Homework 4: due 18:30, Thursday, 11 January 2018

(1) [1 point] Define a language $L$ to be $\text{coNLog}$-complete, if $L \in \text{coNLog}$ and for every language $L' \in \text{coNLog}$, $L' \leq_{\text{log}} L$.

Prove that the language REACH defined in lesson 14 is also $\text{coNLog}$-complete.

(2) [2 points] Prove that the class $\text{NP}$ is closed under union and intersection. That is,

- if $L_1, L_2 \in \text{NP}$, then $L_1 \cup L_2 \in \text{NP}$,
- if $L_1, L_2 \in \text{NP}$, then $L_1 \cap L_2 \in \text{NP}$.

(3) [4 points] Prove that the class $\text{coNP}$ is closed under union and intersection. That is,

- if $L_1, L_2 \in \text{coNP}$, then $L_1 \cup L_2 \in \text{coNP}$,
- if $L_1, L_2 \in \text{coNP}$, then $L_1 \cap L_2 \in \text{coNP}$.

(4) [3 points] Prove that if $\text{SAT} \in \text{coNP}$, then $\text{NP} \subseteq \text{coNP}$, and hence, $\text{NP} = \text{coNP}$.

Hint: Use the fact that $\text{SAT}$ is $\text{NP}$-hard.