Extra exercises for review week on 26 October 2020

Question 1. Give the DFA/NFA/regex for each of the following languages. Here the alphabet is \( \Sigma = \{a, b\} \).

(a) \( L_1 := \{w \mid w \text{ starts with } b \text{ and ends with } a\} \).
(b) \( L_2 := \{w \mid w \text{ contains } aba\} \).
   For example: \( aba \) and \( bbabab \) are in \( L_2 \), since they contain \( aba \). On the other hand, \( bbaabb \) and \( babbabba \) are not in \( L_2 \), since they do not contain \( aba \).
(c) \( L_3 := \Sigma^* - L_2 \).
(d) \( L_4 := \{w \mid \text{if } w \text{ contains } ab \text{ then } w \text{ ends with } bb\} \).

Question 2. Construct the CFG for each of the following languages.

(a) \( L_5 := \{ a^n b^n \mid n \geq 1 \} \).
(b) \( L_6 := \{ a^n x b^n \mid x \in \Sigma^* \text{ and } n \geq 1 \} \).
(c) \( L_7 := \{ a^n b^m a^m b^m \mid n, m \geq 1 \} \).
(d) \( L_8 := \{w \mid w \in \{a, b\}^* \text{ is a palindrome}\} \).
(e) \( L_9 \) consists of the words \( w \in \{a, b\}^* \) where the number of \( a \)'s is the same as the number of \( b \)'s.
(f) \( L_{10} \) consists of the words \( w \in \{a, b\}^* \) where the number of \( a \)'s is the twice as many as the number of \( b \)'s.

Question 3. Prove that each of the following languages is not regular.

(a) \( L_{11} = \{a^n b^m a^n \mid m, n \geq 0 \} \).
(b) \( L_{12} = \{a^n b^m a^m b^n \mid n, m \geq 1 \} \).
(c) \( L_{13} = \{w \mid w \in \{a, b\}^* \text{ is not a palindrome}\} \).
(d) \( L_{14} = \{wxw \mid w, x \in \{a, b\}^* \} \).

Are they CFL?

Question 4. Consider the language \( L := \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1, \text{ then } j = k\} \).

(a) Prove that \( L \) is not regular.
(b) Show that \( L \) acts like a regular language in the pumping lemma. In other words, there is an integer \( N \) such that \( L \) satisfies the conditions of the pumping lemma.
(c) Explain why (a) and (b) above do not contradict pumping lemma.

Question 5. For a language \( L \) and integer \( k \geq 1 \), define the following operators.

\[
\text{HALF}(L) := \{u \mid \text{there is } v \text{ such that } |u| = |v| \text{ and } uv \in L\}.
\]

\[
\text{k-ROOT}(L) := \{u \mid u \text{ is a prefix of word } v \in L \text{ such that } |u|^k = |v|\}.
\]

Prove the following.

(a) If \( L \) is regular, then \( \text{HALF}(L) \) is regular.
(b) If \( L \) is regular, then for every integer \( k \geq 1 \), \( \text{k-ROOT}(L) \) is regular.