Homework 3: due 10:20, 21 December 2017

There are SIX questions altogether.

(1) **[2 points]** Consider the following Turing machine $M_1 = (\Sigma, \Gamma, q_0, q_{acc}, q_{rej}, \delta)$.

- $\Sigma = \{0, 1\}$.
- $\Gamma = \{\sqcup, 0, 1, \sqcap\}$.
- $Q = \{q_0, p_0, p_1, s, t, r_0, r_1, q', q_{acc}, q_{rej}\}$.
- $q_0, q_{acc}, q_{rej}$ are the initial, accepting and rejecting states, respectively.
- $\delta$ is defined as follows.

\[
\begin{align*}
(q_0, \sqcup) &\rightarrow (q_{rej}, \sqcup, \text{Stay}) & (p_0, \sqcup) &\rightarrow (q_{rej}, 0, \text{Stay}) & (p_1, \sqcup) &\rightarrow (s, 1, \text{Stay}) \\
(q_0, 0) &\rightarrow (p_0, \sqcup, \text{Right}) & (p_0, 0) &\rightarrow (p_0, 0, \text{Right}) & (p_1, 0) &\rightarrow (p_0, 1, \text{Right}) \\
(q_0, 1) &\rightarrow (p_1, \sqcup, \text{Right}) & (p_0, 1) &\rightarrow (p_1, 0, \text{Right}) & (p_1, 1) &\rightarrow (p_1, 1, \text{Right}) \\
(q_0, \sqcap) &\rightarrow (q_{rej}, \sqcap, \text{Stay}) & (p_0, \sqcap) &\rightarrow (q_{rej}, \sqcap, \text{Stay}) & (p_1, \sqcap) &\rightarrow (q_{rej}, \sqcap, \text{Stay}) \\
(s, \sqcup) &\rightarrow (q_{rej}, \sqcup, \text{Stay}) & (t, \sqcup) &\rightarrow (q_{rej}, \sqcup, \text{Stay}) & (q', \sqcup) &\rightarrow (q', 0, \text{Left}) \\
(s, 0) &\rightarrow (t, 1, \text{Left}) & (t, 0) &\rightarrow (t, 0, \text{Left}) & (q', 0) &\rightarrow (r_0, 0, \text{Left}) \\
(s, 1) &\rightarrow (s, 0, \text{Left}) & (t, 1) &\rightarrow (t, 1, \text{Left}) & (q', 1) &\rightarrow (r_1, 0, \text{Left}) \\
(s, \sqcap) &\rightarrow (r_1, \sqcap, \text{Right}) & (t, \sqcap) &\rightarrow (q_{acc}, \sqcap, \text{Stay}) & (q', \sqcap) &\rightarrow (q_{acc}, \sqcap, \text{Right}) \\
(r_0, \sqcup) &\rightarrow (t, 0, \text{Left}) & (r_1, \sqcup) &\rightarrow (t, 1, \text{Left}) \\
(r_0, 0) &\rightarrow (r_0, 0, \text{Right}) & (r_1, 0) &\rightarrow (r_0, 1, \text{Right}) \\
(r_0, 1) &\rightarrow (r_1, 0, \text{Right}) & (r_1, 1) &\rightarrow (r_1, 1, \text{Right}) \\
(r_0, \sqcap) &\rightarrow (q_{rej}, \sqcap, \text{Stay}) & (r_1, \sqcap) &\rightarrow (q_{rej}, \sqcap, \text{Stay})
\end{align*}
\]

Determine the run of $M$ on each of the following input words.

(i) $\epsilon$.
(ii) 011.
(iii) 100.
(iv) 111.

(2) **[1 point]** Construct a Turing machine $M_2$ that accepts only the configurations of $M_1$ defined above. That is, $M_2$ accepts an input word $w$ if and only if $w$ is a configuration of $M_1$.

(3) **[1 point]** Describe an algorithm for the following problem.

**Input:** A string $w$ which is a configuration of $M_1$.

**Output:** A string $w'$ such that $w'$ is the subsequent configuration of $w$, i.e., $w \rightarrow w'$.

(4) **[2 points]** Recall that for a Turing machine $M$, we denote by $L(M)$ the language that consists of all words accepted by $M$. That is, $L(M) = \{w \mid M \text{ accepts } w\}$.

Consider the following language:

\[
L_{\text{fin}} := \{[M] \mid L(M) \text{ is finite}\}.
\]

That is, $L_{\text{fin}}$ consists of all descriptions of Turing machines that accepts only finitely many words. Prove that $L_{\text{fin}}$ is undecidable.
(5) [2 points] Prove that the following problem is undecidable.

**Input:** A CFG \( G \) and a DFA \( A \).

**Task:** Decide whether \( L(G) = L(A) \). That is, return True, if \( L(G) = L(A) \). Otherwise, return False.

(6) [2 points] Prove that the following problem is undecidable.

**Input:** A Turing machine description \( [M] \).

**Task:** Decide whether the language \( L(M) \) is a CFL. That is, return True, if \( L(M) \) is a CFL. Otherwise, return False.