Homework 2: due before midterm exam

(1) [2 points] Consider the following grammar $G = (\Sigma, V, R, S)$:

- $\Sigma = \{a, b\}$.
- $V = \{S\}$, and $S$ is the start variable.
- $R$ consists of the following rules:

$$S \rightarrow aS \mid aSbS \mid \epsilon$$

Determine which of the following words are in $L(G)$.

(i) $a^3b^3$, i.e., $aaabbb$.
(ii) $a^2b^3$, i.e., $aabb$.
(iii) $abba$.
(iv) $a^2b^2a^3b^3$, i.e., $aabbaabbb$.
(v) $baababa$.

Please substantiate your claim, i.e., if you claim a word is in $L(G)$, you should provide its derivation tree. If you claim it is not, then state your reason why.

(2) [2 points] Construct the CFG for each of the following languages.

(i) $L_1 = \{a^mb^n \mid m > n\}$.
(ii) $L_2 = \{a^mb^n \mid n > m\}$.
(iii) $L_3 = \{a^{2^n}b^n \mid n \geq 0\}$.
(iv) $L_4 = \{w\$w^R\$ \mid w \in \{a,b\}^*\}$.

Here $L_4$ is a language over the alphabet $\{a, b, \$\}$, and $w^R$ denotes the reverse of $w$. For example, if $w = aabb$, then $w^R = bbaa$. If $w = abababa$, then $w^R = abababa$, which is the same as $w$ itself. Likewise, if $w = \epsilon$, then $w^R = \epsilon$.

(v) $L_5$ is the complement of the language $\{a^nb^n \mid n \geq 0\}$ over the alphabet $\{a, b\}$. More formally, $L_5 = \Sigma^* \setminus \{a^nb^n \mid n \geq 0\}$, where $\Sigma = \{a, b\}$.

(3) [2 points] Show that the following languages are not CFL.

(i) $L_1 = \{a^kb^mc^n \mid k \leq m \leq n\}$.
(ii) $L_2 = \{a^n \mid n \text{ is a prime number}\}$.

(4) [2 points] Consider the grammar defined in (1). Prove that $w \in L(G)$ if and only if every prefix of $w$ has at least as many $a$’s as $b$’s.

(5) [2 points] Prove or disprove that if $L_1$ is a CFL and $L_2$ is a regular language, then $L_1 \cap L_2$ is a CFL.