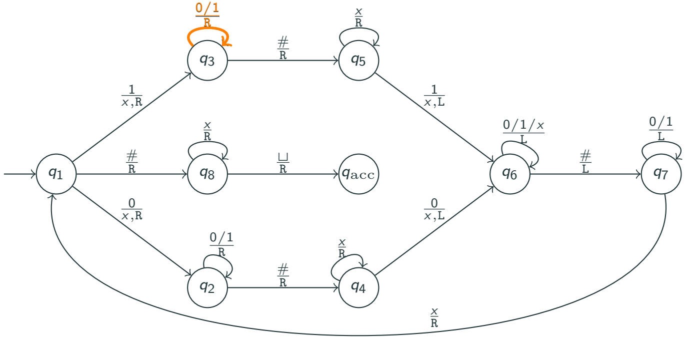
$\triangleleft$	x	1	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



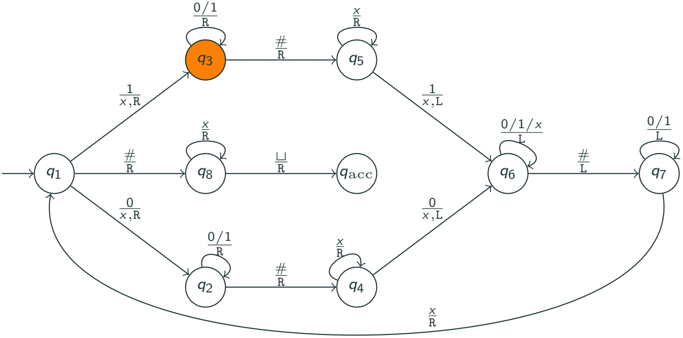
◁	x	x	1	0	1	#	x	1	1	0	1	⊔	⊔	···

An example of a Turing machine on input 01101#01101



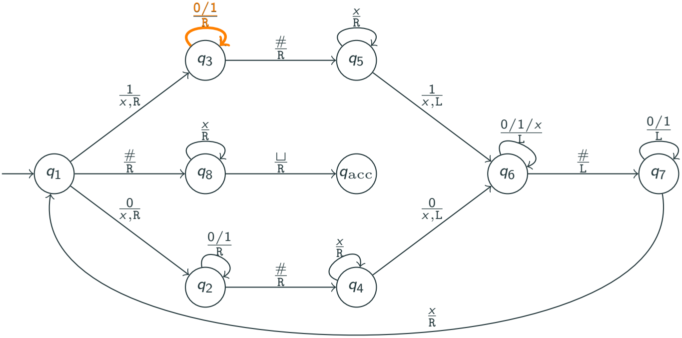
$\triangleleft$	x	x	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

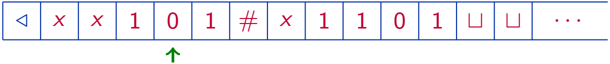
An example of a Turing machine on input 01101#01101



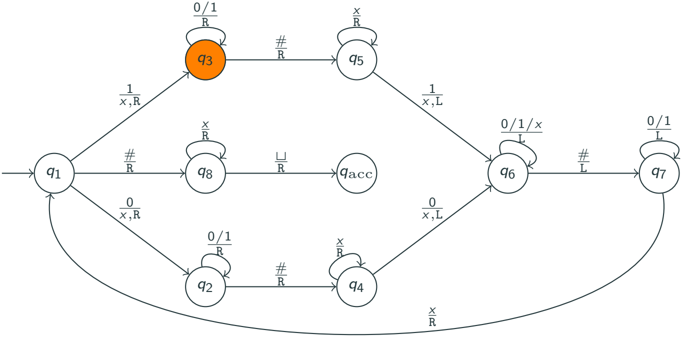
$\triangleleft$	x	x	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



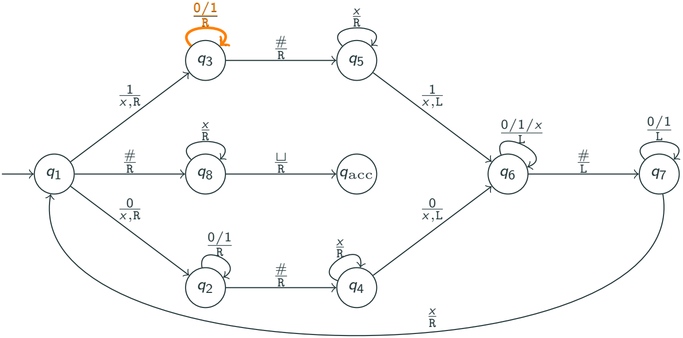


An example of a Turing machine on input 01101#01101



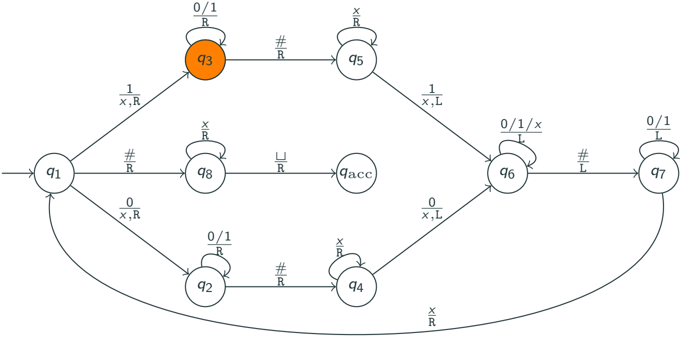
$\triangleleft$	x	x	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



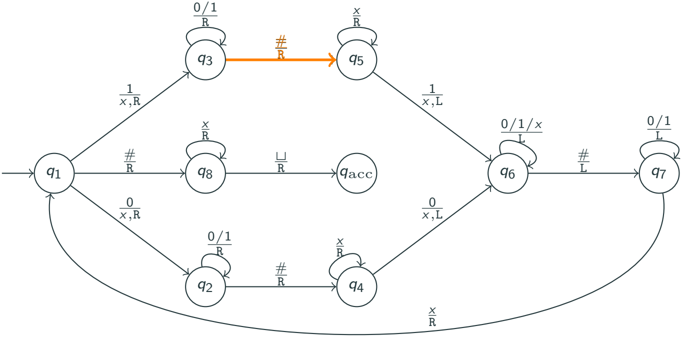
$\triangleleft$	x	x	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



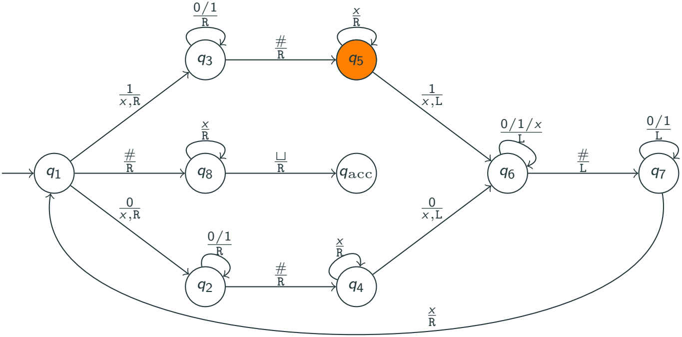
$\triangleleft$	x	x	1	0	1	#	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$
						↑								

An example of a Turing machine on input 01101#01101



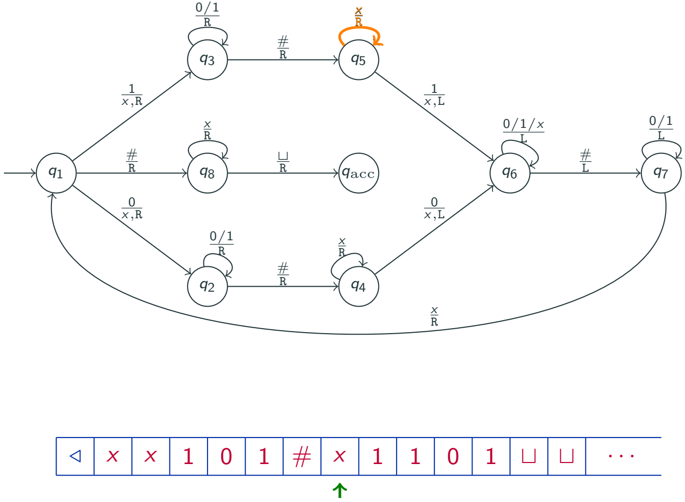
◁	x	x	1	0	1	#	x	1	1	0	1	⊔	⊔	⋯

An example of a Turing machine on input $01101\#01101$

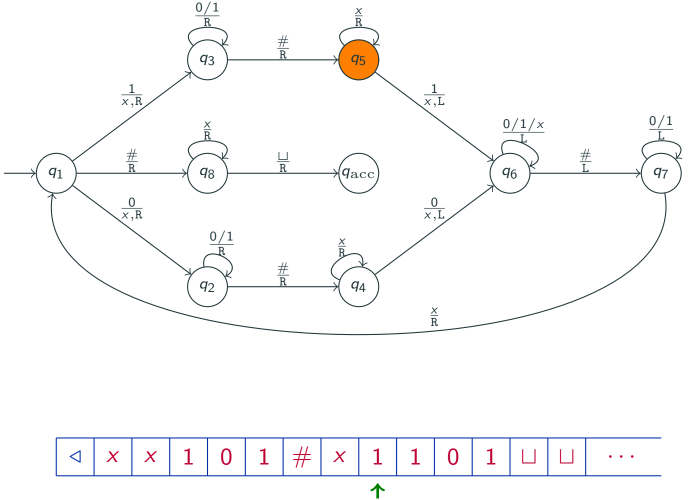


$\triangleleft$	x	x	1	0	1	$\#$	x	1	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

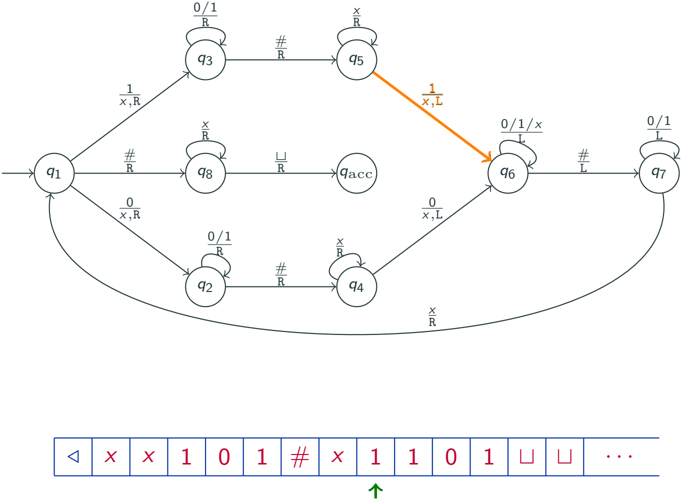
An example of a Turing machine on input 01101#01101



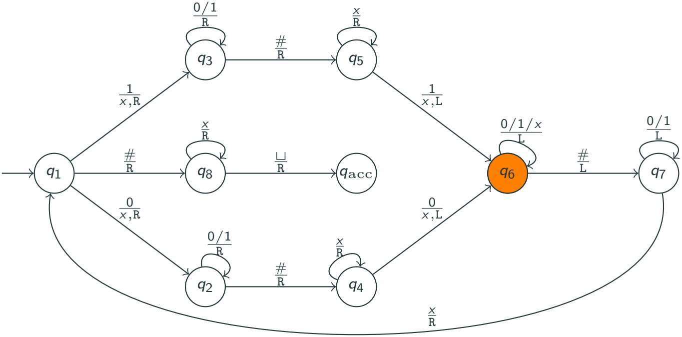
An example of a Turing machine on input 01101#01101



An example of a Turing machine on input 01101#01101

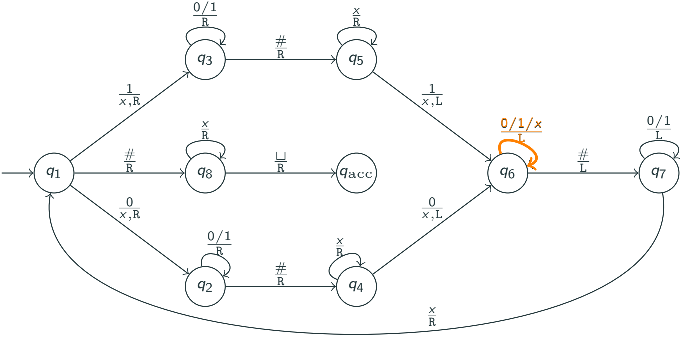


An example of a Turing machine on input 01101#01101



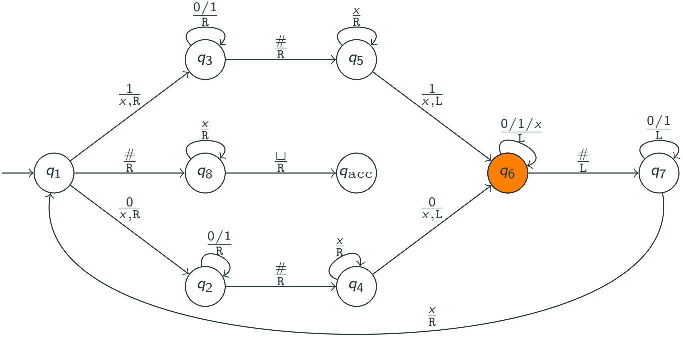
$\triangleleft$	x	x	1	0	1	#	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



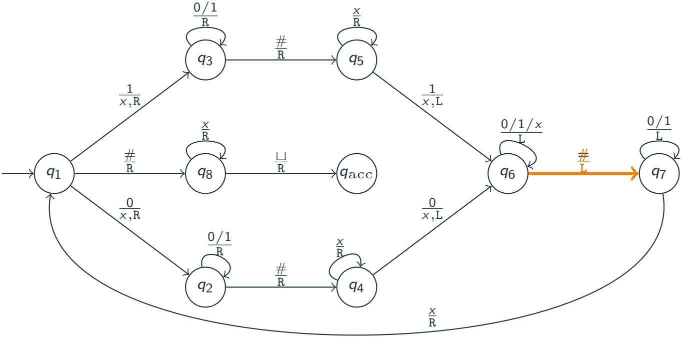
◁	x	x	1	0	1	#	x	x	1	0	1	⊔	⊔	⋯

An example of a Turing machine on input 01101#01101



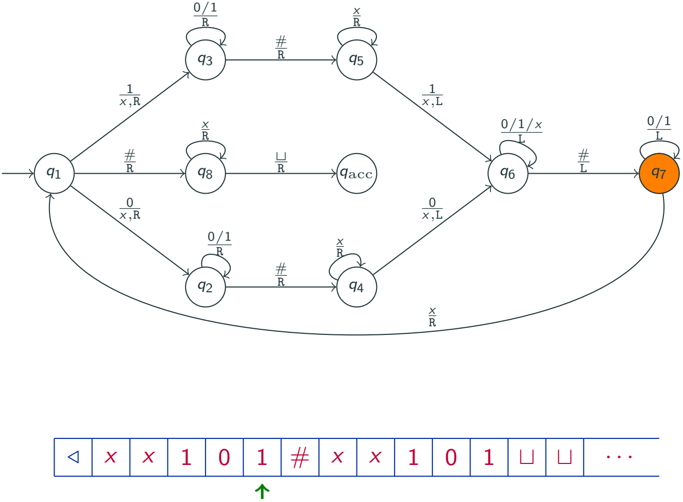
$\triangleleft$	x	x	1	0	1	#	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

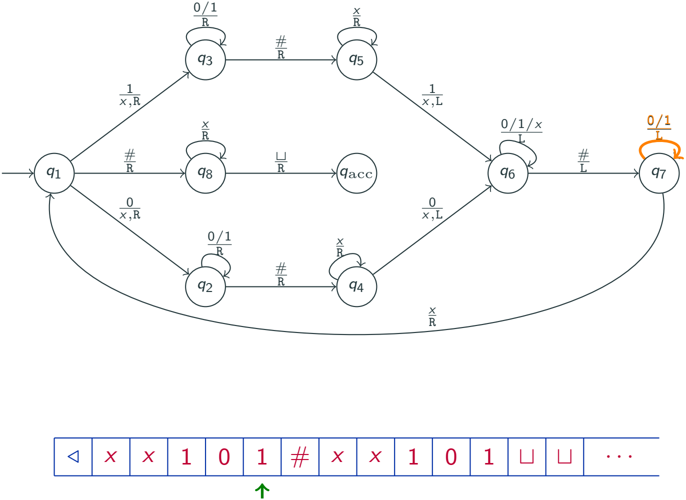


◁	x	x	1	0	1	#	x	x	1	0	1	⊔	⊔	···

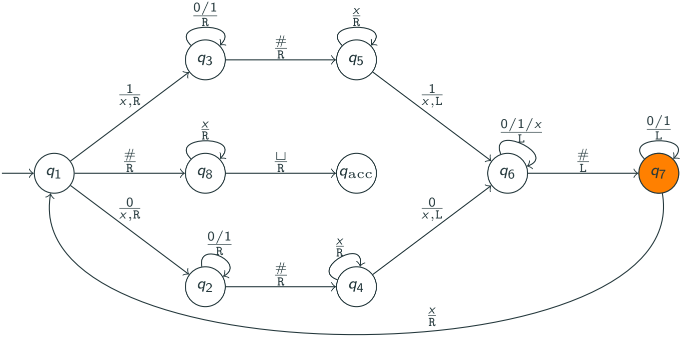
An example of a Turing machine on input 01101#01101



An example of a Turing machine on input 01101#01101

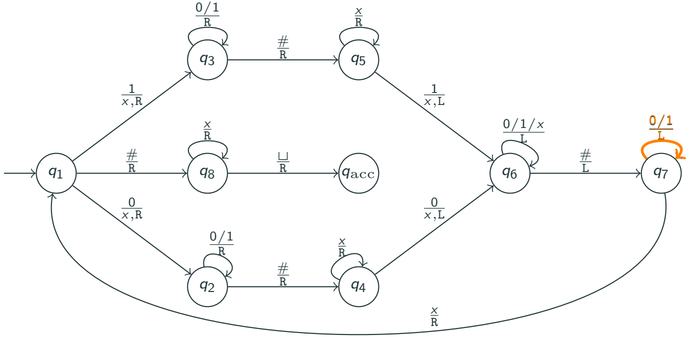


An example of a Turing machine on input 01101#01101



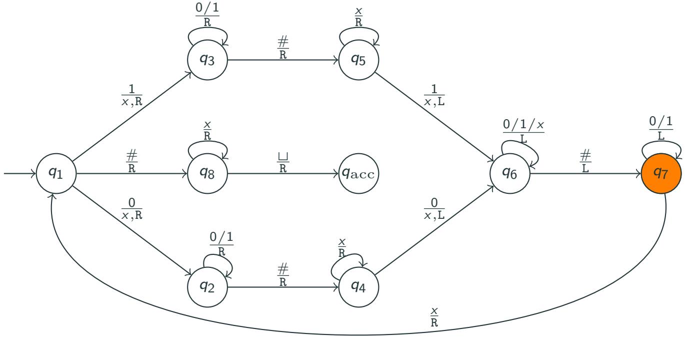
$\triangleleft$	x	x	1	0	1	#	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

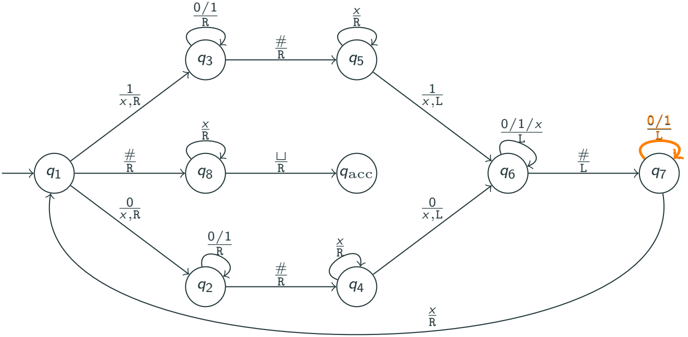


◁	x	x	1	0	1	#	x	x	1	0	1	⊔	⊔	···

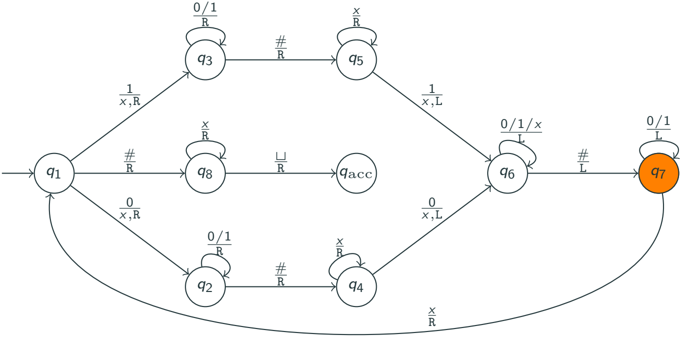
An example of a Turing machine on input 01101#01101



An example of a Turing machine on input 01101#01101

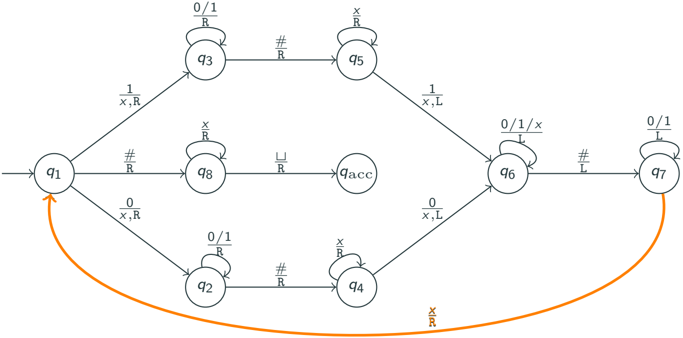


An example of a Turing machine on input 01101#01101



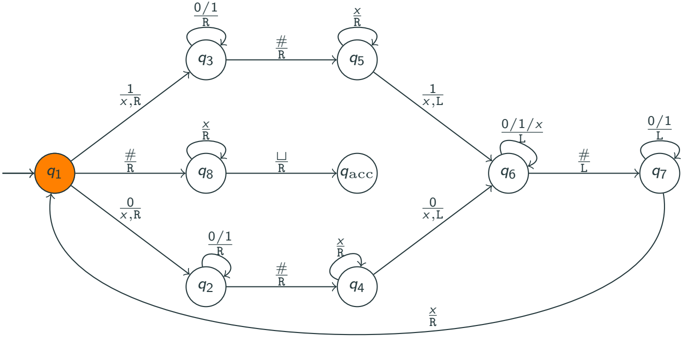
$\triangleleft$	x	x	1	0	1	#	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$
		↑												

An example of a Turing machine on input 01101#01101



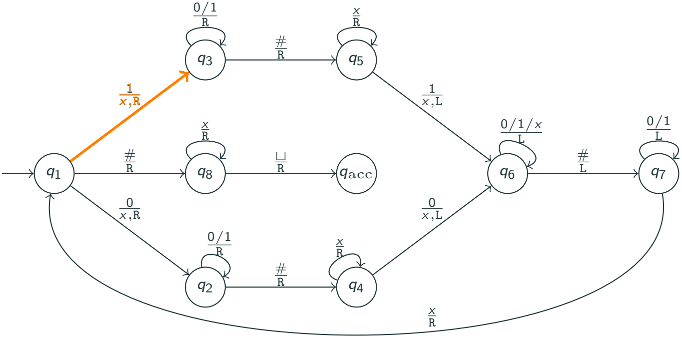
$\triangleleft$	x	x	1	0	1	#	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input $01101\#01101$



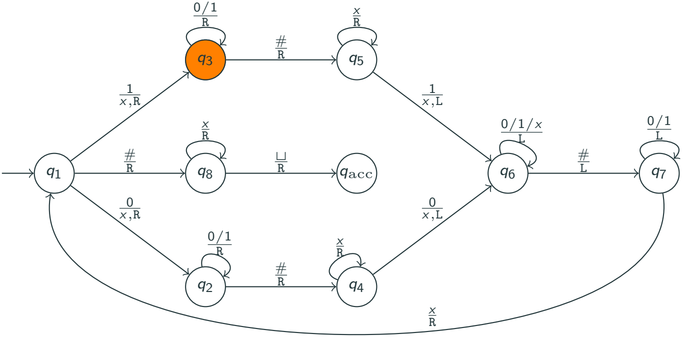
$\triangleleft$	x	x	1	0	1	$\#$	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$
			$\uparrow$											

An example of a Turing machine on input $01101\#01101$

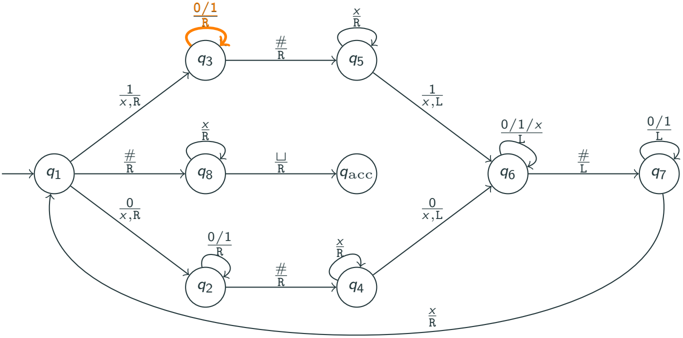


$\triangleleft$	x	x	1	0	1	$\#$	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$
			$\uparrow$											

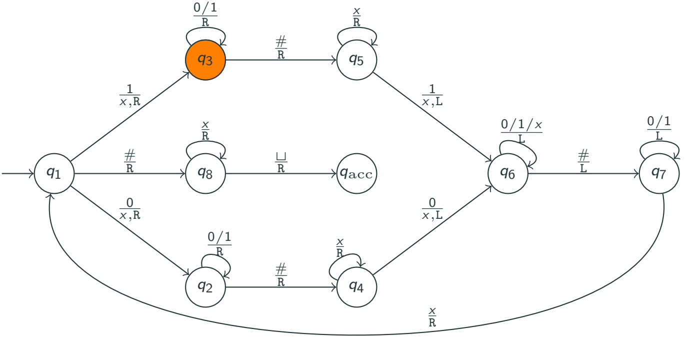
An example of a Turing machine on input 01101#01101



An example of a Turing machine on input 01101#01101

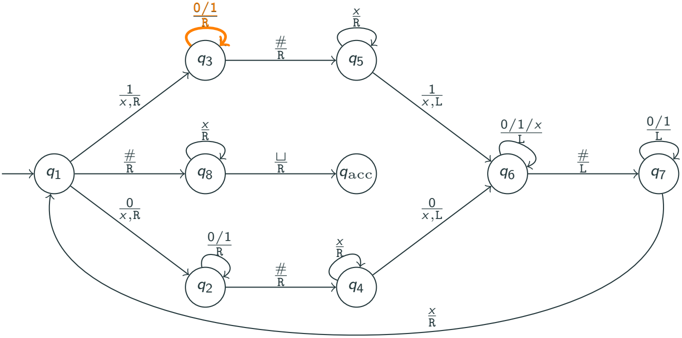


An example of a Turing machine on input 01101#01101



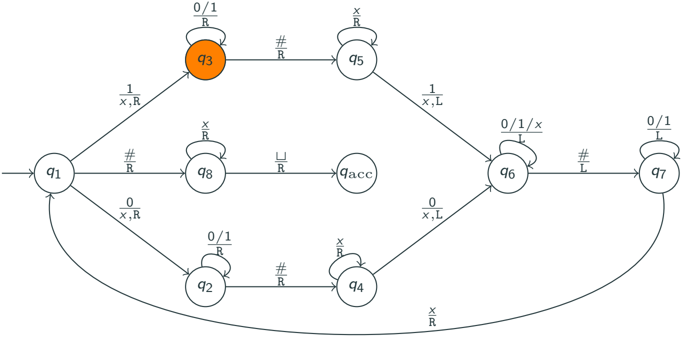
$\triangleleft$	x	x	x	0	1	#	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



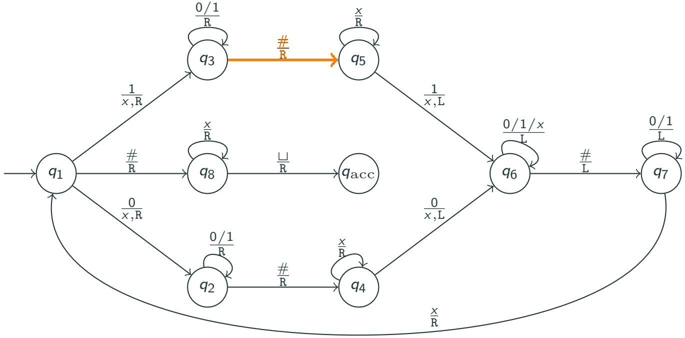
$\triangleleft$	x	x	x	0	1	#	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



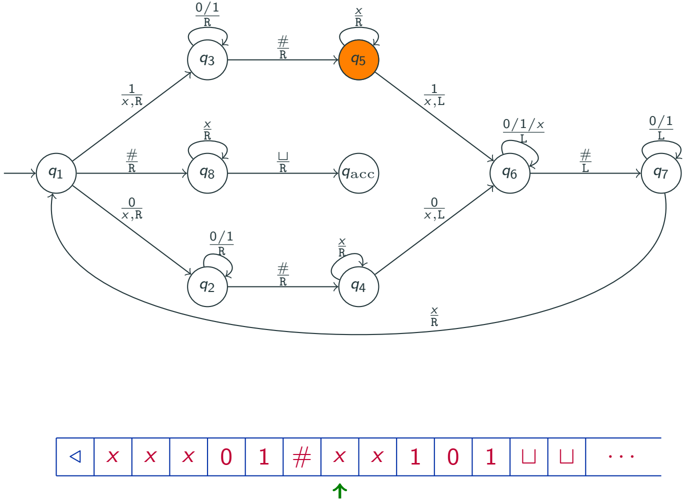
$\triangleleft$	x	x	x	0	1	#	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input $01101\#01101$

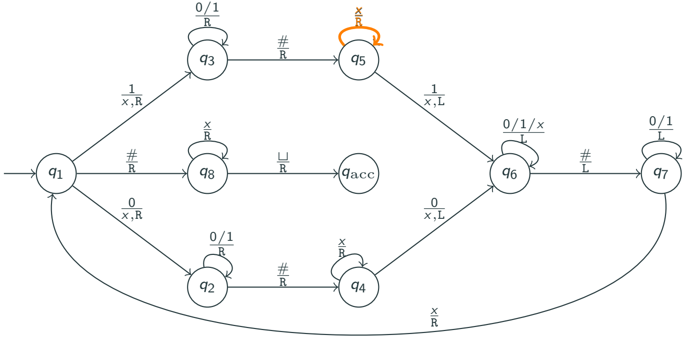


$\triangleleft$	x	x	x	0	1	$\#$	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

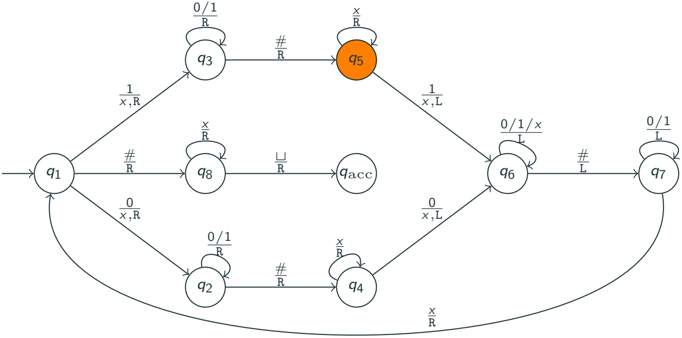


An example of a Turing machine on input 01101#01101



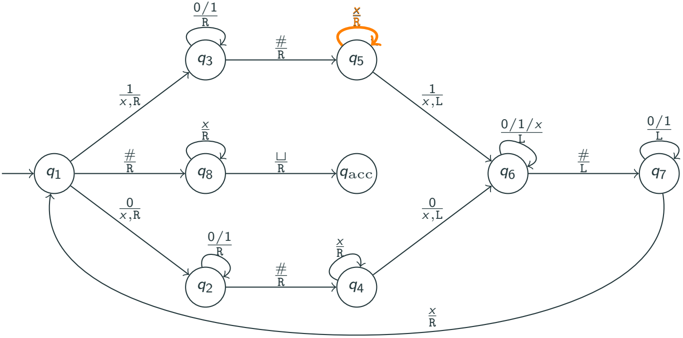
$\triangleleft$	x	x	x	0	1	#	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



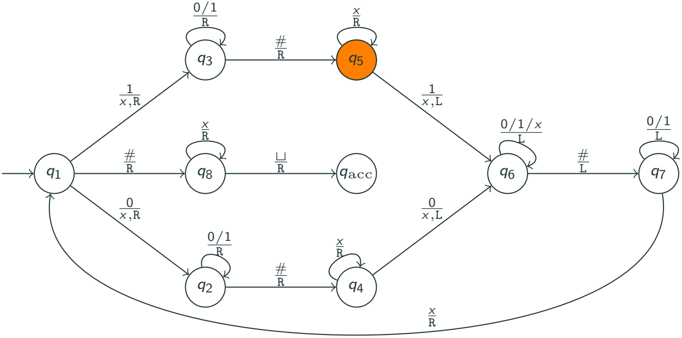
◁	x	x	x	0	1	#	x	x	1	0	1	⊔	⊔	···

An example of a Turing machine on input $01101\#01101$



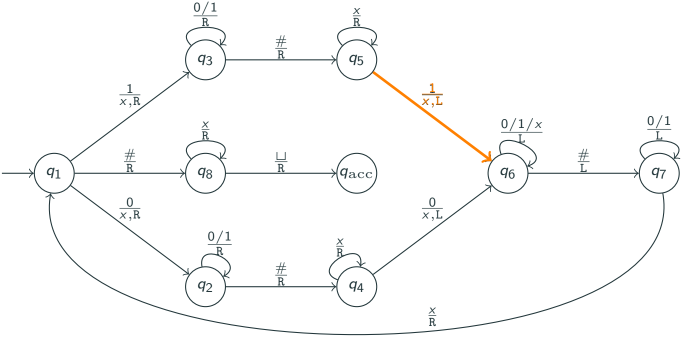
$\triangleleft$	x	x	x	0	1	$\#$	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



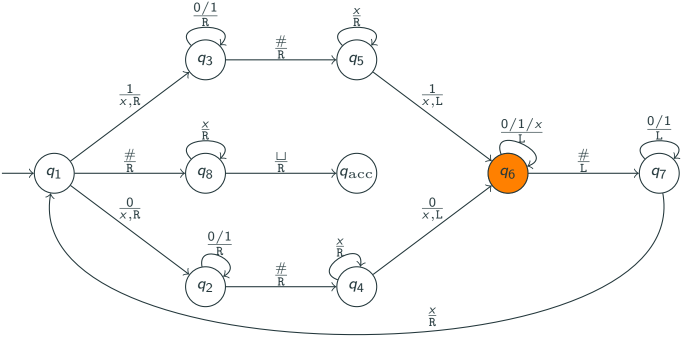
$\triangleleft$	x	x	x	0	1	#	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input $01101\#01101$



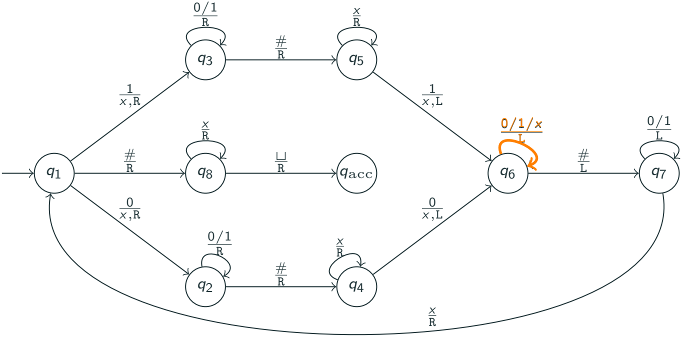
$\triangleleft$	x	x	x	0	1	$\#$	x	x	1	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input $01101\#01101$



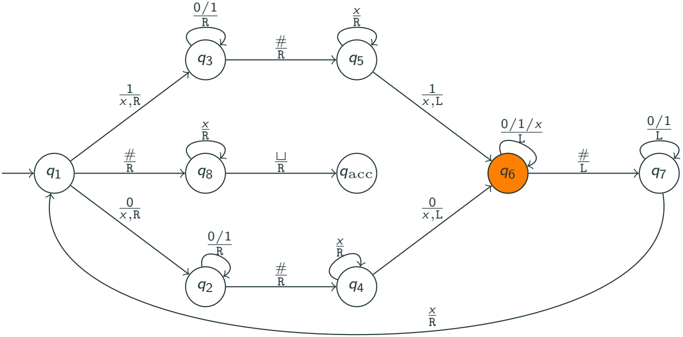
$\triangleleft$	x	x	x	0	1	$\#$	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input $01101\#01101$



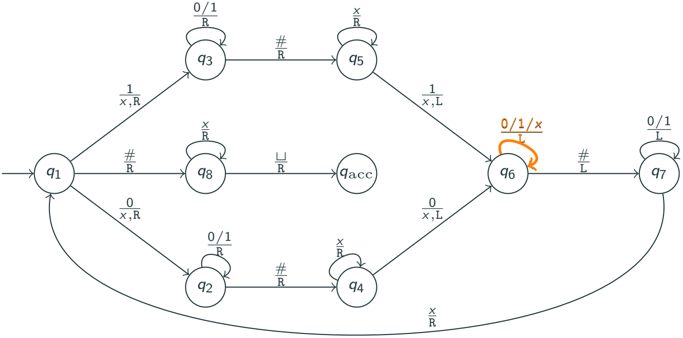
$\triangleleft$	x	x	x	0	1	$\#$	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



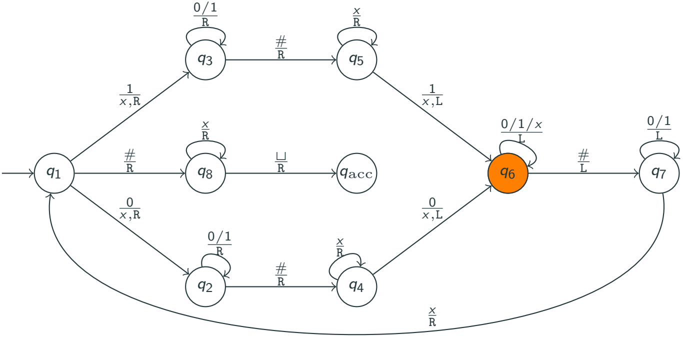
$\triangleleft$	x	x	x	0	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



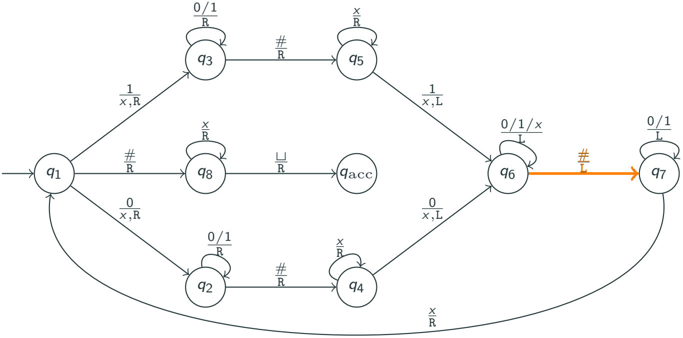
$\triangleleft$	x	x	x	0	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

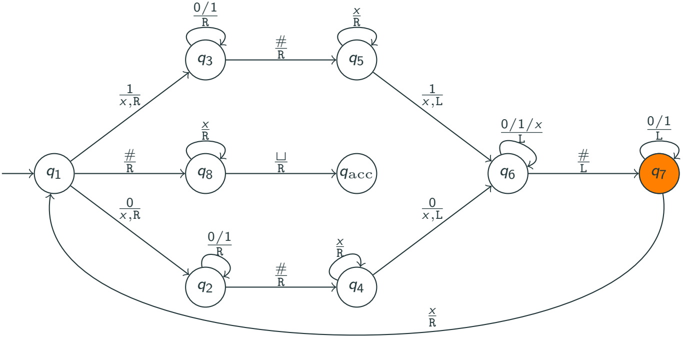


◁	x	x	x	0	1	#	x	x	x	0	1	⊔	⊔	⋯

An example of a Turing machine on input 01101#01101

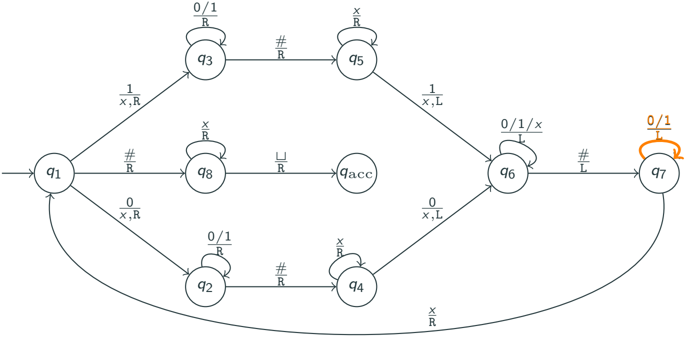


An example of a Turing machine on input 01101#01101

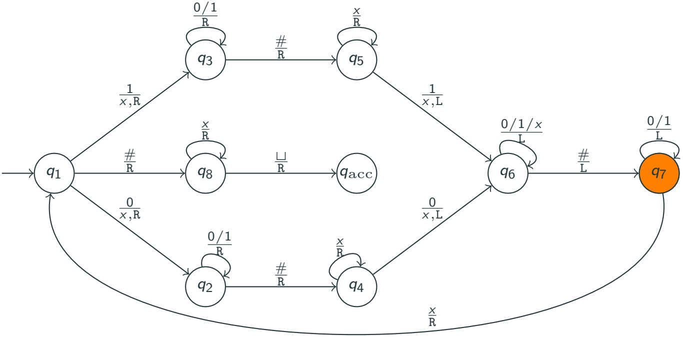


$\triangleleft$	x	x	x	0	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

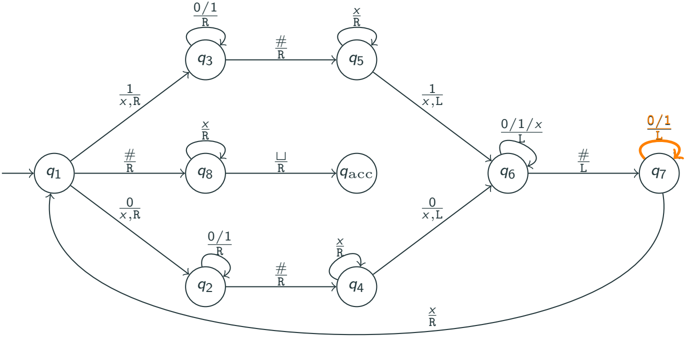


An example of a Turing machine on input 01101#01101

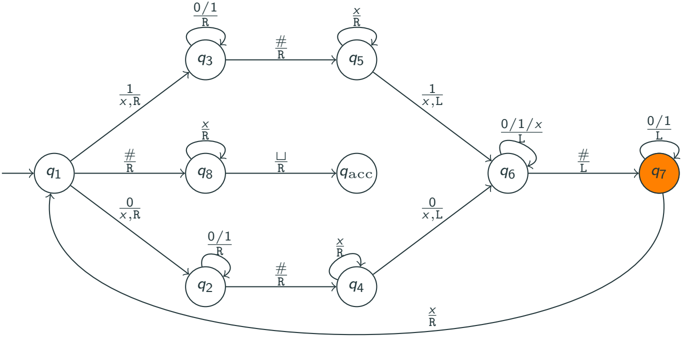


◁	x	x	x	0	1	#	x	x	x	0	1	⊔	⊔	···
				↑										

An example of a Turing machine on input $01101\#01101$

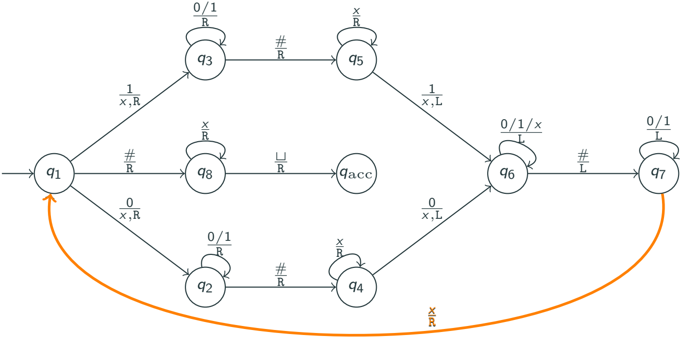


An example of a Turing machine on input 01101#01101



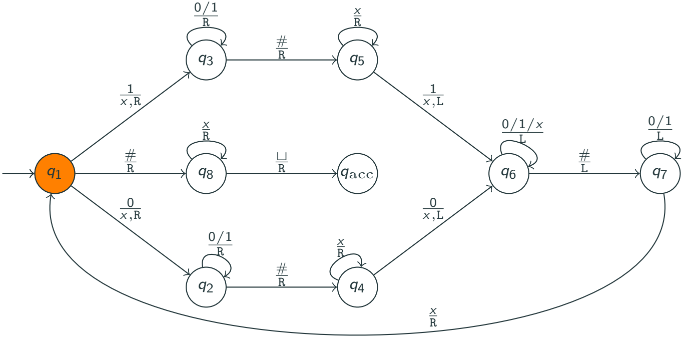
$\triangleleft$	x	x	x	0	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

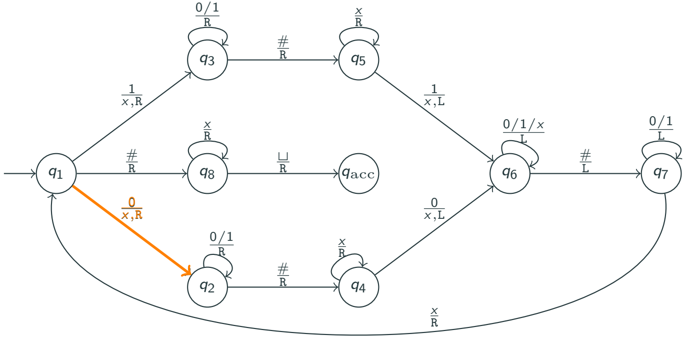


$\triangleleft$	x	x	x	0	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

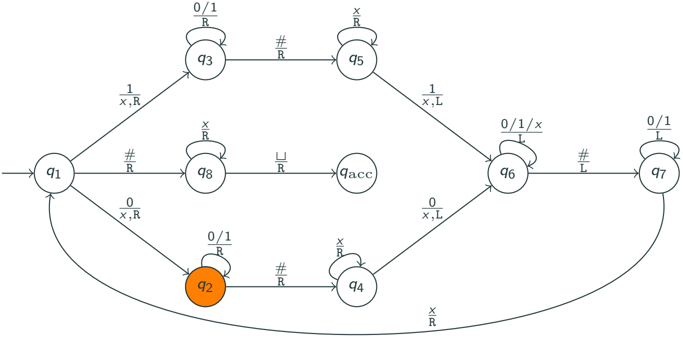


An example of a Turing machine on input 01101#01101



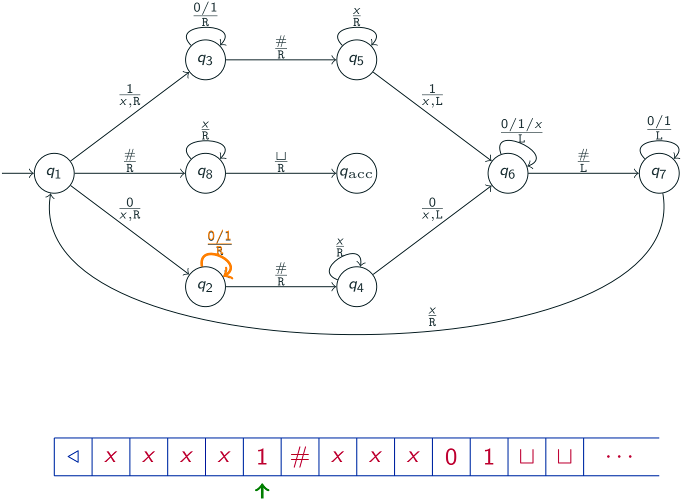
◁	x	x	x	0	1	#	x	x	x	0	1	⊔	⊔	···

An example of a Turing machine on input 01101#01101

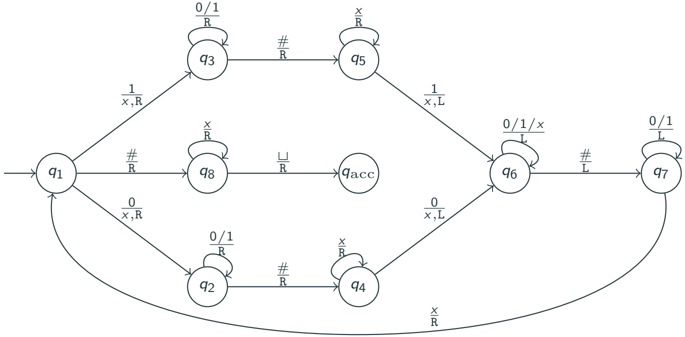


$\triangleleft$	x	x	x	x	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

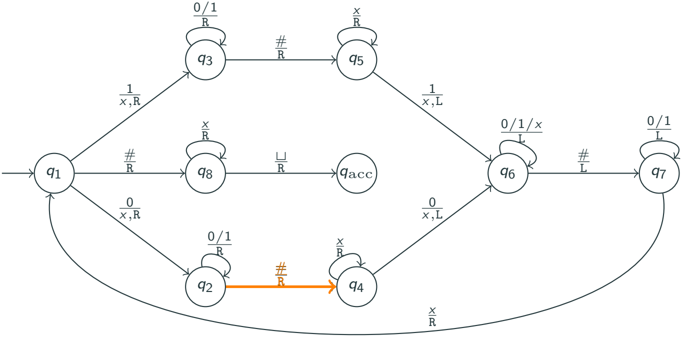


An example of a Turing machine on input 01101#01101



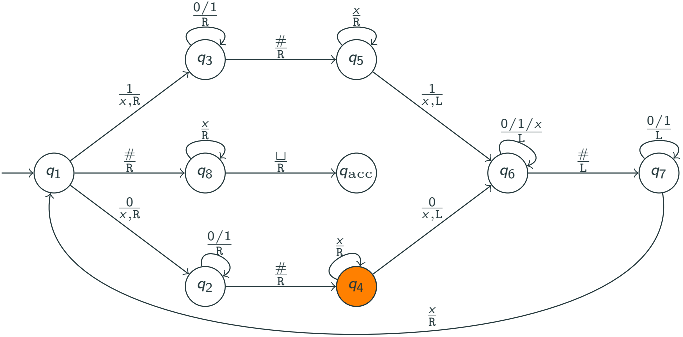
$\triangleleft$	x	x	x	x	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



$\triangleleft$	x	x	x	x	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

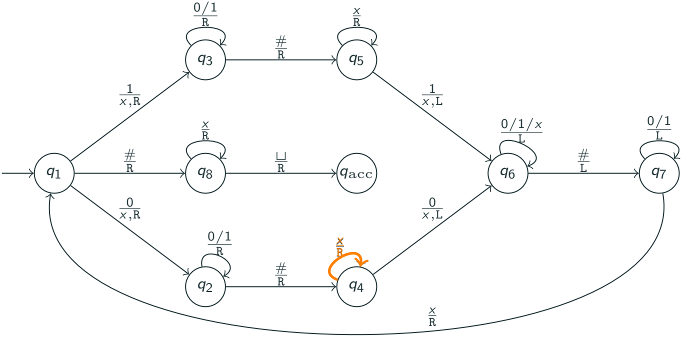
An example of a Turing machine on input 01101#01101



$\triangleleft$	x	x	x	x	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

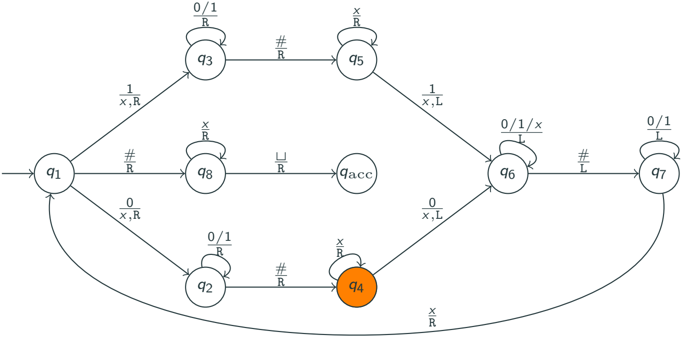
$\uparrow$

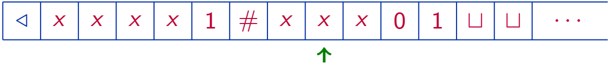
An example of a Turing machine on input 01101#01101



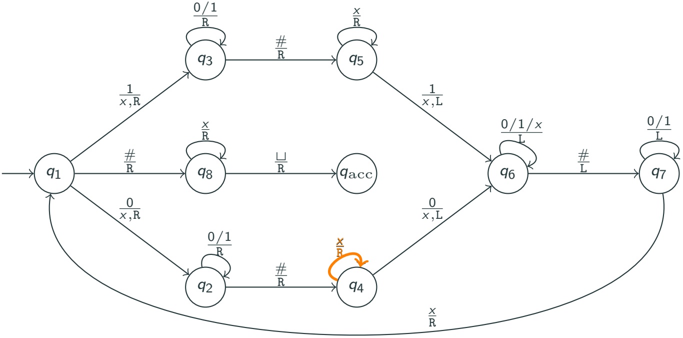
◁	x	x	x	x	1	#	x	x	x	0	1	⊔	⊔	···

An example of a Turing machine on input 01101#01101



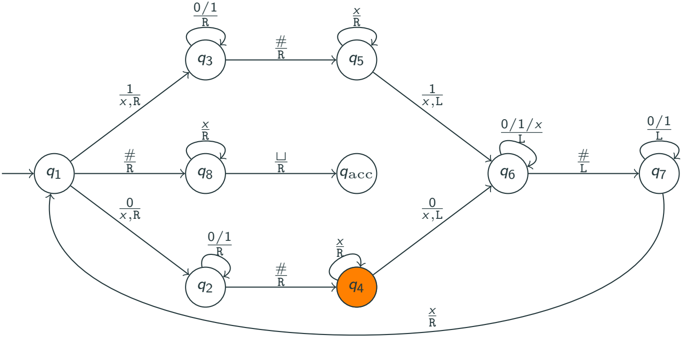


An example of a Turing machine on input 01101#01101



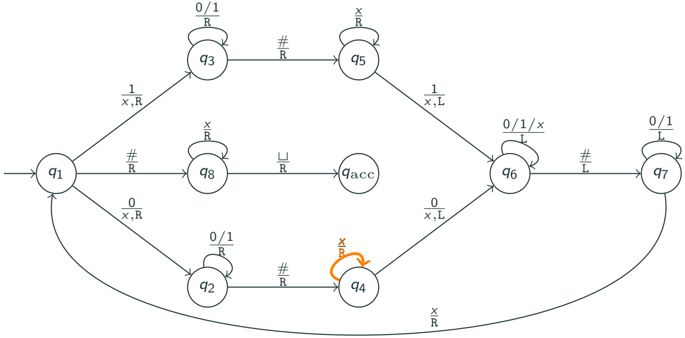
◁	x	x	x	x	1	#	x	x	x	0	1	⊔	⊔	···

An example of a Turing machine on input 01101#01101

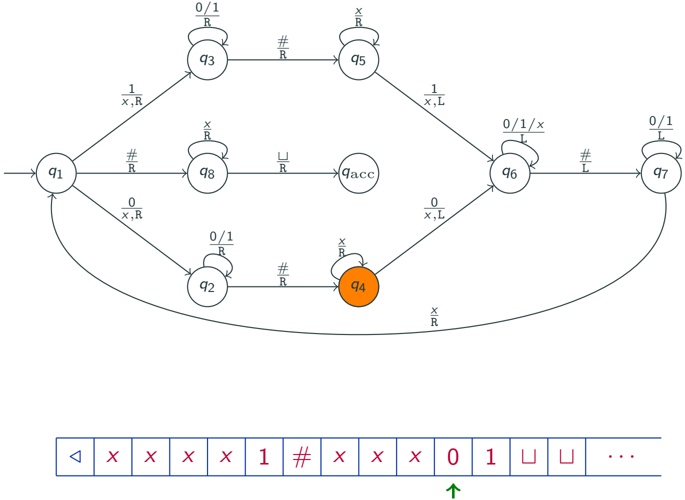


$\triangleleft$	x	x	x	x	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

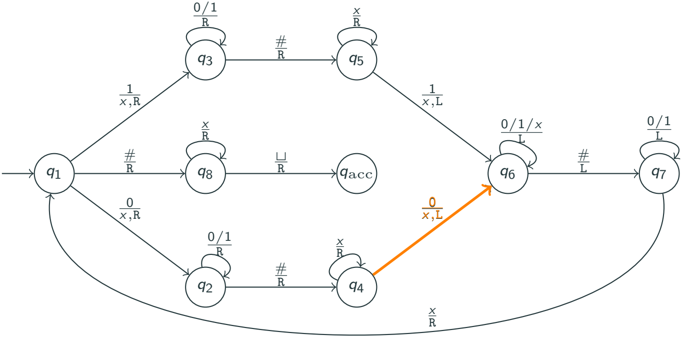
An example of a Turing machine on input 01101#01101



An example of a Turing machine on input 01101#01101

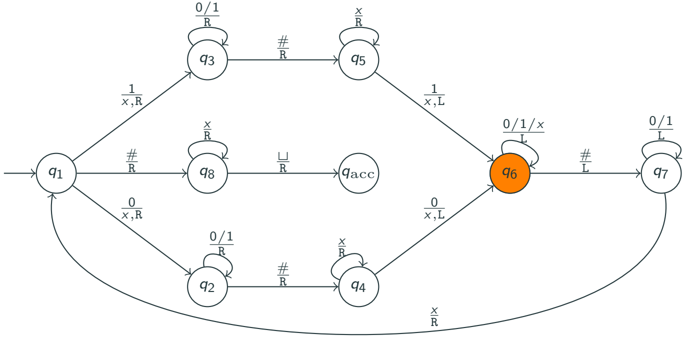


An example of a Turing machine on input 01101#01101



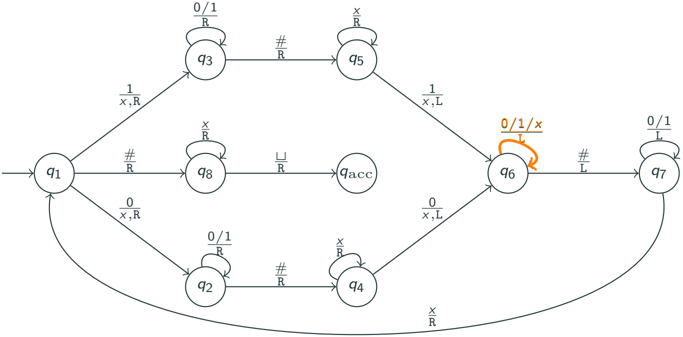
$\triangleleft$	x	x	x	x	1	#	x	x	x	0	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



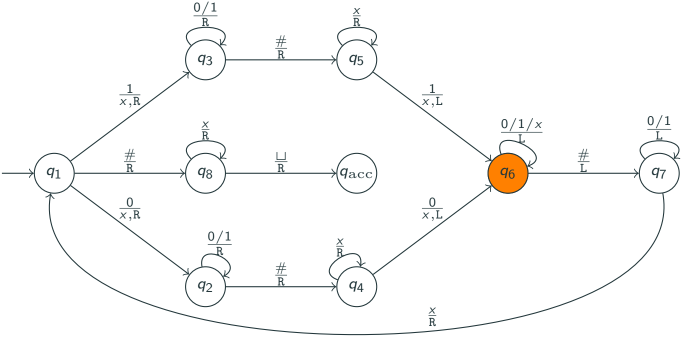
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



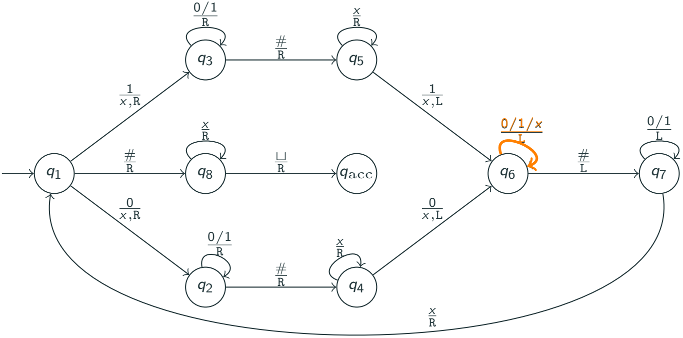
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



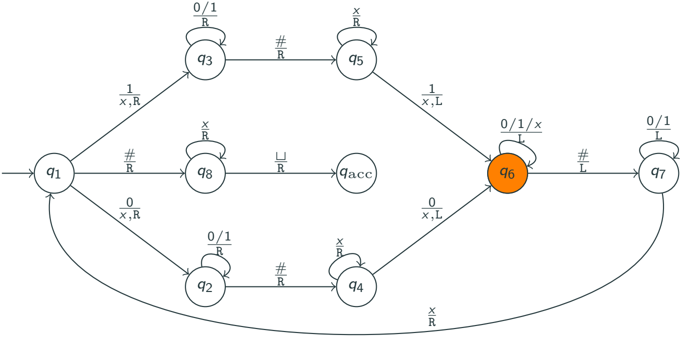
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



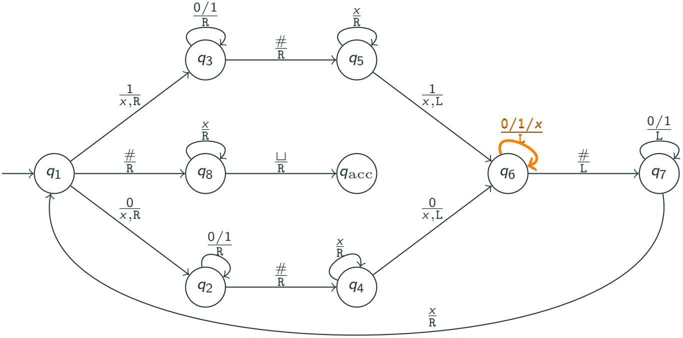
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



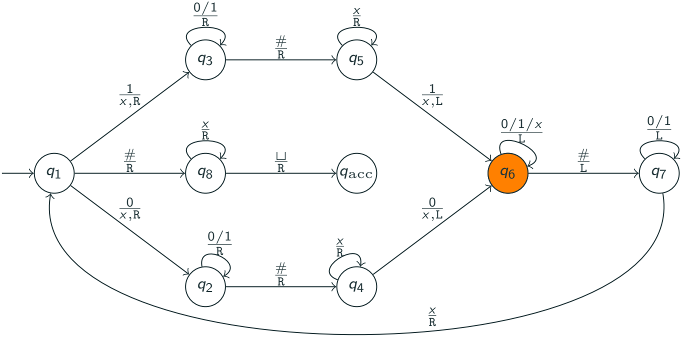
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



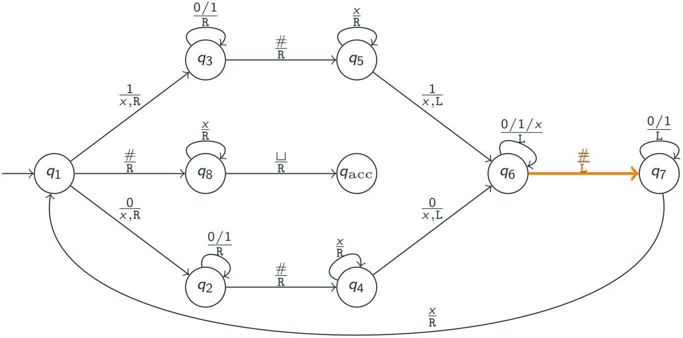
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



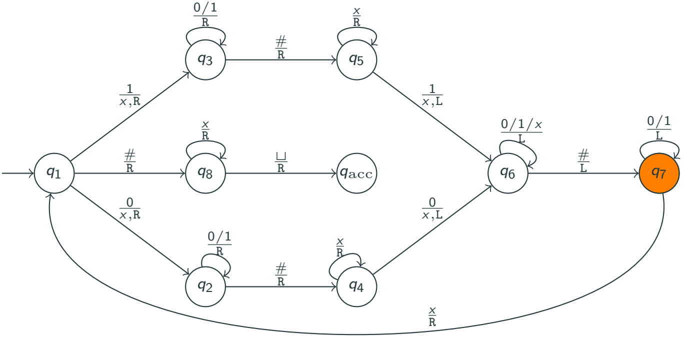
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



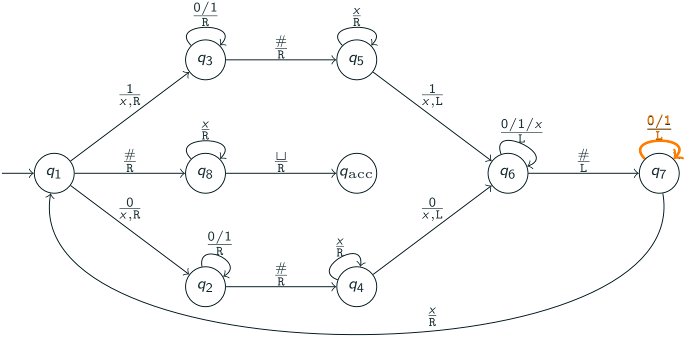
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



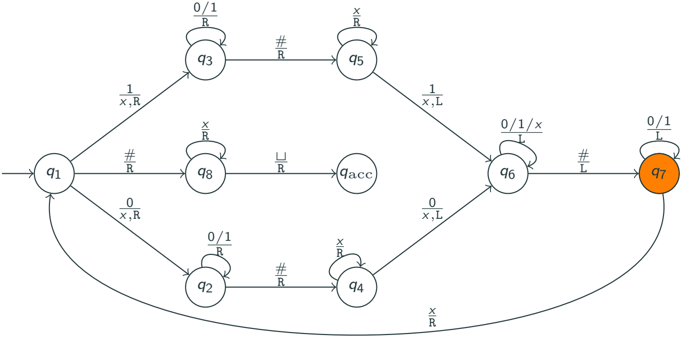
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$
					$\uparrow$									

An example of a Turing machine on input 01101#01101

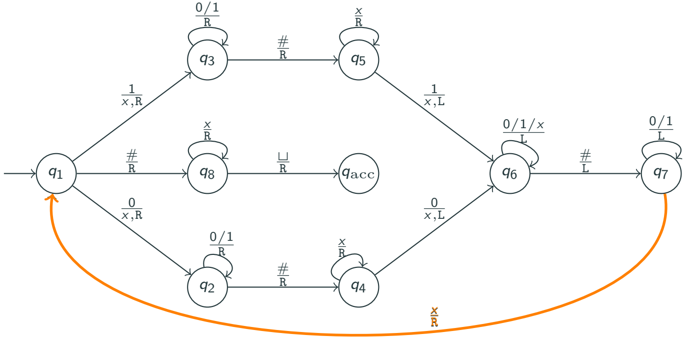


$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

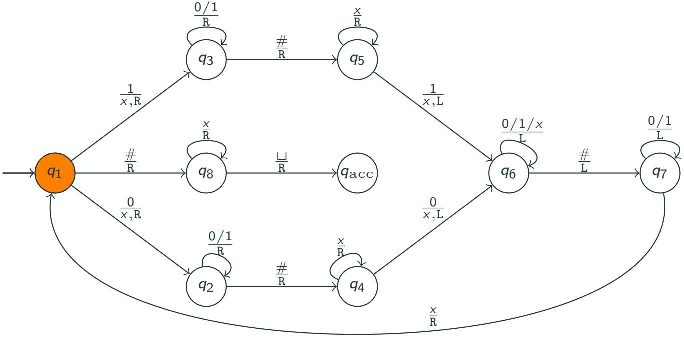


An example of a Turing machine on input 01101#01101



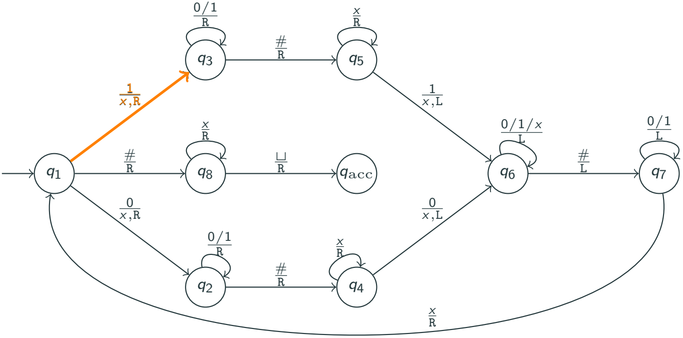
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



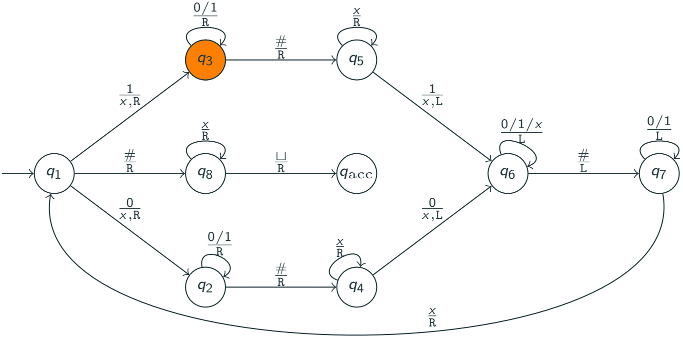
$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

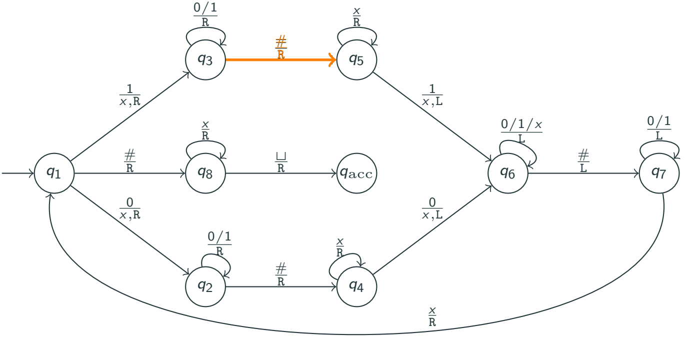


$\triangleleft$	x	x	x	x	1	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

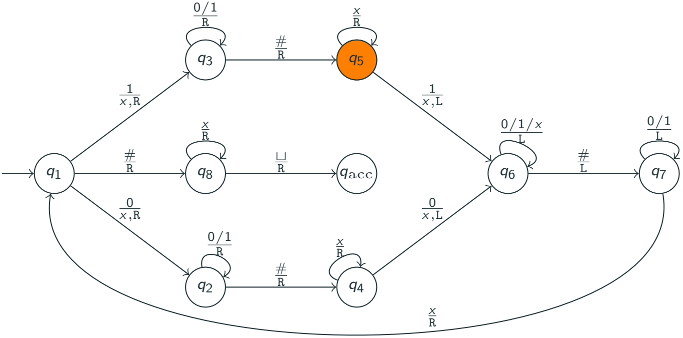


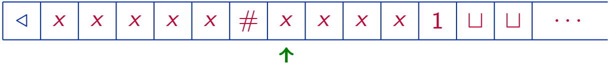
An example of a Turing machine on input 01101#01101



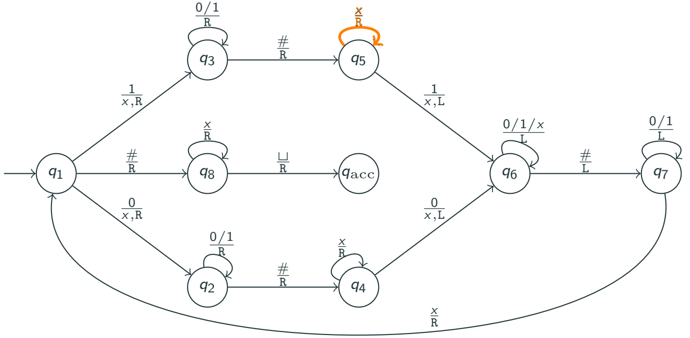
$\triangleleft$	x	x	x	x	x	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



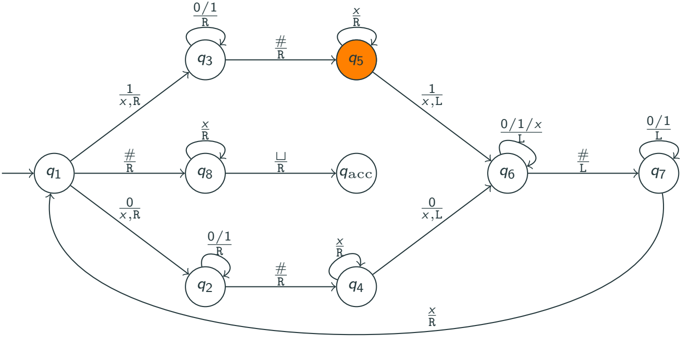


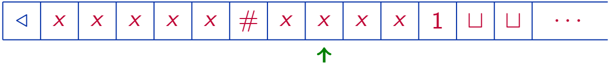
An example of a Turing machine on input 01101#01101



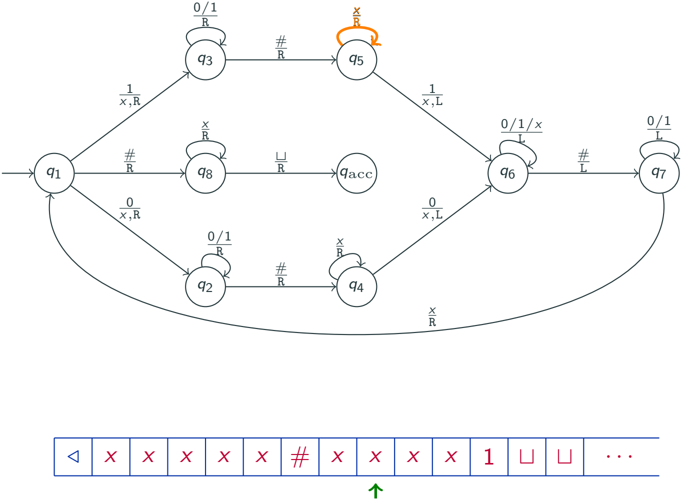
$\triangleleft$	x	x	x	x	x	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

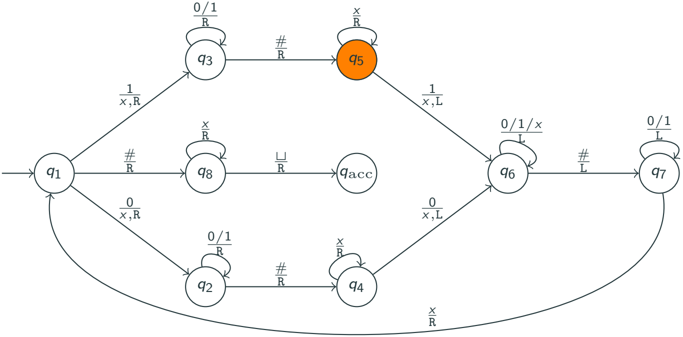




An example of a Turing machine on input 01101#01101

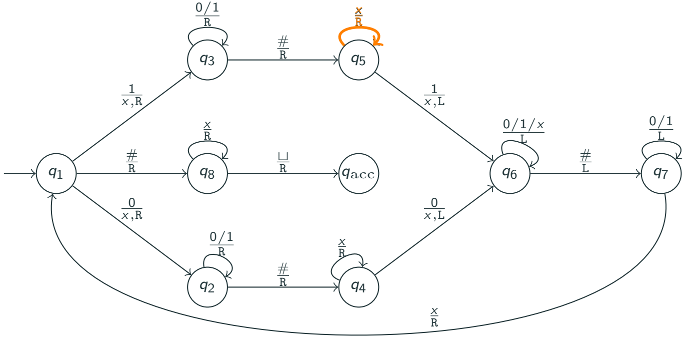


An example of a Turing machine on input 01101#01101

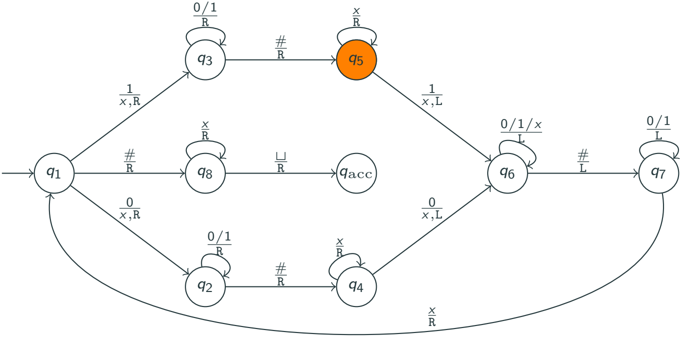


$\triangleleft$	x	x	x	x	x	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

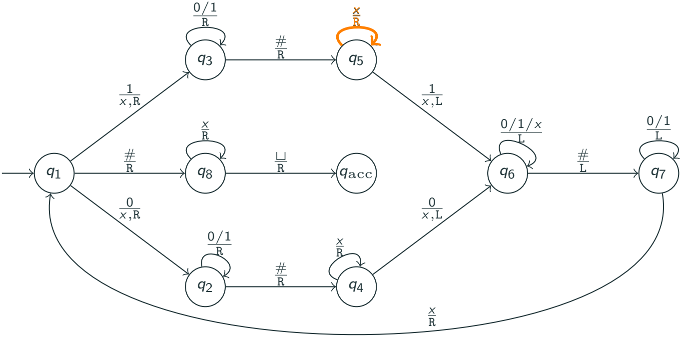


An example of a Turing machine on input $01101\#01101$



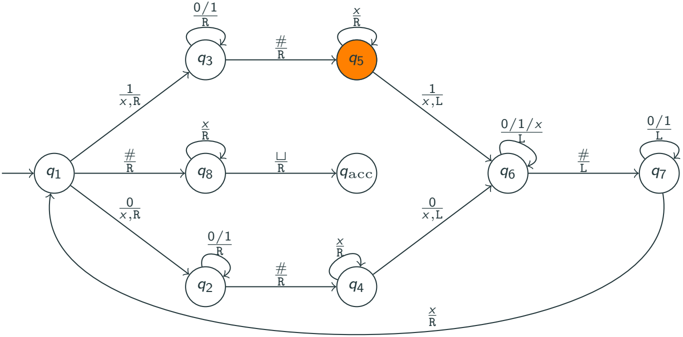
$\triangleleft$	x	x	x	x	x	$\#$	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



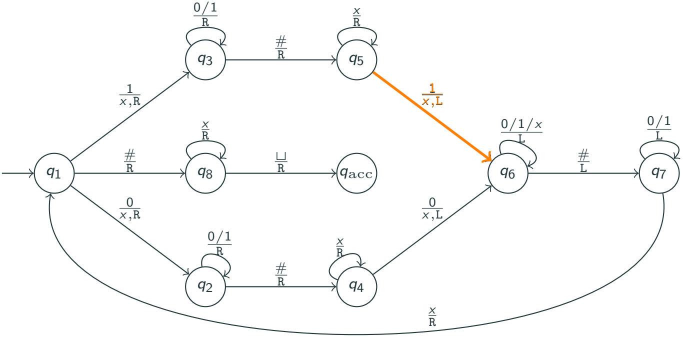
◁	x	x	x	x	x	#	x	x	x	x	1	⊔	⊔	···

An example of a Turing machine on input 01101#01101

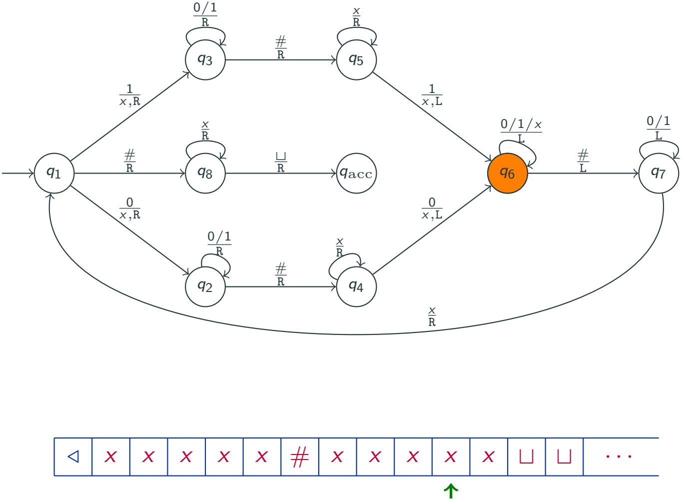


$\triangleleft$	x	x	x	x	x	#	x	x	x	x	1	$\sqcup$	$\sqcup$	$\cdots$

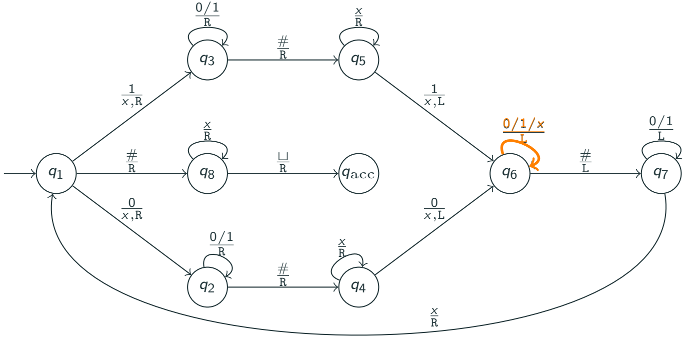
An example of a Turing machine on input 01101#01101



An example of a Turing machine on input $01101\#01101$

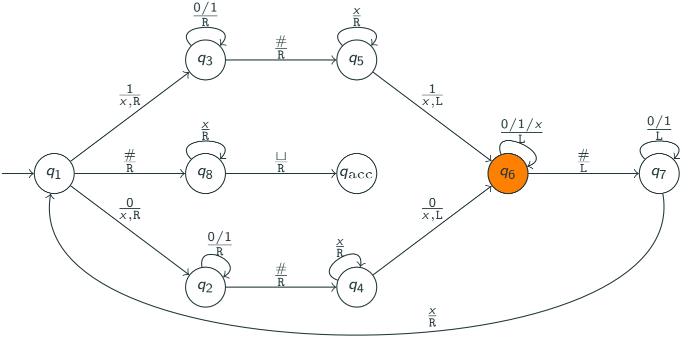


An example of a Turing machine on input $01101\#01101$

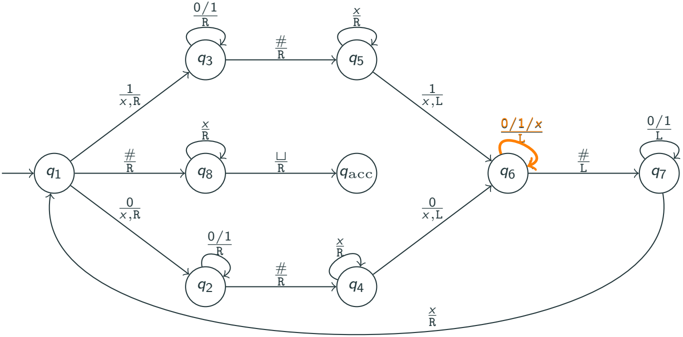


$\triangleleft$	x	x	x	x	x	$\#$	x	x	x	x	x	$\sqcup$	$\sqcup$	$\cdots$

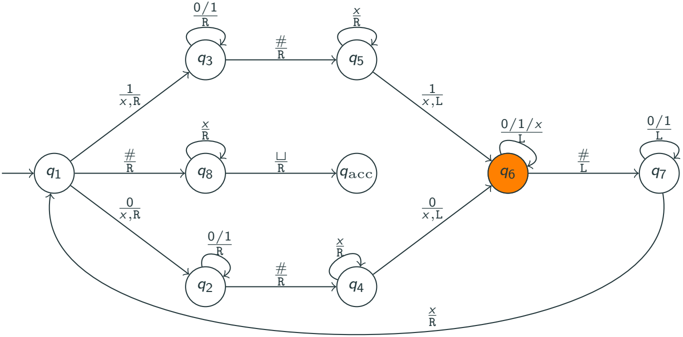
An example of a Turing machine on input 01101#01101



An example of a Turing machine on input 01101#01101

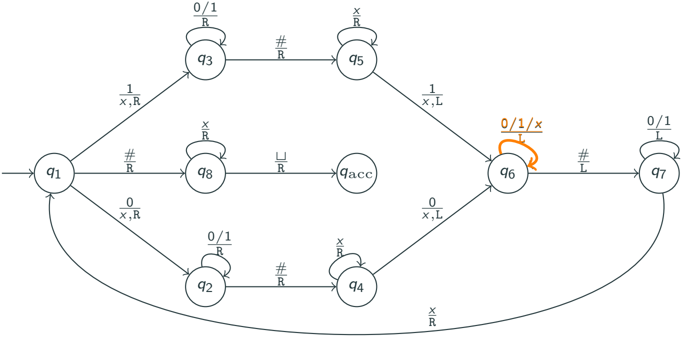


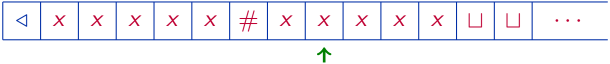
An example of a Turing machine on input $01101\#01101$



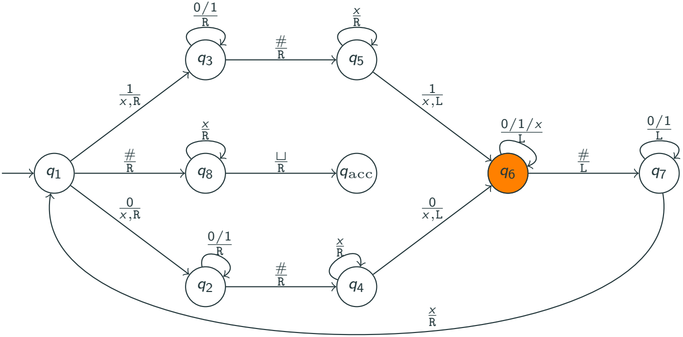
$\triangleleft$	x	x	x	x	x	$\#$	x	x	x	x	x	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input $01101\#01101$



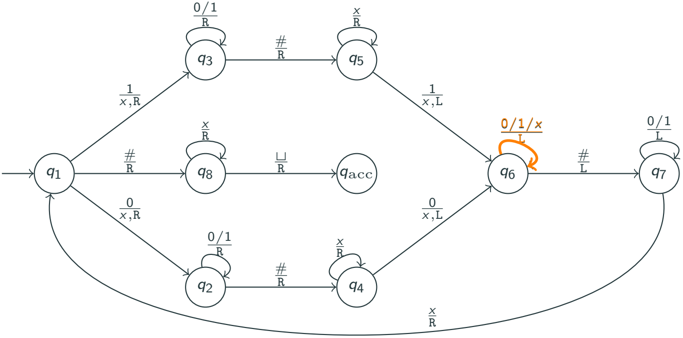


An example of a Turing machine on input $01101\#01101$



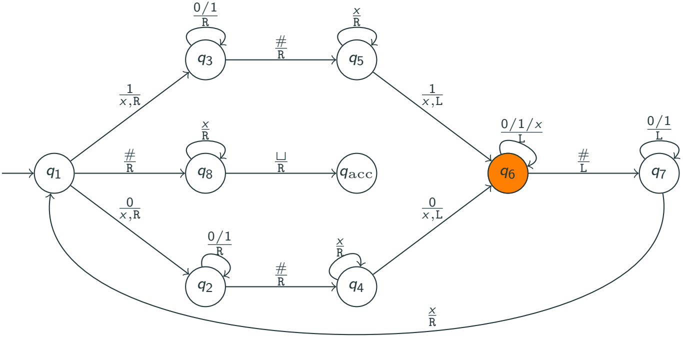
$\triangleleft$	x	x	x	x	x	$\#$	x	x	x	x	x	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



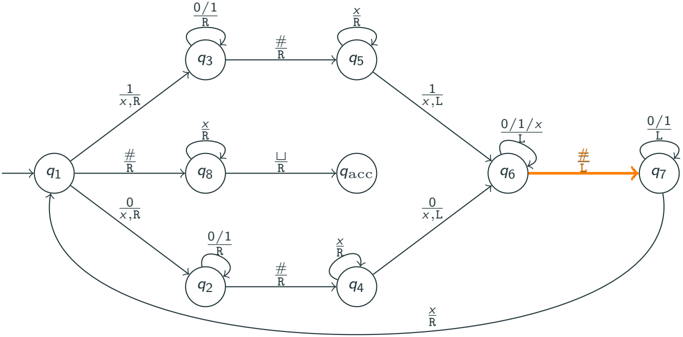
$\triangleleft$	x	x	x	x	x	$\#$	x	x	x	x	x	$\sqcup$	$\sqcup$	$\cdots$
							$\uparrow$							

An example of a Turing machine on input 01101#01101

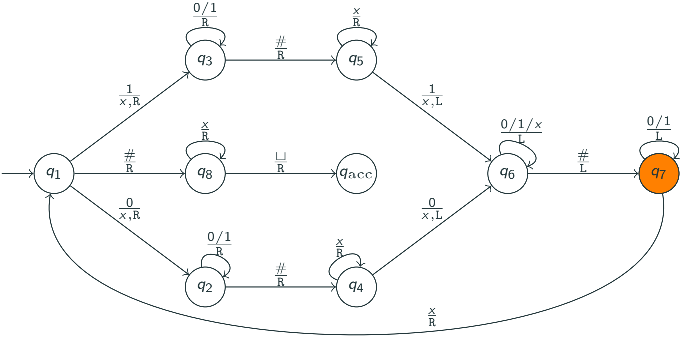


$\triangleleft$	x	x	x	x	x	$\#$	x	x	x	x	x	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

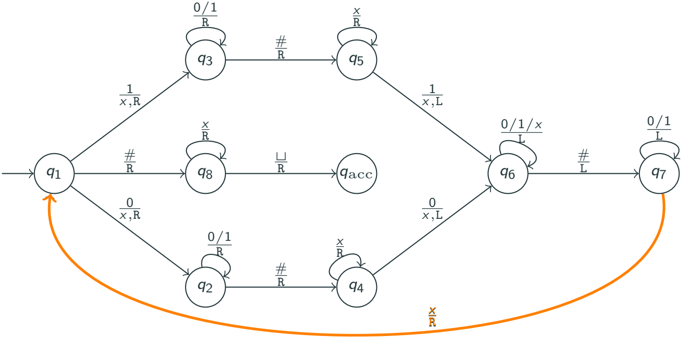


An example of a Turing machine on input $01101\#01101$



$\triangleleft$	x	x	x	x	x	$\#$	x	x	x	x	x	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101



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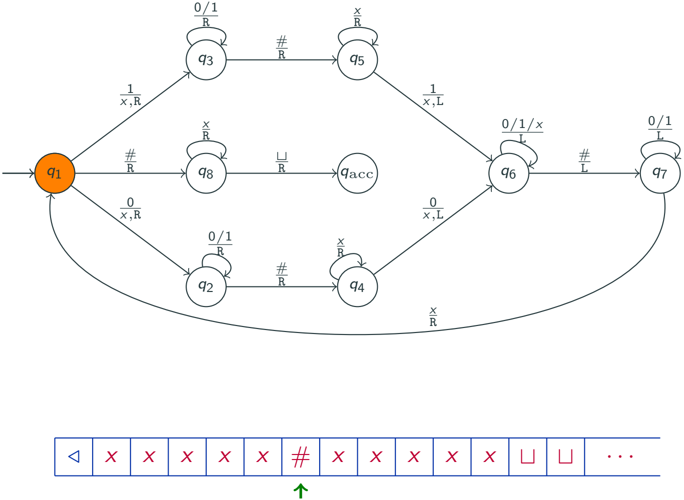

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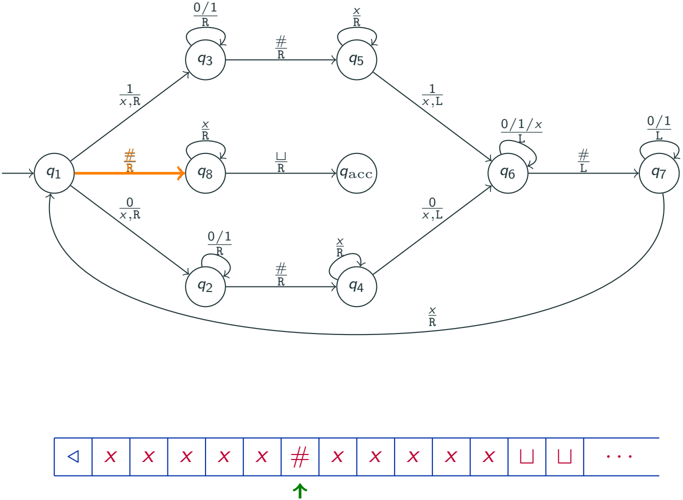
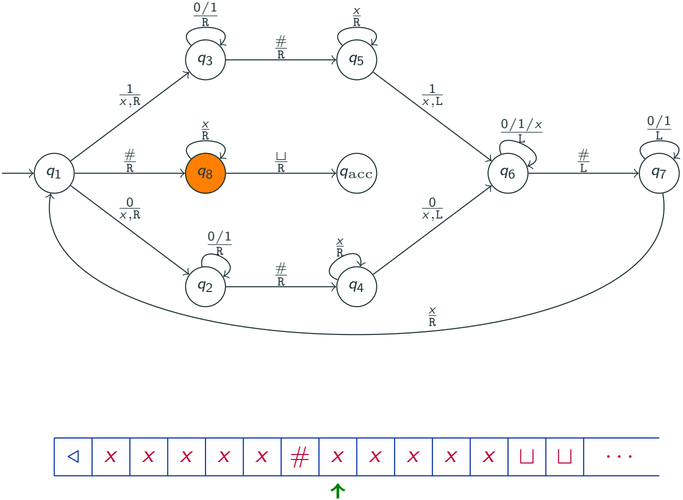
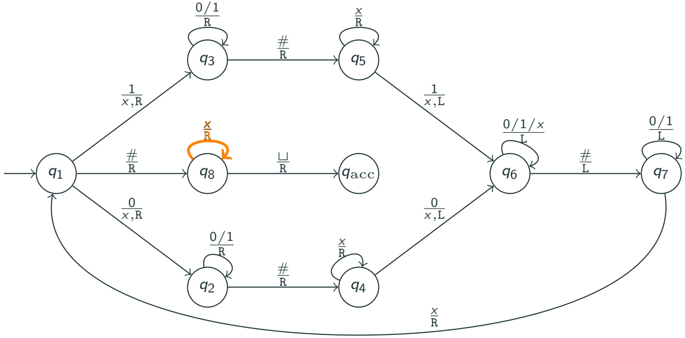
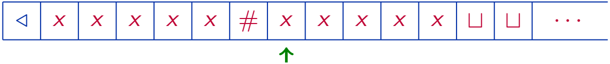


An example of a Turing machine on input 01101#01101

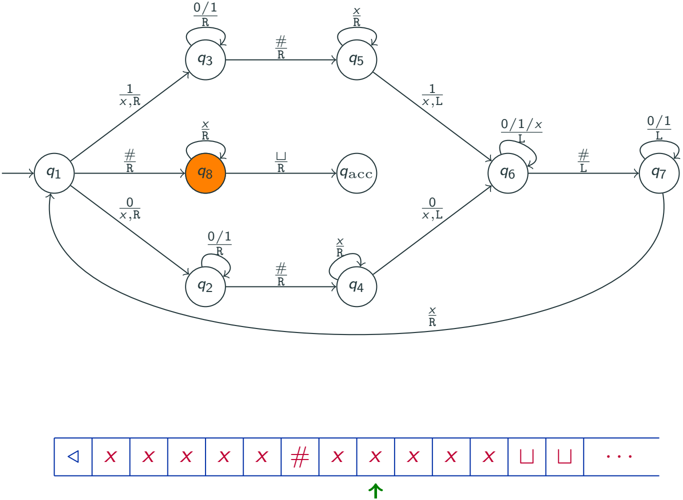


An example of a Turing machine on input 01101#01101

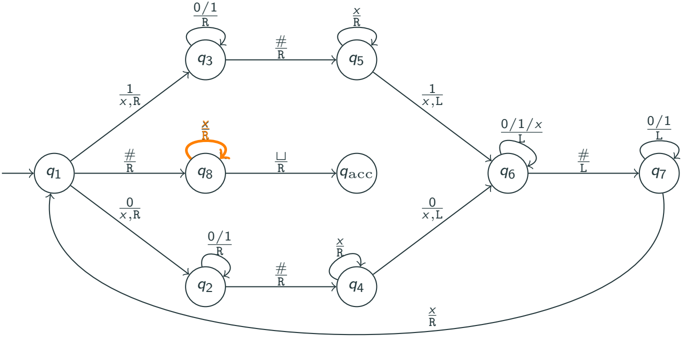


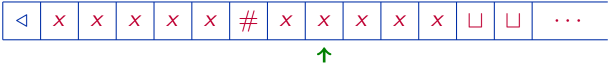


An example of a Turing machine on input 01101#01101

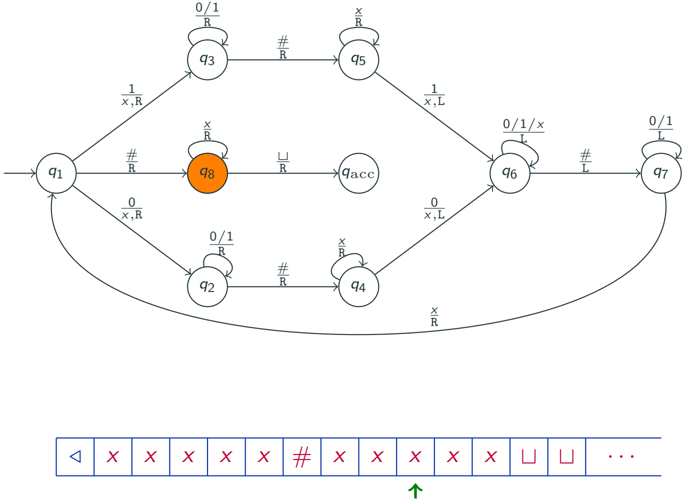


An example of a Turing machine on input 01101#01101

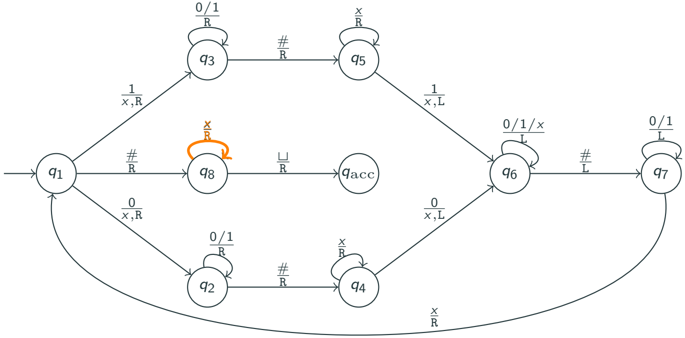




An example of a Turing machine on input 01101#01101

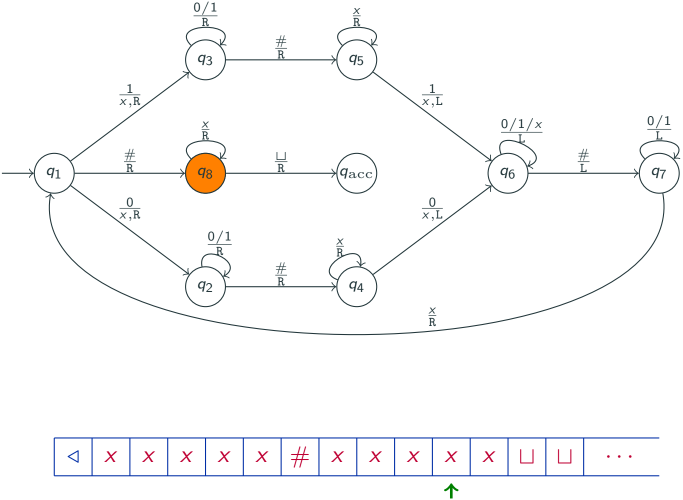


An example of a Turing machine on input 01101#01101

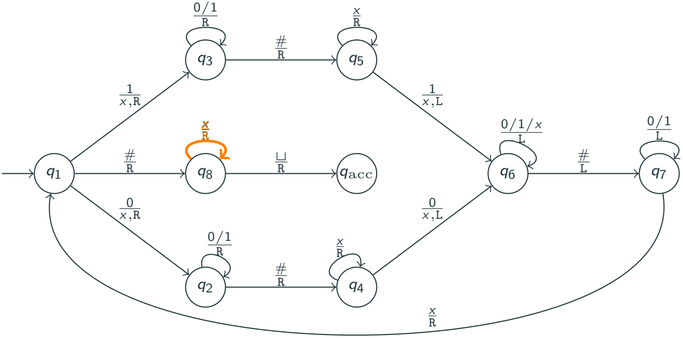


$\triangleleft$	x	x	x	x	x	$\#$	x	x	x	x	x	$\sqcup$	$\sqcup$	$\cdots$
									$\uparrow$					

An example of a Turing machine on input 01101#01101

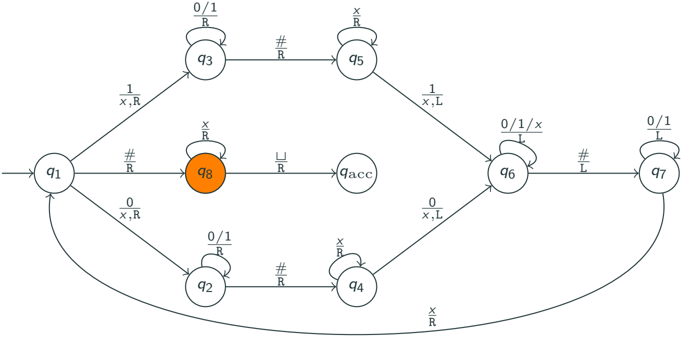


An example of a Turing machine on input 01101#01101

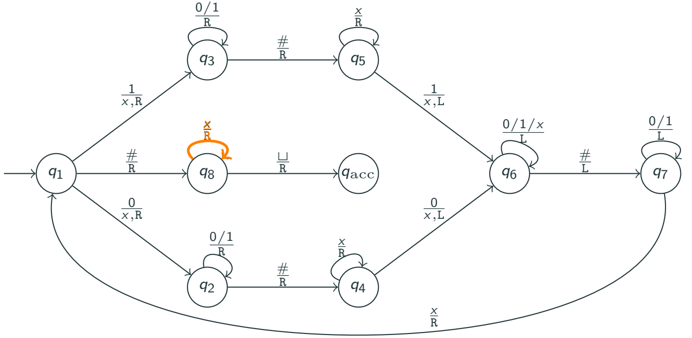


$\triangleleft$	x	x	x	x	x	#	x	x	x	x	x	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input 01101#01101

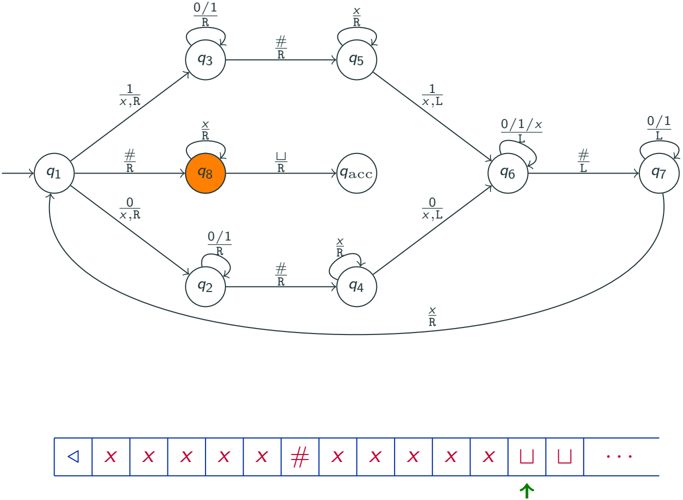


An example of a Turing machine on input 01101#01101

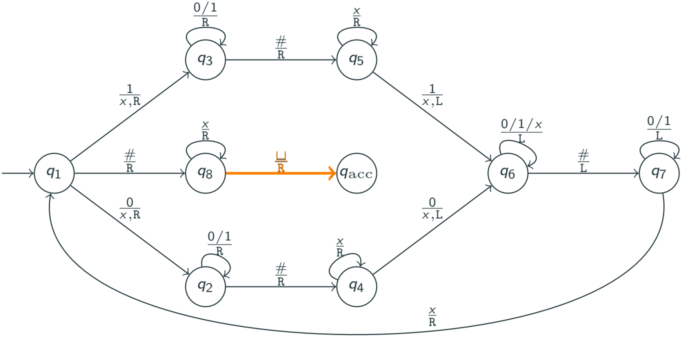


$\triangleleft$	x	x	x	x	x	$\#$	x	x	x	x	x	$\sqcup$	$\sqcup$	$\cdots$

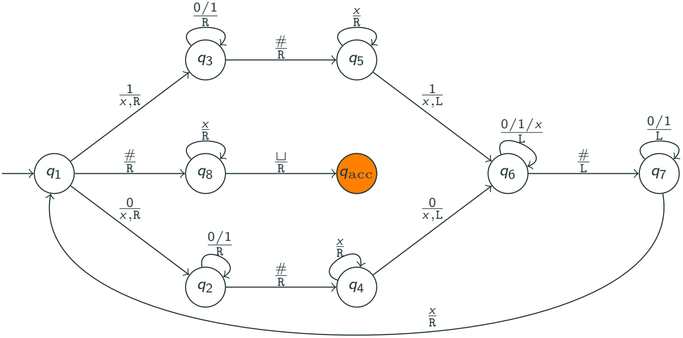
An example of a Turing machine on input 01101#01101



An example of a Turing machine on input 01101#01101

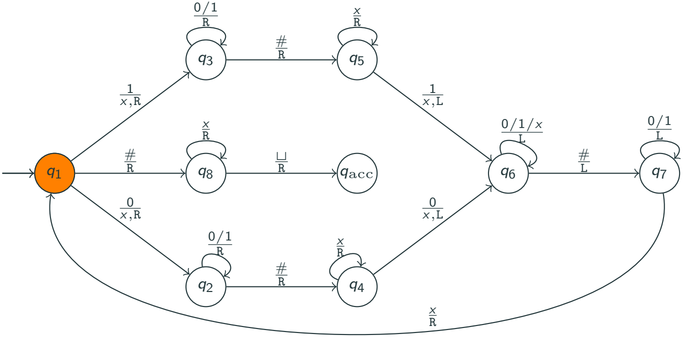


An example of a Turing machine on input 01101#01101



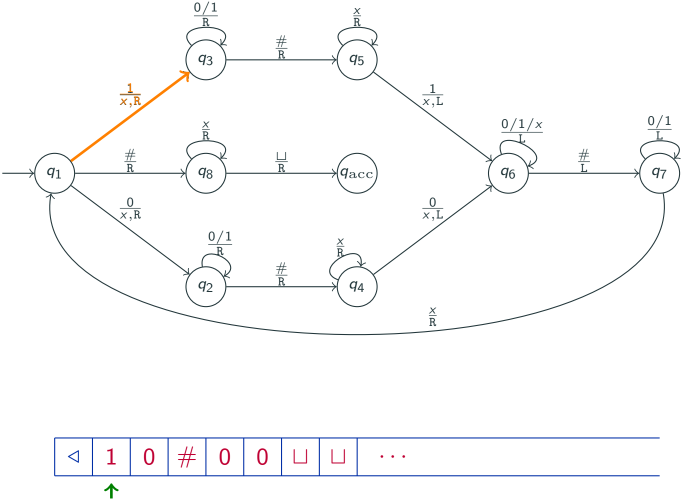
$\triangleleft$	x	x	x	x	x	$\#$	x	x	x	x	x	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input $10\#00$

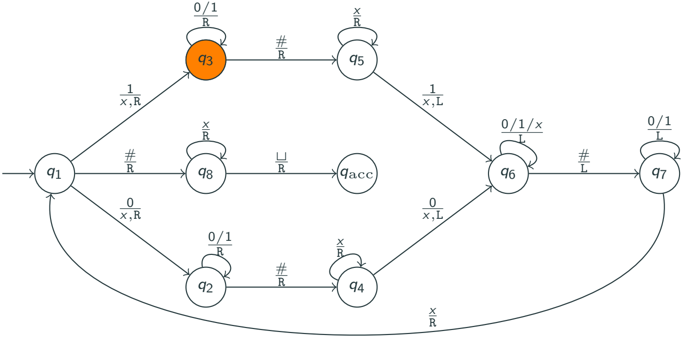


$\triangleleft$	1	0	#	0	0	$\sqcup$	$\sqcup$	$\cdots$

An example of a Turing machine on input $10\#00$

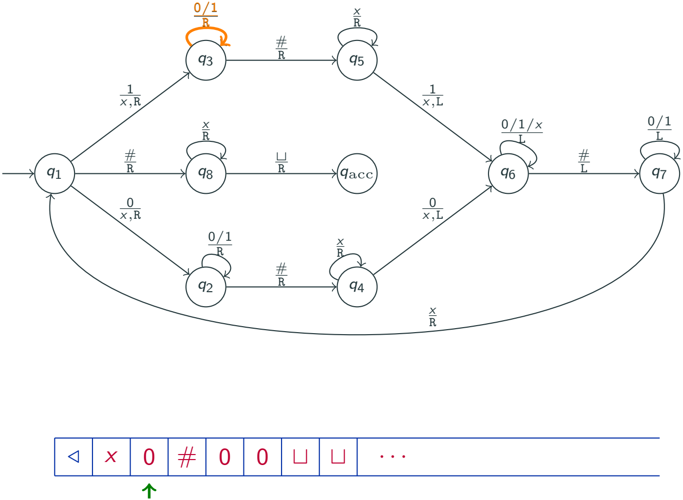


An example of a Turing machine on input $10\#00$

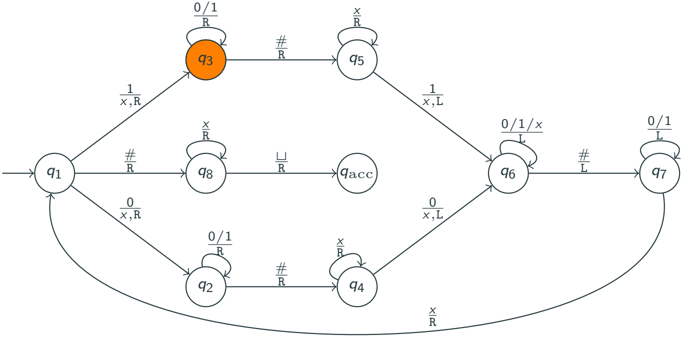


$\triangleleft$	x	0	#	0	0	$\sqcup$	$\sqcup$	$\cdots$

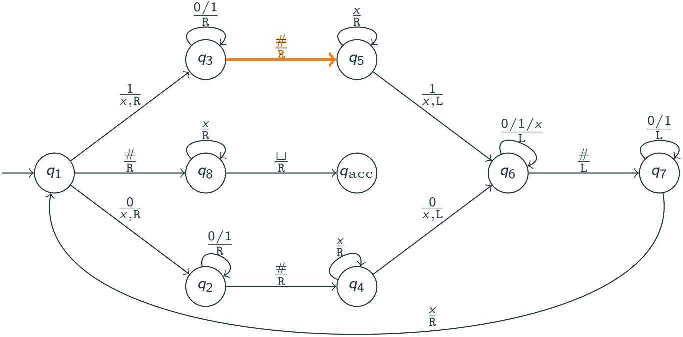
An example of a Turing machine on input $10\#00$



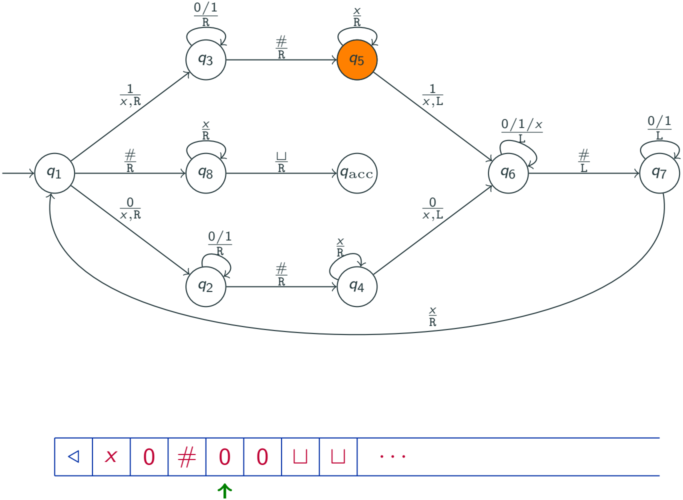
An example of a Turing machine on input $10\#00$



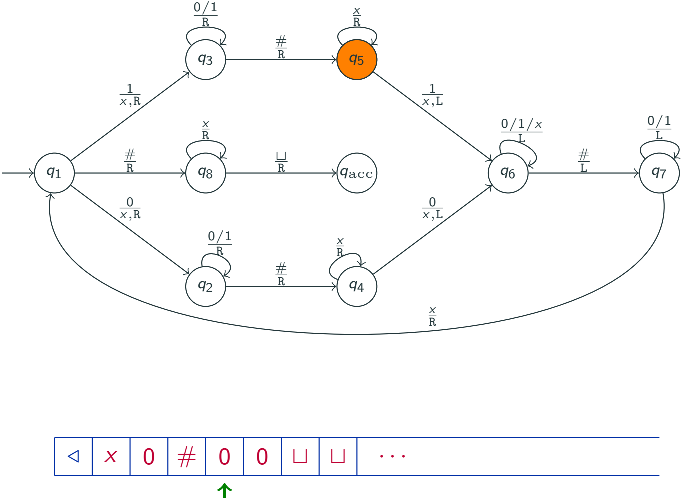
An example of a Turing machine on input $10\#00$



An example of a Turing machine on input $10\#00$

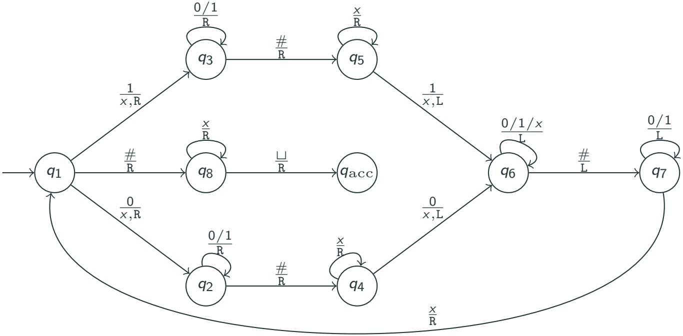


An example of a Turing machine on input $10\#00$



From state q_5, it enters q_{rej}.

An example of a Turing machine



This TM accepts the input word iff it is of the form: $w \# w$ where $w \in \{0,1\}^*$.

The algorithmic meaning of the TM

It works as follows:

$$\triangleleft \; \xleftarrow{\hspace{6em}} w_1 \xrightarrow{\hspace{6em}} \# \xleftarrow{\hspace{6em}} w_2 \xrightarrow{\hspace{6em}}$$

$\uparrow$

The algorithmic meaning of the TM

It works as follows:

$$\begin{array}{l} \triangleleft \;\longleftarrow\!\!\!-\!\!-\!\!-\; w_1 \;-\!\!-\!\!-\!\!\!\longrightarrow \# \longleftarrow\!\!\!-\!\!-\!\!-\; w_2 \;-\!\!-\!\!-\!\!\!\longrightarrow \\ \quad\;\uparrow \end{array}$$

- Read the "first" symbol of w_1, "mark" it and "remember" it.

The algorithmic meaning of the TM

It works as follows:

$$\triangleleft \quad \overset{0}{\longleftarrow}\!\!-\!\!-\!\!-\; w_1 \;\longrightarrow \quad \# \quad \longleftarrow\; w_2 \;\longrightarrow$$
$$\uparrow$$

- Read the “first” symbol of w_1, “mark” it and “remember” it.

The algorithmic meaning of the TM

It works as follows:

- Read the “first” symbol of w_1, “mark” it and “remember” it.

The algorithmic meaning of the TM

It works as follows:

$$\triangleleft \quad \overset{\cancel{0}}{\longleftarrow}\!\!\longleftarrow w_1 \longrightarrow \;\#\; \longleftarrow w_2 \longrightarrow$$

$$\uparrow$$

- Read the "first" symbol of w_1, "mark" it and "remember" it.
- Go to the "first" symbol of w_2 and check if it is the same (as the remembered symbol).

The algorithmic meaning of the TM

It works as follows:

$$\triangleleft \quad \cancel{0} \longleftarrow w_1 \longrightarrow \# \longleftarrow w_2 \longrightarrow$$

$$\uparrow$$

- Read the "first" symbol of w_1, "mark" it and "remember" it.
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$$\triangleleft \;\overset{\cancel{0}}{\longleftarrow}\!\!\longrightarrow w_1 \longrightarrow \;\#\; \longleftarrow w_2 \longrightarrow$$

$$\uparrow$$

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$$\triangleleft \quad \overset{\cancel{0}}{\longleftarrow\!\!-\!\!-\!\!-} \; w_1 \; \longrightarrow \quad \# \quad \longleftarrow \; w_2 \; \longrightarrow$$

$$\uparrow$$

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$$\triangleleft \quad \overset{\cancel{0}}{\longleftarrow\!\!-\!\!-\!\!-} \; w_1 \longrightarrow \# \longleftarrow w_2 \longrightarrow$$

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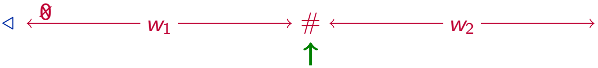

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The algorithmic meaning of the TM

It works as follows:

- Read the "first" symbol of w_1, "mark" it and "remember" it.
- Go to the "first" symbol of w_2 and check if it is the same (as the remembered symbol).
- If it is different, reject immediately.

The algorithmic meaning of the TM

It works as follows:

- Read the “first” symbol of w_1, “mark” it and “remember” it.
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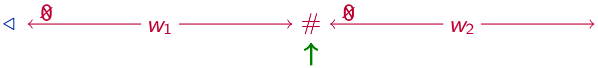
- If it is different, reject immediately.
- If it is the same, "mark" it.
- Repeat until there is no more "unmarked" symbol on both sides.
  The "first" symbol means the "first unmarked" symbol.

The algorithmic meaning of the TM

It works as follows:



- Read the "first" symbol of w_1, "mark" it and "remember" it.
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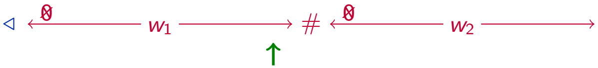

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The algorithmic meaning of the TM

It works as follows:

$$\triangleleft \quad \overset{\cancel{0}}{\longleftarrow\!\!-\!\!-} \; w_1 \; \longrightarrow \; \# \; \overset{\cancel{0}}{\longleftarrow\!\!-\!\!-} \; w_2 \; \longrightarrow$$

$$\uparrow$$

- Read the "first" symbol of w_1, "mark" it and "remember" it.
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- If it is different, reject immediately.
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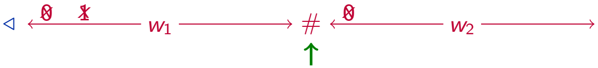

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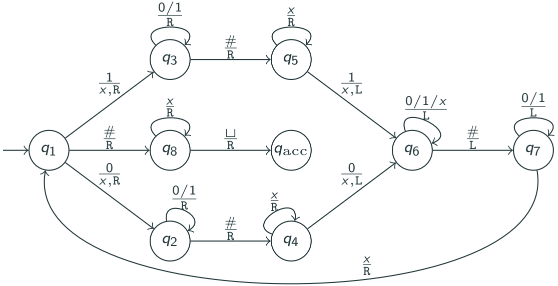
Comparison between the Turing machine and the algorithm, part 1



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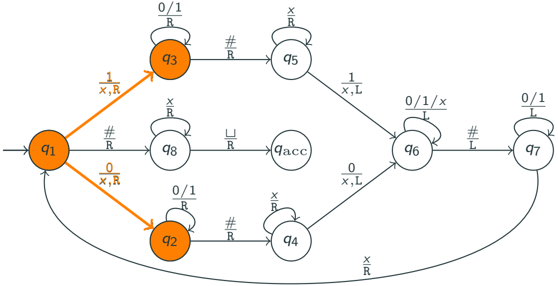

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It goes to q_2 to "remember" that the symbol is 0.

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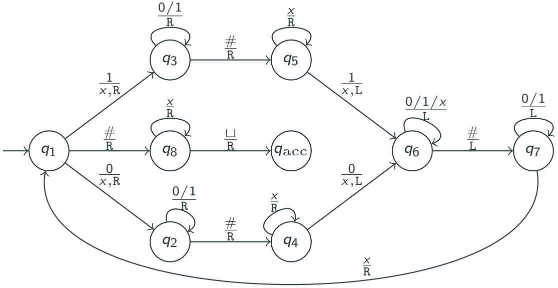
Comparison between the Turing machine and the algorithm, part 2



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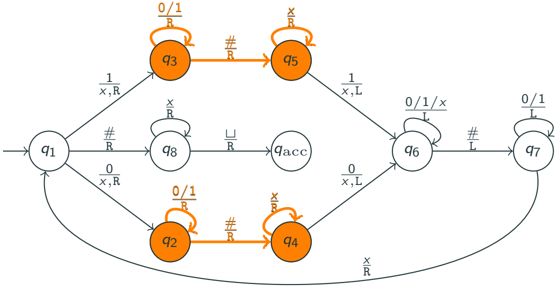

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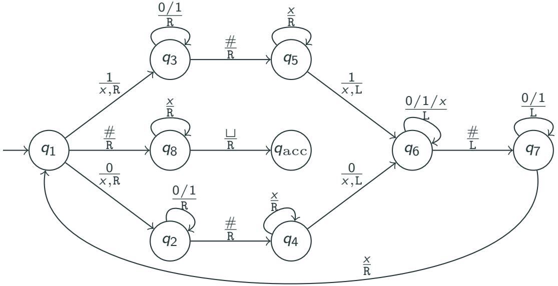

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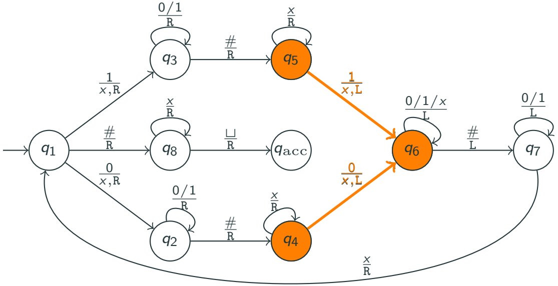

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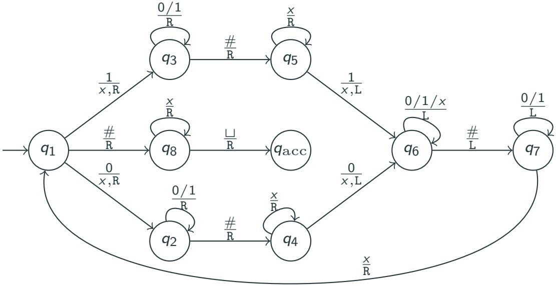

- Repeat until there is no more "unmarked" symbol on both sides.

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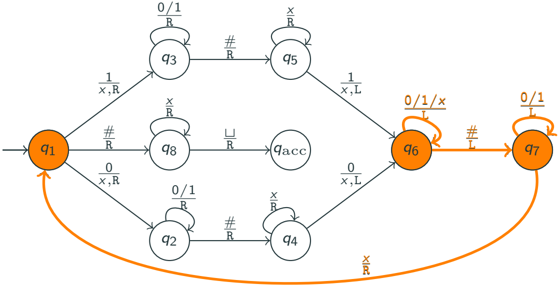

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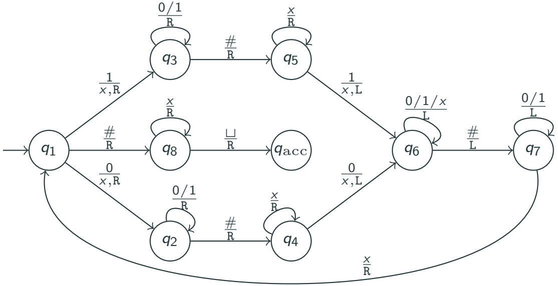

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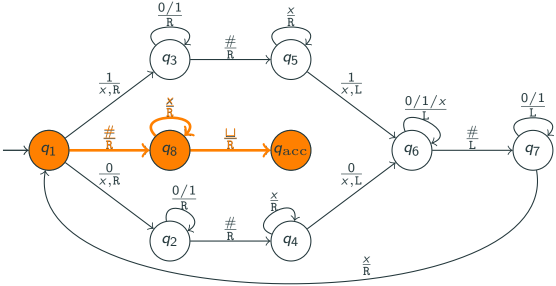

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When we want to describe a Turing machine for a certain problem, it is sufficient to describe an algorithm in some "reasonable" format/pseudo-code.

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Church-Turing thesis
Every "algorithm" is equivalent to a Turing machine.

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(Def.) A configuration of $\mathcal{M}$ is a string C over $Q \cup \Gamma$ which contains *exactly one symbol* from Q, i.e., a string of the form:

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(Note) We use the symbol p to indicate the state of the TM and the position of the head.

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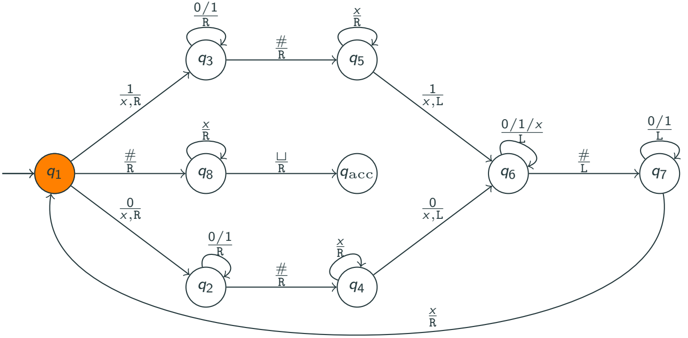
The run is finite when it ends with a halting configuration, i.e., when the TM reaches either q_{acc} or q_{rej}.

Example: The run of our TM on input 10#00



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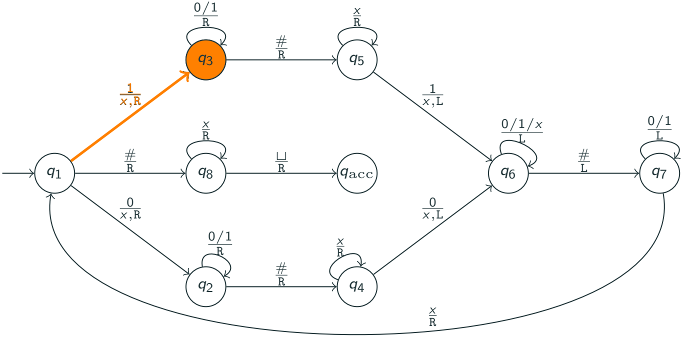

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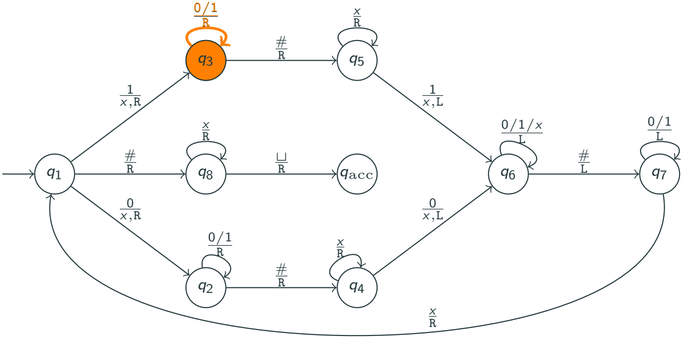

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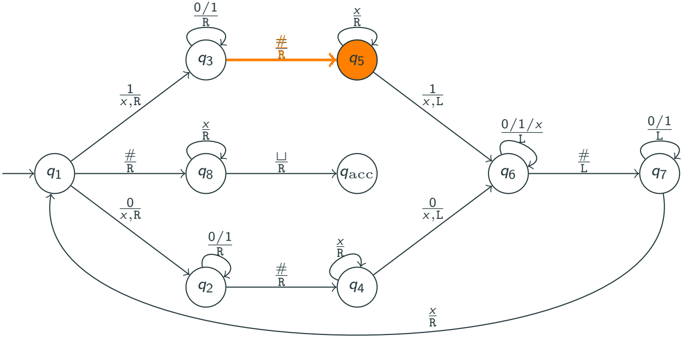

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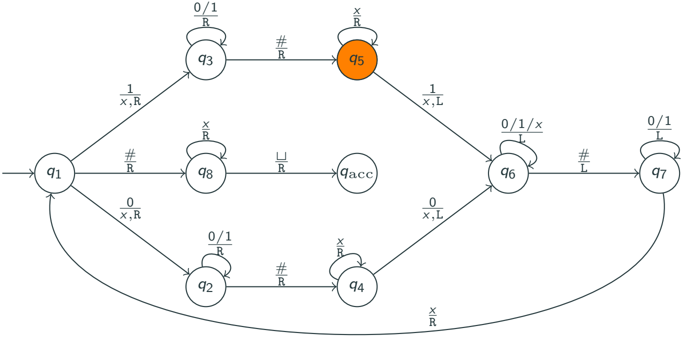

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Recognizable languages

(Def.) We say that $\mathcal{M}$ *recognizes* a language L, if for every input word w:

- if $w \in L$, then $\mathcal{M}$ accepts w,
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(Remark) Every decidable language is a recognizable language, but there is a recognizable language that is not decidable. (We will see this language in the next few weeks.)

End of Lesson 5