Homework 1
Due on Tuesday, 5:00 pm, 15 October 2019 (2019/10/15)

Name :
Student ID :

Note:
1. Write down clearly your name and student ID in the space above.
2. There are FOUR questions altogether.
3. Write your solution for each question in the space provided.
4. Submit your solution in the appropriate box outside room 516 according to your class.
   If you belong to class 1, put it in box 1. If you belong to class 2, put it in box 2.
(2.5 points) Question 1. Construct an NFA for each of the following languages.

(a) The language $L_1$ that consists of all the words in which $b$ appears at least twice.

(b) The language $L_2$ that consists of all the words that starts with $bb$ and ends with $ab$.

(c) The language $L$ that consists of all the words that contains $aba$.
   For example: $aba$ and $ababaa$ are in $L$, since they contain $aba$. On the contrary, $aaaaa$ and $abbabbabb$ are not in $L$, since they do not contain $aba$.

(d) The language $L$ that consists of all the words that do not contain $bb$.

(e) The language $L$ that consists of all the words $w$ such that if $w$ contains $bb$, then $w$ ends with $ab$.

Solutions for question 1.
Solutions for question 1.
(2.5 points) Question 2. Construct the regular expression for each of the languages above.

Solutions for question 2.
(2 points) Question 3. A string $w \in \{0, 1\}^*$ represents an integer in a standard way. For example, the string 000 represents the integer 0, and so do 0 and 000000. The string 00100 and 100 both represent the integer 4.

Construct a DFA for the following language over the alphabet $\{0, 1\}$:

$$L_0 := \{ w \mid w \text{ represents an integer divisible by 3}\}$$

Hint: Consider $(2i + j) \mod 3$, for every $i \in \{0, 1, 2\}$ and $j \in \{0, 1\}$.

Solution for question 3.
Question 4. For a language $L \subseteq \Sigma^*$ (not necessarily regular), we define the equivalence relation
$\sim_L$ on $\Sigma^*$, where $u \sim_L v$ if for every $w \in \Sigma^*$, $uw \in L$ if and only if $vw \in L$.
Equivalently, we can say that $u \sim_L v$, if one of the following holds.

- Both $uw$ and $vw$ are in $L$.
- Both $uw$ and $vw$ are not in $L$.

(a) (1 points) Prove that $\sim_L$ is an equivalence relation.

(b) (2 points) In the following, let $\#(\sim_L)$ (read: the index of $\sim_L$) denote the number of
equivalence classes of $\sim_L$.
Prove that $L$ is a regular language if and only if $\#(\sim_L)$ is finite.

Solutions for question 4.
Solutions for question 4.