Midterm exam: 18:00–21:00, Monday, 5 November 2018

Instructions

• This is a closed book exam.
• Write down your name and student number clearly.
• Write down your solutions clearly.
• There are FIVE questions altogether.
• Discussions/collaborations are NOT allowed.
• All electronic devices must be switched off during the exam.
• You don’t need to do the questions in the same order as written here.
• You can use any result discussed, or stated in the class or in the homework.
• However, if you use results never stated in the class or in the homework before, you must supply their complete proofs.
• You can also freely use pumping lemma (for both regular languages and CFL).
In the following, the alphabet is always $\Sigma = \{a, b\}$.

**Question 1. [2 points]** Construct the NFA/regex for each of the following languages.

(a) $L_1 = \{a^{2n} \mid n \geq 1\}$.
(b) $L_2 = \{w \mid$ every $a$ in $w$ is followed immediately by $b\}$.
(c) $L_3 = \{w \mid w$ contains three consecutive $a$’s$\}$.
(d) $L_4 = \{w \mid w$ does not contain three consecutive $a$’s$\}$.

You don’t need to prove your NFA/regex is correct, but too complicated solutions will be considered wrong.

**Question 2. [2 points]** Prove or disprove the following for every regular expression $r$ and $s$.

(a) $L(r^* \cup s^*) = L((r \cup s)^*)$.
(b) $L((r^* \cdot s^*)^*) = L((r \cdot s)^*)$.
(c) $L((r^* \cup s)^* ) = L((r \cup s)^*)$.
(d) $L((r^* \cup s)^* ) = L((r \cup s)^*)$.

**Question 3. [2 points]** Construct the CFG for the following languages.

(a) $K_1 = \{a^nb^n \mid n \geq 1\}$.
(b) $K_2 = \{a^nxb^n \mid x \in \Sigma^* \text{ and } n \geq 1\}$.
(c) $K_3 = \{a^nb^n a^m b^m \mid n, m \geq 1\}$.
(d) $K_4 = \{a^na^mb^n b^n \mid m, n \geq 1\}$.

You don’t need to prove your CFG is correct, but too complicated grammars will be considered wrong.

**Question 4. [2 points]** A grammar $G = \langle \Sigma, V, R, S \rangle$ is called left-linear, if each of its rule in $R$ is in one of the following forms:

$$A \rightarrow aB,$$

$$A \rightarrow a,$$

where $A, B$ are variables and $a$ is a terminal. That is, on the right hand side of a rule, there is one terminal and at most one variable. Moreover, if a variable appears, it appears at the end. A language $L$ is called left-linear, if it can be generated by a left-linear grammar. Prove that every left-linear language is regular.

**Question 5. [2 points]** Prove that if $L$ is regular and $K$ is CFL, then $L \cap K$ is CFL.