Homework 4

Due on Tuesday, 10:59 am, 31 December 2019 (2019/12/31)

Name : 
Student ID : 
Class no : Class 1 (Friday) or Class 2 (Tuesday)

Note:

1. Write down clearly your name and student ID in the space above.
2. Circle the class where you belong.
   - If you belong to class 1, circle “Class 1 (Friday)”.
   - If you belong to class 2, circle “Class 2 (Tuesday)”.
3. There are FIVE questions altogether.
4. Write your solution for each question in the space provided.
5. Submit your solution in the appropriate box outside room 516 according to your class.
   If you want to submit it before Tuesday, you can slip it under the door.
(2 points) Question 1. Let $L$ be a language over the alphabet $\Sigma = \{0, 1\}$. Let $u \in \Sigma^*$ be a word.

Consider the following “algorithm” $A$ which is defined in terms of $L$ and $u$:

**INPUT:** $w$.
- If $u \in L$, ACCEPT.
- If $u \notin L$, REJECT.

Note that $L$ is not necessarily a decidable language. In fact, $L$ can be any language.

(a) Can $A$ be considered an algorithm in the sense that there is a Turing machine that is equivalent to it?

(b) What is $L(A)$? Is $L(A)$ decidable?

Here $L(A)$ is defined as the language $\{ w \mid A$ accepts $w \}$.

Solutions for Question 1.
(2 points) Question 2. Recall the language \( \text{HALT} := \{ [M]w \mid M \text{ accepts } w \} \). Let \( L \) be a decidable language. Prove that \( L \leq_m \text{HALT} \).

Solution for question 2.
(2 points) Question 3. Consider the following problem.

<table>
<thead>
<tr>
<th>CFL-REG</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> A CFG $G$ and a DFA $A$</td>
</tr>
<tr>
<td><strong>Task:</strong> Output True, if $L(G) = L(A)$. Otherwise, output False.</td>
</tr>
</tbody>
</table>

Prove that CFL-REG is undecidable.

Solution for Question 3.
(2 points) Question 4. Prove that the class $\text{coNP}$ is closed under union and intersection. That is, prove the following.

- If $L_1, L_2 \in \text{coNP}$, then $L_1 \cup L_2 \in \text{coNP}$.
- If $L_1, L_2 \in \text{coNP}$, then $L_1 \cap L_2 \in \text{coNP}$.

Hint: Use the definition of $\text{coNP}$ and de Morgan’s law.

Solution for Question 4.
(2 points) Question 5. Consider the following problem.

<table>
<thead>
<tr>
<th>CFL-Reversal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input: A CFG $G$.</td>
</tr>
<tr>
<td>Task: Output True, if $L(G) = L(G)^r$, i.e., $L(G)$ is closed under reversal. Otherwise, output False.</td>
</tr>
</tbody>
</table>

Prove that CFL-Reversal is undecidable.

Note: $L(G)$ is closed under reversal, if for every word $w \in L(G)$, $w^r \in L(G)$, where $w^r$ denotes the reverse of $w$.

Solution for Question 5.
Solution for Question 5 (continued).