Homework 1

Due on Monday, 10:30 am, 26 October 2020 (2020/10/26)

Name : 
Student ID : 

Note:

1. Write down clearly your name and student ID in the space above.
2. There are FIVE questions altogether.
3. Write your solution for each question in the space provided.
4. Submit your solution before the lesson on 26 October 2020. If you want to submit it earlier, you can slip it under the door of my office.
Question 1 (2 points). In the following, the alphabet is $\Sigma = \{a, b\}$. Construct a DFA/NFA for each of the following languages.

- $L_1 := \{a^{2n} \mid n \geq 0\}$.
- $L_2 := \{w \mid w \text{ contains } abb\}$.
  
  For example, the word $bb$ and $abab$ are not in $L_2$, because they do not contain $abb$. On the other hand, $aabbab$, $abb$ and $abbabb$ are in $L_2$, because they contain $abb$.

- $L_3 := \Sigma^* - L_1$.
- $L_4 := \Sigma^* - L_2$.

For this solution, your DFA/NFA can only have up to 4 states. You don’t need to prove your DFA/NFA is correct.

Solution for question 1.
Solution for question 1.
Question 2 (2 points). Construct regular expressions for each of the languages $L_1$-$L_4$ in question 1.

Solution for question 2.
Question 3 (2 points). Construct CFG for each of the following languages.

- \( L_5 := \{ a^n b^n \mid n \geq 1 \} \).
- \( L_6 := \{ a^m b^n \mid m \neq n \} \).

You don’t need to prove that your CFG is correct, but too complicated grammars will be considered wrong.

Solution for question 3.
Question 4 (2 points). Let $L_7$ be the following language.

$$L_7 := \{a^n \mid n \text{ is a perfect square}\}.$$ 

For example, $\epsilon, a^4, a^9$ all belong to $L_7$, since 0, 4, 9 are all perfect square, i.e., $0 = 0^2$, $4 = 2^2$ and $9 = 3^2$. On the other hand, $a^5$ and $a^8$ do not belong to $L_7$, since the square roots of 5 and 8 are not integers.

Prove that $L_7$ is not CFL.

Solution for question 4.
**Question 5 (2 points).** For a language $L$, define the *square root of $L$*, denoted by $\text{SQRT}(L)$, as follows.

$$\text{SQRT}(L) := \{ u \mid \text{there is } v \in L \text{ such that } |u|^2 = |v| \}.$$ 

Prove that if $L$ is regular, then $\text{SQRT}(L)$ is regular.

**Solution for question 5.**
Solution for question 5.