Homework 1
Due on Tuesday, 1:30 pm, 30 March 2021 (110/03/30)

Question 1 (1 point). Consider the proof of Theorem 1.6.
• Prove that the function $H$ is non-decreasing, i.e., for every $n \leq n'$, $H(n) \leq H(n')$.
• Prove that Algorithm 1 runs in time $O(n^3)$.

Question 2 (2 points). Give a parsimonious (polynomial time) reduction from SAT to 3-SAT.

Question 3 (2 points). Prove that $P = NP$ if and only if there is a polynomial time DTM for the following problem.

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<th>FIND-SOL</th>
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<td><strong>Input:</strong> A propositional formula $\varphi$ in CNF.</td>
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<td><strong>Task:</strong> Output a satisfying assignment for $\varphi$, if it exists. Otherwise, output 0.</td>
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Question 4 (2 points). Prove that if there is a unary language $L$ that is $NP$-hard, then $SAT \in P$, and hence, $P = NP$.
Def: A language $L$ is a unary language, if $L \subseteq \{1\}^*$, i.e., every word $w \in L$ contains only 1.

Question 5 (2 points). Consider the following language CYCLE.

$\text{CYCLE} \overset{\text{def}}{=} \{G : G$ is a directed graph and it contains a cycle$\}$

• Prove that $\text{CYCLE}$ is $\text{NL}$-complete.
• Give a logarithmic space NTM for $\text{CYCLE}$.
  Here $\text{CYCLE}$ is the complement of $\text{CYCLE}$, defined as follows.

$\text{\overline{CYCLE}} \overset{\text{def}}{=} \{G : G$ is a directed graph and does not contain cycle$\}$

Question 6 (1 point). Consider the following language $K$.

$K \overset{\text{def}}{=} \{(M, w, 1^n) : M$ is a DTM that accepts $w$ in space $n\}$

Prove that $K$ is $\text{PSPACE}$-complete. Is its complement $\overline{K}$ $\text{PSPACE}$-complete?