Midterm exam: 18:30–21:30, Thursday, 9 November 2016

Instructions

- This is a closed book exam.
- Write down your name and student number clearly.
- Write down your solutions clearly.
- There are FIVE questions altogether.
- Discussions/collaborations are NOT allowed.
- All electronic devices must be switched off during the exam.
- You don’t need to do the questions in the same order as written here.
- You can use any result discussed, or stated in the class or in the homework.
- However, if you use results never stated in the class or in the homework before, you must supply their complete proofs.
- You can also freely use pumping lemma (for both regular languages and CFL).
Questions

(1) [2 points] Consider the following automaton $A$ over the alphabet $\Sigma = \{a, b\}$.

(i) Is $A$ deterministic or non-deterministic?

(ii) Is $aaa$ accepted by $A$?

(iii) Is $abababababa$ accepted by $A$?

(iv) Construct the deterministic automaton for $A$.

(2) [2 points] A string $w \in \{0, 1\}^*$ represents an integer in a standard way. For example, the string $000$ represents the integer zero, and so do $0$ and $000000$. The string $00100$ and $100$ both represent the integer $4$.

Construct a DFA for the following language over the alphabet $\{0, 1\}$:

$L_0 := \{w \mid w \text{ represents an integer divisible by } 3\}$

Hint: Consider $(2i + j) \mod 3$, for every $i \in \{0, 1, 2\}$ and $j \in \{0, 1\}$.

(3) [2 points] In this question the alphabet is $\Sigma = \{0, 1, \#\}$.

For a word $w \in \{0, 1\}^*$, we denote by $w^R$ the string obtained by reversing $w$. For example, if $w$ is $011$, then $w^R$ is $110$. Likewise, if $w$ is $000110$, then $w^R$ is $011000$. By default, if $w$ is the empty string $\epsilon$, then $w^R$ is $\epsilon$.

Construct the CFG for each of the following languages.

- $L_1 := \{w \# w^R \# \mid w \in \{0, 1\}^*\}$.
  That is, $L_1$ consists of all the words of the form:
  
  $u \# v \#$

  where $u, v \in \{0, 1\}^*$ and $v = u^R$.

- $L_2 := \{w_1 \# w_1^R \# w_2 \# w_2^R \# \cdots \# w_k \# w_k^R \# \mid \text{each } w_i \in \{0, 1\}^* \text{ for some } k \geq 1\}$.
  That is, $L_2$ consists of all the words of the form:
  
  $u_1 \# v_1 \# u_2 \# v_2 \# \cdots \# u_k \# v_k \#$

  for some $k \geq 1$ and for each $1 \leq i \leq k$, $u_i, v_i \in \{0, 1\}^*$ and $v_i^R = u_i$.

(4) [2 points] Prove or disprove the following.

- If $L$ is regular and $K$ is CFL, then $L \cap K$ is regular.

- If $L$ is regular and $K$ is CFL, then $L \cup K$ is regular.

(5) [2 points] For a language $L \subseteq \{a, b\}^*$, we define $\text{half}(L)$ to be the following language:

$\text{half}(L) := \{x \mid \text{there is } y \text{ such that } |x| = |y| \text{ and } xy \in L\}$.

That is, $\text{half}(L)$ consists of the first halves of words in $L$. Prove that if $L$ is regular, then $\text{half}(L)$ is also regular.