Homework 1

Due on Friday, 10:30 am, 24 March 2023 (112/03/24)

Question 1 (5 points). Prove the second statement of the Padding theorem (Theorem 1.2 in Note 1), i.e., if **NP** = **coNP**, then **NEXP** = **coNEXP**.

Question 2 (5 points). Prove that the two definitions of **NP** in Note 2 are indeed equivalent. That is, prove that for every language L, L is in **NP** in the sense of Def. 2.1 if and only if L is in **NP** in the sense of Def. 2.2.

Question 3 (5 points). Consider the proof of Theorem 2.8 in Note 2. Is the reduction from SAT to 3-SAT parsimonious? Justify your answer. If you think the reduction is not parsimonious, can you supply a parsimonious reduction from SAT to 3-SAT?

Question 4 (5 points). Prove Theorem 2.10 in Note 2. That is, for every language K over the alphabet Σ, K is **NP**-complete if and only if its complement $\overline{K}$ is **coNP**-complete, where $\overline{K} \stackrel{\mathsf{def}}{=} \Sigma^* - K$.

Question 5 (5 points). Prove that $\mathbf{P} = \mathbf{NP}$ if and only if there is a polynomial time DTM for the following problem.

FIND-SOL	
Input:	A propositional formula φ in CNF.
Task:	Output a satisfying assignment for φ, if it exists. Otherwise, output 0.

Note that FIND-SOL is *not* an **NP**-complete problem. Recall that **NP**-complete problems are defined only on "decision" problems, i.e., determining whether a word w belongs to a certain language L.

Question 6 (5 points). Prove that if there is a unary language L that is **NP**-hard, then SAT $\in \mathbf{P}$, and hence, $\mathbf{P} = \mathbf{NP}$.

Def: A language L is a unary language, if $L \subseteq \{1\}^*$, i.e., every word $w \in L$ contains only 1.