Communication Optimization for Parallel Processing

Lecture 4

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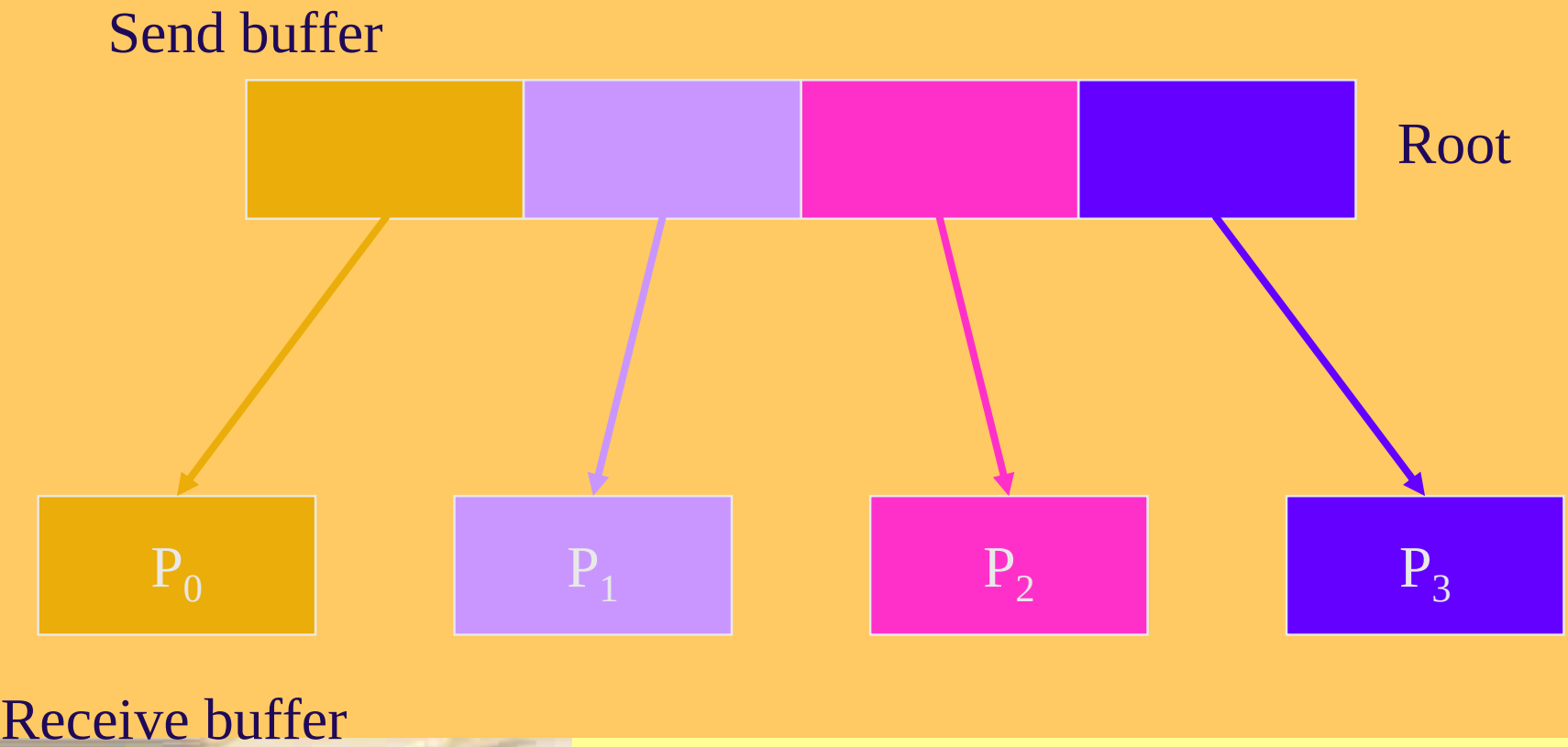
Scatter the Data

- MPI_Scatter
  - This routine distributes data to all processors.
  - The root process sends the data via a send buffer.
  - All the others provide a receive buffer for the incoming data from the root.

Scatter the Data

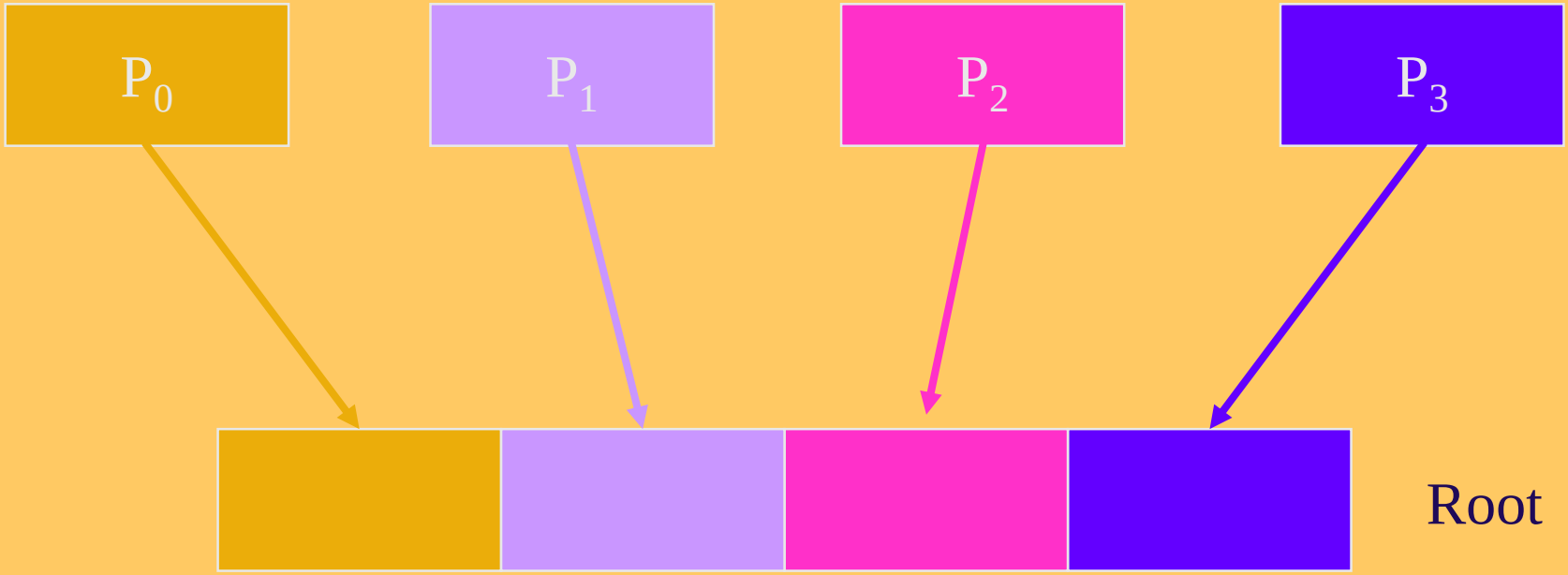


Gather the Data

- MPI_Gather
  - This routine works the opposite way to MPI_Scatter. It collects data from all processes to the root process.
- MPI_Allgather
  - Similar to MPI_Gather but all the processes receive the result.

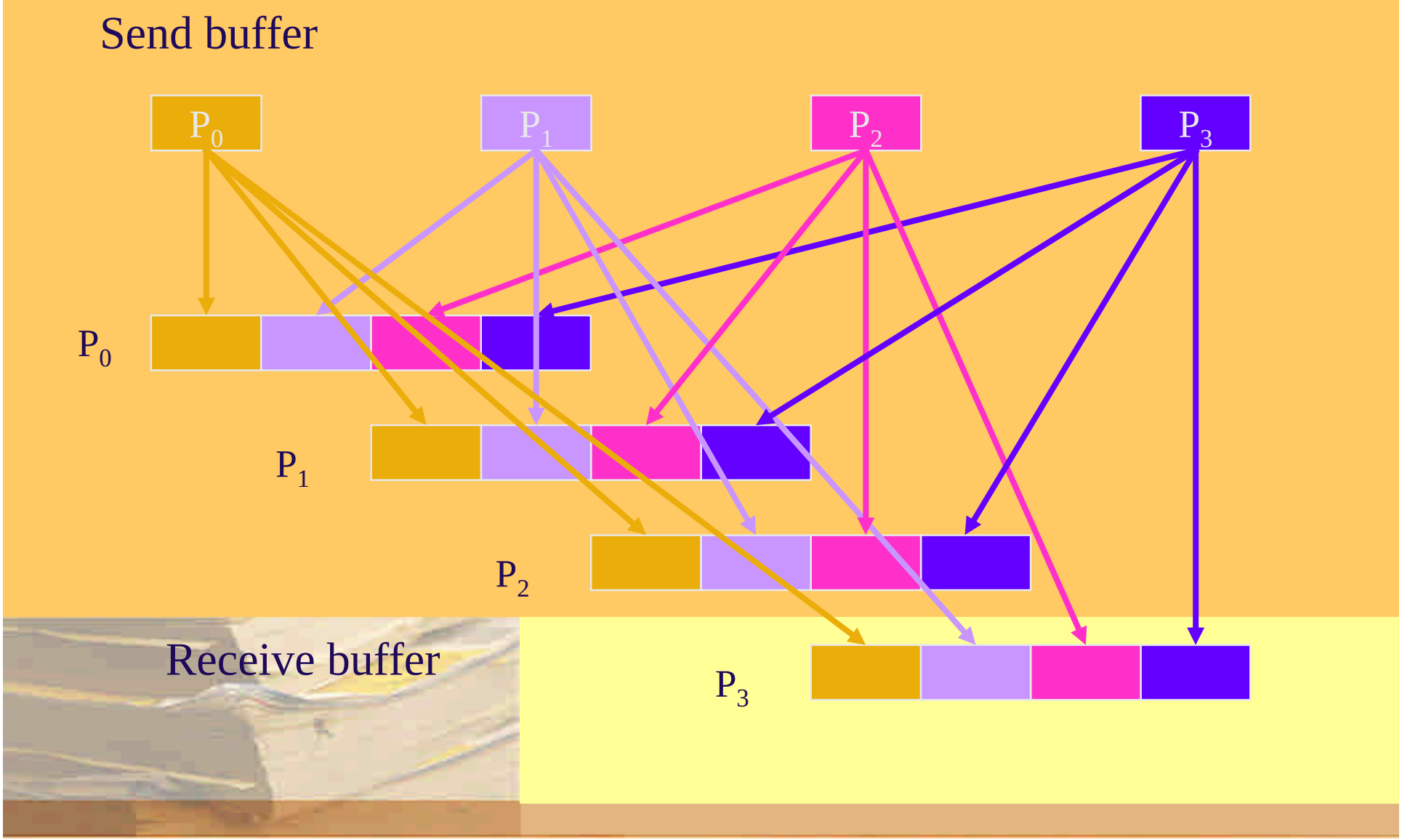
Gather the Data

Send buffer



Receive buffer

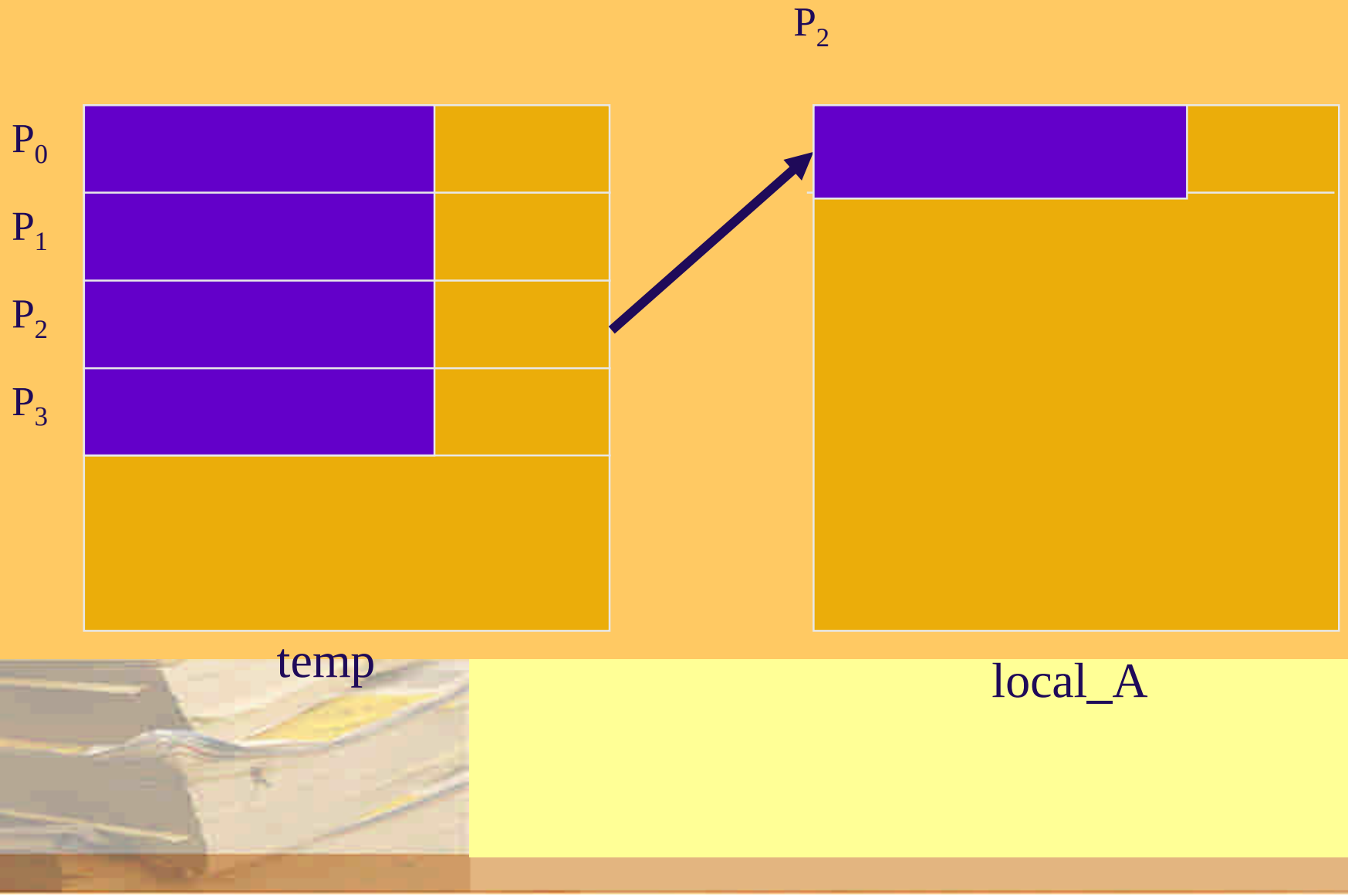
Everyone Gathers the Data



Matrix-Vector Multiplication

- Use MPI_Scatter to distribute the rows of the matrix and the segments of the input vector to all processes.
- Use MPI_Allgather to collect the entire vector in order to perform multiplication.
- Finally the process 0 uses MPI_Gather to collect all the fragments from other processes and prints the result.

Matrix Partition



Variables and Types

```
#define MAX_ORDER 100

typedef float LOCAL_MATRIX_T[MAX_ORDER][MAX_ORDER];

main(int argc, char* argv[])
{
   int my_rank;
   int p;
   LOCAL_MATRIX_T  local_A;
   float global_x[MAX_ORDER];
   float local_x[MAX_ORDER];
   float local_y[MAX_ORDER];
   int m, n;
   int local_m, local_n;

   MPI_Init(&argc, &argv);
   MPI_Comm_size(MPI_COMM_WORLD, &p);
   MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);

   if (my_rank == 0) {
      printf("Enter the order of the matrix (m x n)\n");
      scanf("%d %d", &m, &n);
   }
```

Matrix-Vector Multiplication

```
   MPI_Bcast(&m, 1, MPI_INT, 0, MPI_COMM_WORLD);
   MPI_Bcast(&n, 1, MPI_INT, 0, MPI_COMM_WORLD);

   local_m = m/p;
   local_n = n/p;

   Read_matrix("Enter the matrix", local_A, local_m, n, my_rank, p);
   Print_matrix("We read", local_A, local_m, n, my_rank, p);

   Read_vector("Enter the vector", local_x, local_n, my_rank, p);
   Print_vector("We read", local_x, local_n, my_rank, p);

   Parallel_matrix_vector_prod(local_A, m, n, local_x, global_x,
      local_y, local_m, local_n);
   Print_vector("The product is", local_y, local_m, my_rank, p);

   MPI_Finalize();

}  /* main */
```

Read the Matrix

```
void Read_matrix(char* prompt, LOCAL_MATRIX_T  local_A, int local_m,
    int n, int my_rank, int p)
{
    int i, j;
    LOCAL_MATRIX_T  temp;
    for (i = 0; i < p*local_m; i++)
        for (j = n; j < MAX_ORDER; j++)
            temp[i][j] = 0.0;

    if (my_rank == 0) {
        printf("%s\n", prompt);
        for (i = 0; i < p*local_m; i++)
            for (j = 0; j < n; j++)
                scanf("%f",&temp[i][j]);
    }
    MPI_Scatter(temp, local_m*MAX_ORDER, MPI_FLOAT, local_A, local_m*MAX_ORDER,
        MPI_FLOAT, 0, MPI_COMM_WORLD);
}  /* Read_matrix */
```

Read the Vector

```
void Read_vector(char*  prompt, float  local_x[], int local_n, int my_rank, int p)
{
   int i;
   float temp[MAX_ORDER];
   if (my_rank == 0) {
      printf("%s\n", prompt);
      for (i = 0; i < p*local_n; i++)
         scanf("%f", &temp[i]);
   }
   MPI_Scatter(temp, local_n, MPI_FLOAT, local_x, local_n, MPI_FLOAT, 0,
      MPI_COMM_WORLD);
}  /* Read_vector */
```

Product Computation

```
void Parallel_matrix_vector_prod(LOCAL_MATRIX_T local_A, int m, int n, float local_x[],
    float global_x[], float local_y[], int local_m, int local_n)
{
    /* local_m = m/p, local_n = n/p */

    int i, j;

    MPI_Allgather(local_x, local_n, MPI_FLOAT, global_x, local_n, MPI_FLOAT,
        MPI_COMM_WORLD);
    for (i = 0; i < local_m; i++) {
        local_y[i] = 0.0;
        for (j = 0; j < n; j++)
            local_y[i] = local_y[i] + local_A[i][j]*global_x[j];
    }
}  /* Parallel_matrix_vector_prod */
```

Print the Matrix

```
void Print_matrix(char* title, LOCAL_MATRIX_T  local_A, int local_m, int n , int my_rank, int p)
{
   int i, j;
   float temp[MAX_ORDER][MAX_ORDER];

   MPI_Gather(local_A, local_m*MAX_ORDER, MPI_FLOAT, temp,
      local_m*MAX_ORDER, MPI_FLOAT, 0, MPI_COMM_WORLD);

   if (my_rank == 0) {
      printf("%s\n", title);
      for (i = 0; i < p*local_m; i++) {
         for (j = 0; j < n; j++)
            printf("%4.1f ", temp[i][j]);
         printf("\n");
      }
   }
}  /* Print_matrix */
```

Print the Vector

```
void Print_vector(char*  title, float  local_y[], int local_m, int my_rank, int p)
{
   int i;
   float temp[MAX_ORDER];
   MPI_Gather(local_y, local_m, MPI_FLOAT, temp, local_m, MPI_FLOAT, 0,
      MPI_COMM_WORLD);
   if (my_rank == 0) {
      printf("%s\n", title);
      for (i = 0; i < p*local_m; i++)
         printf("%4.1f ", temp[i]);
      printf("\n");
   }
}  /* Print_vector */
```

Communicator

- A subset of processes
- Intra-communicator
  - Send/receive messages among a collection of processes within the same communicator.
- Inter-communicator
  - Send/receive messages among different communicators.

A Communicator

- A group
  - A set of processes with ranks from 0 to p – 1.
- A context
  - A system defined object that uniquely defines a communicator.
  - Two processes must specify the same context in order to communicate.

MPI Communicator

- MPI_Comm_group
  - Create a MPI_GROUP from an existing one.
- MPI_Group_incl
  - Add a list of processes into a group.
- MPI_Comm_create
  - Create a new MPI_COMM from a MPI_GROUP.

Variables

```
main(int argc, char* argv[]) {
    int p;
    int q; /* = sqrt(p) */
    int my_rank;
    MPI_Group group_world;
    MPI_Group first_row_group;
    MPI_Comm first_row_comm;
    int* process_ranks;
    int proc;
    int test = 0;
    int sum;
    int my_rank_in_first_row;

    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &p);
    MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);

    q = (int) sqrt((double) p);
    /* Make a list of the processes in the new communicator */
    process_ranks = (int*) malloc(q*sizeof(int));
    for (proc = 0; proc < q; proc++)
        process_ranks[proc] = proc;
```

Communicator Creation

```
/* Get the group underlying MPI_COMM_WORLD */
   MPI_Comm_group(MPI_COMM_WORLD, &group_world);

   /* Create the new group */
   MPI_Group_incl(group_world, q, process_ranks, &first_row_group);

   /* Create the new communicator */
   MPI_Comm_create(MPI_COMM_WORLD, first_row_group, &first_row_comm);

   /* Now check whether we can do collective ops in first_row_comm */
   if (my_rank < q) {
      MPI_Comm_rank(first_row_comm, &my_rank_in_first_row);
      if (my_rank_in_first_row == 0) test = 1;
      MPI_Bcast(&test, 1, MPI_INT, 0, first_row_comm);
   }

   MPI_Reduce(&test, &sum, 1, MPI_INT, MPI_SUM, 0, MPI_COMM_WORLD);
   if (my_rank == 0) {
      printf("q = %d, sum = %d\n", q, sum);
   }
   MPI_Finalize();
}  /* main */
```

Split an MPI Communicator

- MPI_Comm_split
  - Int MPI_Comm_split(MPI_Comm old_comm, int split_key, int rank_key, MPI_Comm *new_comm);
  - Splits an MPI communicator into many sub-communicator.
  - Processors having the same split_key are grouped into the same sub-communicator.
  - The new rank within the sub-communicator is determined by rank_key.
  - If we would like to divide a matrix into rows. Each process specifies its row and its rank within that row is determined by its old rank in MPI_COMM_WORLD.

Variables

```
main(int argc, char* argv[])
{
   int p;
   int my_rank;
   MPI_Comm  my_row_comm;
   int my_row;
   int q;
   int test;
   int my_rank_in_row;

   MPI_Init(&argc, &argv);
   MPI_Comm_size(MPI_COMM_WORLD, &p);
   MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);

   q = (int) sqrt((double) p);
```

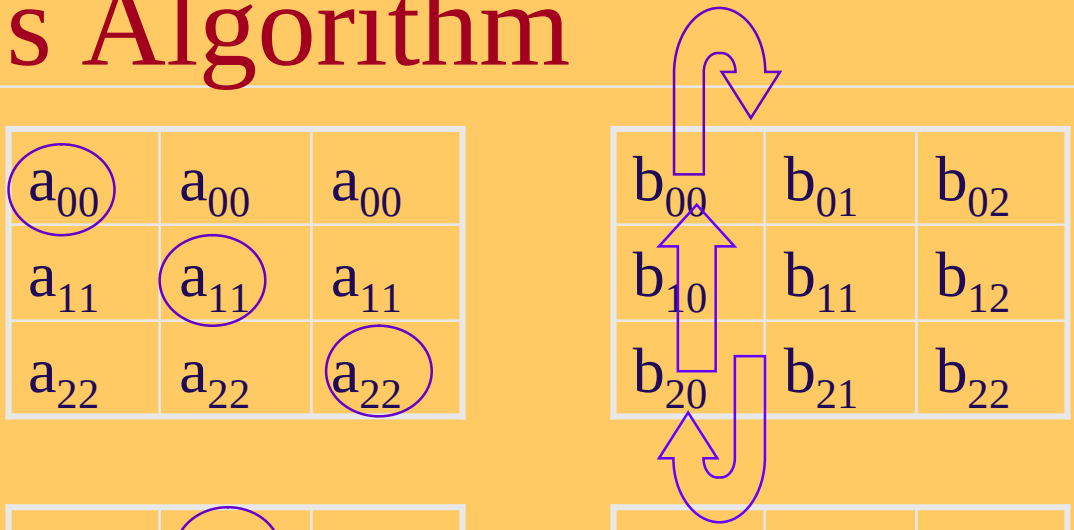
Split a Communicator

```
   /* my_rank is rank in MPI_COMM_WORLD. q*q = p */
   my_row = my_rank/q;
   MPI_Comm_split(MPI_COMM_WORLD, my_row, my_rank, &my_row_comm);

   MPI_Comm_rank(my_row_comm, &my_rank_in_row);
   if (my_rank_in_row == 0)
      test = my_row;
   else
      test = 0;

   MPI_Bcast(&test, 1, MPI_INT, 0, my_row_comm);
   printf("Process %d > my_row = %d, my_rank_in_row = %d, test = %d\n",
      my_rank, my_row, my_rank_in_row, test);

   MPI_Finalize();
}  /* main */
```

A Complete Example

- A parallel matrix multiplication
- We compute the multiplication of two matrices A and B.
- The elements of A are broadcast among the rows of process matrix, and the elements of B are shifted among the column of process matrix.

Fox's Algorithm



Data Types

```
typedef struct {
   int p; /* Total number of processes*/
   MPI_Comm  comm; /* Communicator for entire grid */
   MPI_Comm  row_comm; /* Communicator for my row */
   MPI_Comm  col_comm; /* Communicator for my col */
   int q; /* Order of grid */
   int my_row; /* My row number */
   int my_col; /* My column number */
   int my_rank; /* My rank in the grid comm */
} GRID_INFO_T;
```

```
#define MAX 65536
typedef struct {
   int n_bar;
#define Order(A) ((A)->n_bar)
   float  entries[MAX];
#define Entry(A,i,j) (*(((A)->entries) + ((A)->n_bar)*(i) + (j)))
} LOCAL_MATRIX_T;
```

```
LOCAL_MATRIX_T*  temp_mat;
```

Main Program

```
main(int argc, char* argv[]) {
    int p;
    int my_rank;
    GRID_INFO_T grid;
    LOCAL_MATRIX_T*  local_A;
    LOCAL_MATRIX_T*  local_B;
    LOCAL_MATRIX_T*  local_C;
    int n;
    int n_bar;

    MPI_Init(&argc, &argv);
    MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);

    Setup_grid(&grid);
    if (my_rank == 0) {
        printf("What's the order of the matrices?\n");
        scanf("%d", &n);
    }
    MPI_Bcast(&n, 1, MPI_INT, 0, MPI_COMM_WORLD);
    n_bar = n/grid.q;
```

Multiplication

```
    local_A = Local_matrix_allocate(n_bar);
    Order(local_A) = n_bar;
    Read_matrix("Enter A", local_A, &grid, n);
    Print_matrix("We read A =", local_A, &grid, n);

    local_B = Local_matrix_allocate(n_bar);
    Order(local_B) = n_bar;
    Read_matrix("Enter B", local_B, &grid, n);
    Print_matrix("We read B =", local_B, &grid, n);

    Build_matrix_type(local_A);
    temp_mat = Local_matrix_allocate(n_bar);

    local_C = Local_matrix_allocate(n_bar);
    Order(local_C) = n_bar;
    Fox(n, &grid, local_A, local_B, local_C);

    Print_matrix("The product is", local_C, &grid, n);
    Free_local_matrix(&local_A);
    Free_local_matrix(&local_B);
    Free_local_matrix(&local_C);
    MPI_Finalize();
} /* main */
```

MPI Cartesian Coordinate

- MPI_Cart_create
  - Create a new communicator with a Cartesian coordinate system .
  - The number of dimension, the dimension sizes along each dimension can be specified.
- MPI_Cart_cords
  - Convert a rank into coordinates according to a given Cartesian communicator.
- MPI_Cart_sub
  - Split a grid into sub-grids.
  - We need to specify which dimensions need to be split.

Initializations

```
void Setup_grid (GRID_INFO_T*  grid  /* out */)
{
   int old_rank;
   int dimensions[2];
   int wrap_around[2];
   int coordinates[2];
   int free_coords[2];

   /* Set up Global Grid Information */
   MPI_Comm_size(MPI_COMM_WORLD, &(grid->p));
   MPI_Comm_rank(MPI_COMM_WORLD, &old_rank);

   /* We assume p is a perfect square */
   grid->q = (int) sqrt((double) grid->p);
   dimensions[0] = dimensions[1] = grid->q;

   wrap_around[0] = wrap_around[1] = 1;
   MPI_Cart_create(MPI_COMM_WORLD, 2, dimensions,
      wrap_around, 1, &(grid->comm));
   MPI_Comm_rank(grid->comm, &(grid->my_rank));
   MPI_Cart_coords(grid->comm, grid->my_rank, 2,
      coordinates);
   grid->my_row = coordinates[0];
   grid->my_col = coordinates[1];
```

```
/* Set up row communicators */
   free_coords[0] = 0;
   free_coords[1] = 1;
   MPI_Cart_sub(grid->comm, free_coords,
      &(grid->row_comm));

   /* Set up column communicators */
   free_coords[0] = 1;
   free_coords[1] = 0;
   MPI_Cart_sub(grid->comm, free_coords,
      &(grid->col_comm));
} /* Setup_grid */
```

Fox's Algorithm

```
void Fox(int n, GRID_INFO_T*grid, LOCAL_MATRIX_T*  local_A,
      LOCAL_MATRIX_T*  local_B, LOCAL_MATRIX_T* local_C)
{
   LOCAL_MATRIX_T*  temp_A;
   int stage;
   int bcast_root;
   int n_bar;  /* n/sqrt(p) */
   int source;
   int dest;
   MPI_Status status;

   n_bar = n/grid->q;
   Set_to_zero(local_C);

   /* Calculate addresses for circular shift of B */
   source = (grid->my_row + 1) % grid->q;
   dest = (grid->my_row + grid->q - 1) % grid->q;
```

Fox's Algorithm

```
/* Calculate addresses for circular shift of B */
source = (grid->my_row + 1) % grid->q;
dest = (grid->my_row + grid->q - 1) % grid->q;

/* Set aside storage for the broadcast block of A */
temp_A = Local_matrix_allocate(n_bar);

for (stage = 0; stage < grid->q; stage++) {
   bcast_root = (grid->my_row + stage) % grid->q;
   if (bcast_root == grid->my_col) {
      MPI_Bcast(local_A, 1, local_matrix_mpi_t, bcast_root, grid->row_comm);
      Local_matrix_multiply(local_A, local_B, local_C);
   } else {
      MPI_Bcast(temp_A, 1, local_matrix_mpi_t, bcast_root, grid->row_comm);
      Local_matrix_multiply(temp_A, local_B, local_C);
   }
   MPI_Sendrecv_replace(local_B, 1, local_matrix_mpi_t,
      dest, 0, source, 0, grid->col_comm, &status);
} /* for */
} /* Fox */
```

Allocation/Free Functions

```
LOCAL_MATRIX_T* Local_matrix_allocate(int local_order)
{
   LOCAL_MATRIX_T* temp;

   temp = (LOCAL_MATRIX_T*) malloc(sizeof(LOCAL_MATRIX_T));
   return temp;
}  /* Local_matrix_allocate */


void Free_local_matrix(LOCAL_MATRIX_T** local_A_ptr)
{
   free(*local_A_ptr);
}  /* Free_local_matrix */
```

Read the Matrix

```
void Read_matrix(char* prompt, LOCAL_MATRIX_T* local_A, GRID_INFO_T* grid. int n)
{
   int mat_row, mat_col;    int grid_row, grid_col;
   int dest;    int coords[2];
   float* temp;    MPI_Status status;
    if (grid->my_rank != 0)
      for (mat_row = 0; mat_row < Order(local_A); mat_row++)
         MPI_Recv(&Entry(local_A, mat_row, 0), Order(local_A), MPI_FLOAT, 0, 0, grid->comm, &status);
   else {
      temp = (float*) malloc(Order(local_A)*sizeof(float));
      printf("%s\n", prompt);       fflush(stdout);
      for (mat_row = 0;  mat_row < n; mat_row++) {
         grid_row = mat_row/Order(local_A);
         coords[0] = grid_row;
         for (grid_col = 0; grid_col < grid->q; grid_col++) {
            coords[1] = grid_col;
            MPI_Cart_rank(grid->comm, coords, &dest);
            if (dest == 0) {
               for (mat_col = 0; mat_col < Order(local_A); mat_col++)
                  scanf("%f", (local_A->entries)+mat_row*Order(local_A)+mat_col);
            } else {
               for (mat_col = 0; mat_col < Order(local_A); mat_col++)
                  scanf("%f", temp + mat_col);
               MPI_Send(temp, Order(local_A), MPI_FLOAT, dest, 0,  grid->comm);
            }
         }
      }
      free(temp);
   }
}  /* Read_matrix */
```

Print the Matrix

```
void Print_matrix(char* title, LOCAL_MATRIX_T*  local_A, GRID_INFO_T* grid, int n)
{
   int mat_row, mat_col;    int grid_row, grid_col;
   int source;    int coords[2];
   float* temp;    MPI_Status status;
   if (grid->my_rank == 0) {
      temp = (float*) malloc(Order(local_A)*sizeof(float));
      printf("%s\n", title);
      for (mat_row = 0;  mat_row < n; mat_row++) {
         grid_row = mat_row/Order(local_A);
         coords[0] = grid_row;
         for (grid_col = 0; grid_col < grid->q; grid_col++) {
            coords[1] = grid_col;
            MPI_Cart_rank(grid->comm, coords, &source);
            if (source == 0) {
               for(mat_col = 0; mat_col < Order(local_A); mat_col++)
                  printf("%4.1f ", Entry(local_A, mat_row, mat_col));
            } else {
               MPI_Recv(temp, Order(local_A), MPI_FLOAT, source, 0,  grid->comm, &status);
               for(mat_col = 0; mat_col < Order(local_A); mat_col++)
                  printf("%4.1f ", temp[mat_col]);
            }
         }
         printf("\n");
      }
      free(temp);
   } else
      for (mat_row = 0; mat_row < Order(local_A); mat_row++)
         MPI_Send(&Entry(local_A, mat_row, 0), Order(local_A),  MPI_FLOAT, 0, 0, grid->comm);
}  /* Print_matrix */
```

MPI Type Construction

- Generate portable types for data transmission among different machine architectures.
- MPI_Type_contiguous
  - Declare the data type to consist of a given number of contiguous elements.
- MPI_Address
  - Calculate the offset.

MPI Type Construction

- MPI_Type_struct
  - Construct a data type using the following.
    - The Number of data blocks
    - The number of data in each block
    - The data type of each element in a block.
    - The offset of each element in a block
- MPI_Type_commit
  - Commit a type to the runtime system.

Build Matrix Data Type

```
void Build_matrix_type(LOCAL_MATRIX_T*  local_A  )
{
   MPI_Datatype  temp_mpi_t;
   int block_lengths[2];
   MPI_Aint displacements[2];
   MPI_Datatype typelist[2];
   MPI_Aint start_address;
   MPI_Aint address;

   MPI_Type_contiguous(Order(local_A)*Order(local_A), MPI_FLOAT, &temp_mpi_t);
   block_lengths[0] = block_lengths[1] = 1;
   typelist[0] = MPI_INT;
   typelist[1] = temp_mpi_t;

   MPI_Address(local_A, &start_address);
   MPI_Address(&(local_A->n_bar), &address);
   displacements[0] = address - start_address;

   MPI_Address(local_A->entries, &address);
   displacements[1] = address - start_address;

   MPI_Type_struct(2, block_lengths, displacements, typelist, &local_matrix_mpi_t);
   MPI_Type_commit(&local_matrix_mpi_t);
}  /* Build_matrix_type */
```

Local Computation

```
void Local_matrix_multiply(LOCAL_MATRIX_T*  local_A, LOCAL_MATRIX_T*  local_B,
    LOCAL_MATRIX_T*  local_C)
{
    int i, j, k;

    for (i = 0; i < Order(local_A); i++)
        for (j = 0; j < Order(local_A); j++)
            for (k = 0; k < Order(local_B); k++)
                Entry(local_C,i,j) = Entry(local_C,i,j) + Entry(local_A,i,k)*Entry(local_B,k,j);

}  /* Local_matrix_multiply */
```

Print the Local Matrix

```
void Print_local_matrices(char* title, LOCAL_MATRIX_T*  local_A, GRID_INFO_T* grid)
{
   int  coords[2];    int  i, j;
   int  source;    MPI_Status  status;

   if (grid->my_rank == 0) {
      printf("%s\n", title);
      printf("Process %d > grid_row = %d, grid_col = %d\n", grid->my_rank, grid->my_row, grid->my_col);
      for (i = 0; i < Order(local_A); i++) {
         for (j = 0; j < Order(local_A); j++)
            printf("%4.1f ", Entry(local_A,i,j));
         printf("\n");
      }
      for (source = 1; source < grid->p; source++) {
         MPI_Recv(temp_mat, 1, local_matrix_mpi_t, source, 0, grid->comm, &status);
         MPI_Cart_coords(grid->comm, source, 2, coords);
         printf("Process %d > grid_row = %d, grid_col = %d\n",
            source, coords[0], coords[1]);
         for (i = 0; i < Order(temp_mat); i++) {
            for (j = 0; j < Order(temp_mat); j++)
               printf("%4.1f ", Entry(temp_mat,i,j));
            printf("\n");
         }
      }
      fflush(stdout);
   } else
      MPI_Send(local_A, 1, local_matrix_mpi_t, 0, 0, grid->com
}  /* Print_local_matrices */
```