Lesson 0. Preliminary

CSIE 3110 – Formal Languages and Automata Theory

Tony Tan
Department of Computer Science and Information Engineering
College of Electrical Engineering and Computer Science
National Taiwan University

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1. Introduction
2. Some words about mathematical proofs
3. The halting problem in C++
4. The notion of alphabets and languages
5. Concluding remarks

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The most important information!

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- Lecture notes, slides and homework will be posted there.

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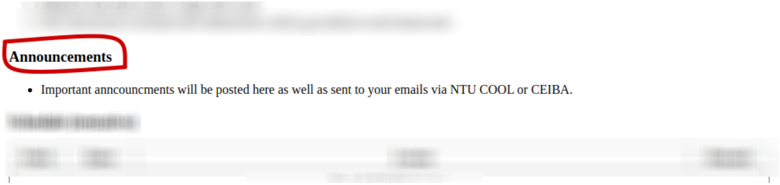
Pay special attention to the "Announcement" part:



Announcements

- Important anncouncments will be posted here as well as sent to your emails via NTU COOL or CEIBA.

Staff

Instructor:

- Tony Tan (陳偉松)
  `tonytan@csie.ntu.edu.tw`

TA:

- Lu Chia-Hsuan (呂佳軒)
  `r09922064@csie.ntu.edu.tw`
- Lu Yu-Cheng (呂侑承)
  `r109220304@csie.ntu.edu.tw`
- Mailing list: `automata@csie.ntu.edu.tw`.

Syllabus and schedule (can be found in the course website)

Week	Dates	Lesson	Remark
Part 0: Preliminaries			
I	27 Sep.	Lesson 0. Preliminaries	--
Part 1: Regular languages and context-free languages			
II	4 Oct.	Lesson 1. Finite state automata	--
III	11 Oct.	**Holiday (National day)**	--
IV	18 Oct.	Lesson 2. Regular expressions	--
V	25 Oct.	Lesson 3. Context-free languages	--
VI	1 Nov.	Lesson 4. Push-down automata	--
Reading week and midterm exam			
VII	8 Nov.	Reading week	--
VIII	15 Nov.	**Midterm exam:**Monday, 10:30-12:30, room: To be determined.	--
Part 2: Turing machines and some basic complexity classes			
IX	22 Nov.	Lesson 5. Turing machines and decidable languages	--
X	29 Nov.	Lesson 6. Turing machines and the notion of algorithm	--
XI	6 Dec.	Lesson 7. Universal Turing machines and halting problem	--
XII	13 Dec.	Lesson 8. Reducibility	--
XIII	20 Dec.	Lesson 9. Non-deterministic Turing machines	--
XIV	27 Dec.	Lesson 10. Basic complexity classes	--
Reading week and final exam			
XV	3 Jan.	Reading week	--
XVI	10 Jan.	**Final exam:** Monday, 10:30--12:30, room: To be determined.	--

How this course will be conducted

- The note for a lesson in a particular week will be posted in the course website a week before.

II	4 Oct.	Lesson 1. Finite state automata (tex, pdf, slides)

For example, for lesson 1 it will be posted 1 week before 4 October.

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  For now, we use Google meet.
  Check the announcement "Our Google meet link" in NTU COOL for the link.

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- Depending on the situation, we may be able to conduct the lesson in the physical class, but nothing is certain yet.

  It all depends on the future instruction from the university.

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- Since this course is supposed to be in English, the discussion is also in English.

Some details about videos

Usually each lesson will be divided into a few videos.

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For example, for lesson 1:

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There will be three videos, one video for each of the following topics.

- Deterministic finite state automata.
- Non-deterministic finite state automata.
- Pumping lemma.

Textbook, homework and exams

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- We are still deciding how to conduct the exams.

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Definitions, theorems, proofs[1]

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A **lemma** is a statement that we prove to assist in the proof of a theorem or another lemma.

A **corollary** (of a theorem) is a statement that follows easily from a theorem or its proof.

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Formally we can view a proof as a (finite) sequence of statements:

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such that each statement s_i is either an assumption or it can be trivially deduced from earlier statements $s_1, \ldots, s_{i-1}$.

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statement_n is the theorem/lemma that we want to prove.

Some examples of simple deductions

Example 1:

$$\begin{array}{l} \text{If it is raining, John stays at home.} \\ \text{John is not at home today.} \\ \hline \therefore \text{ It is not raining today.} \end{array}$$

Some examples of simple deductions

Example 1:

If it is raining, John stays at home.
John is not at home today.

$\therefore$ It is not raining today.

Example 2:

If it is sunny, John goes to the beach.
When John is at the beach, he swims in the sea.
It is sunny today.

$\therefore$ Today John swims in the sea.

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See Appendix B for some other examples of deductions.

Types of proofs

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Types of proofs

Types of proofs that normally occur in this course.[2]

- Proofs by construction.
- Proofs by contradiction.
- Proofs by induction.

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Tips to write proofs[3]

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Be patient. If you don't see how to do it right away, don't worry. Researchers sometimes work for weeks or even years to find a single proof.

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> *Genius is the patience of thoughts, concentrated in a certain direction.*
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> — *Isaac Newton*

Come back to it. Look over the statement you want to prove, think about it a bit, leave it, and then return a few minutes or hours later. Let the unconscious, intuitive part of your mind have a chance to work.

[3] Taken from Chapter 0 in Sipser's textbook.

Tips to write proofs[4]

Be neat! When you are building your intuition for the statement you are trying to prove, use simple, clear pictures and/or text. You are trying to develop your insight into the statement, and sloppiness gets in the way of insight. Furthermore, when you are writing a solution for another person to read, neatness will help that person understand it.

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Be concise. Brevity helps you express high-level ideas without getting lost in details. Good mathematical notation is useful for expressing ideas concisely. But be sure to include enough of your reasoning when writing up a proof so that the reader can easily understand what you are trying to say

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Tips to write proofs[5]

Be patient.

Come back to it.

Be neat!

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After you have written down your proof....[6]

Reread, reconsider and reexamine your proof, even after you are convinced that your proof is correct.

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Doing so could consolidate your knowledge and develop your problem solving skill.

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After you have written down your proof....[6]

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Doing so could consolidate your knowledge and develop your problem solving skill.

Ask the following questions.

- What is the main idea of the proof?
- Can the proof be simplified?
- Can the result be derived differently?
- Can the result/method be used for some other problem?
- How does it relate to other results that you know?

All these obviously take time and energy, but worth the effort.

[6] Adopted from the book *How to solve it* by George Pólya.

Some references

On general problem solving skill:

- *How to Solve it* by George Pólya.
- *Princeton Companion to Mathematics*, part VIII. *Final Perspectives*, Timothy Gowers, editor.

On the more technical side:

- *Mathematical discovery on understanding, learning, and teaching problem solving* (volumes I and II) by George Pólya.
- *Solving mathematical problems: A personal perspective* by Terrence Tao.

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An example of impossible problem for computer

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Consider the following problem denoted by **Problem-A**.

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Input: Two files:
The first file is a C++ program, denoted by `file-1.cpp`.
The second file is a file with arbitrary extension, denoted by `file-2`.

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(Important!) Notice how we define **Problem-A**.

It is clear what the input and output should be!

This is not acceptable:

- We want to decide if a C++ program output `True` on an input.
- We want to decide the outcome of a C++ program on an input.
- **Problem-A** is to determine the outcome of a C++ program on an input.

A little analysis: What's wrong with these specifications?

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Problem-X(`P.cpp`)

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Task: Output `True`, if the C++ program `P.cpp` returns `True` on input `input-file`. Otherwise, output `False`.

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Problem-Y(`input-file`)

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Problem-Z(`P.cpp`, `input-file`)

Input: Nothing.
Task: Output `True`, if the C++ program `P.cpp` returns `True` on input `input-file`. Otherwise, output `False`.

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All of them are very different from our original **Problem-A**.

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For example, we can use this format:

Problem XYZ	
Input:	...
Task:	...

and

Algorithm XYZ	
Input:	...
Task:	...

Other format is also acceptable as long as the input and output is clear.

Coming back to Problem-A

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Note that it requires that on <u>every</u> input `file-1.cpp` and `file-2`, it should output `True` or `False`.

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We would like to show:

Theorem 0.1

There is no C++ program for **Problem-A***.*

In other words, it is impossible to write a C++ program for **Problem-A**.

Reductions....

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So, we reduce it to:

Problem-B

Input: One files: A C++ program, denoted by `file.cpp`.

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Note that if there is a C++ program for **Problem-A**, then there is a C++ program for **Problem-B**.

Reductions....

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Note that if there is a C++ program for **Problem-A**, then there is a C++ program for **Problem-B**.

In some sense, **Problem-A** is more "general" than **Problem-B**.

Reductions....

Problem-B

Input: One files: A C++ program, denoted by `file.cpp`.

Task: Output <u>`True`</u>, if the C++ program file.cpp returns `True` when its input is the content of `file.cpp` itself.

Otherwise, output <u>`False`</u>.

Reductions....

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Still we don't know how to show that there is no C++ program for **Problem-B**.

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So, we consider the following problem:

Problem-C

Input: One files: A C++ program, denoted by `file.cpp`.

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Reductions....

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Note that the output of **Problem-B** is just the complement of the output of **Problem-C**.

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Still we don't know how to show that there is no C++ program for **Problem-B**.

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Note that the output of **Problem-B** is just the complement of the output of **Problem-C**.

There is a C++ program for **Problem-B**, if and only if there is a C++ program for **Problem-C**.

Mathematical proof that there is no C++ program for Problem-C

Problem-C

Input: One files: A C++ program, denoted by `input.cpp`.

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The proof is by contradiction. Suppose there is a C++ program for **Problem-C**, which we denote by `myprog.cpp`.

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  Since `myprog.cpp` is a program for **Problem-C**, by definition of **Problem-C**, `myprog.cpp` does not return `True` on `myprog.cpp` itself. A contradiction.

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Now we run `myprog.cpp` with input `myprog.cpp` itself. There are two cases.

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- The output is `False`.
  Since `myprog.cpp` is a program for **Problem-C**, by definition of **Problem-C**, `myprog.cpp` returns `True` on `myprog.cpp` itself. A contradiction.

Therefore, there is no such C++ program `myprog.cpp` for **Problem-C**.

Proof of Theorem 0.1

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Since there is no C++ program for **Problem-B**, there is no C++ program for **Problem-A**.

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Thus, we complete the proof of Theorem 0.1.

Theorem 0.1

There is no C++ program for **Problem-A***.*

Proof of Theorem 0.1

Since there is no C++ program for **Problem-C**, there is no C++ program for **Problem-B**.

Since there is no C++ program for **Problem-B**, there is no C++ program for **Problem-A**.

Thus, we complete the proof of Theorem 0.1.

Theorem 0.1

There is no C++ program for **Problem-A**.

Problem-A is usually known as the "Halting" problem.

We will come back to it again when we discuss Turing machines in the next few months.

Table of contents

1. Introduction

2. Some words about mathematical proofs

3. The halting problem in C++

4. The notion of alphabets and languages

5. Concluding remarks

Language theoretic terminology

In this course we assume familiarity with basic terminology from discrete mathematics, especially set-theoretic terminology.

See Appendix A in Note 0 for some that we will often use.

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(Def.) We write ε to denote the *empty string/word*, i.e., the word of length 0.

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(Def.) Σ^* denotes the set of all *finite* words over Σ, i.e., $\Sigma^* = \bigcup_{n \geqslant 0} \Sigma^n$.

(Def.) A *language* L over Σ is a subset of Σ^*.

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- $\{w \in \Sigma^* | \text{the length of } w \text{ is a prime number}\}$ is a language over Σ.

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Concluding remarks

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4. The notion of alphabets and languages.

   It will be used throughout the course.

End of Lesson 0