Applied Deep Learning

Policy Gradient
& Actor-Critic

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http://adl.miulab.tw

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Reinforcement Learning Approach

- Value-based RL
  - Estimate the optimal value function $Q^*(s, a)$

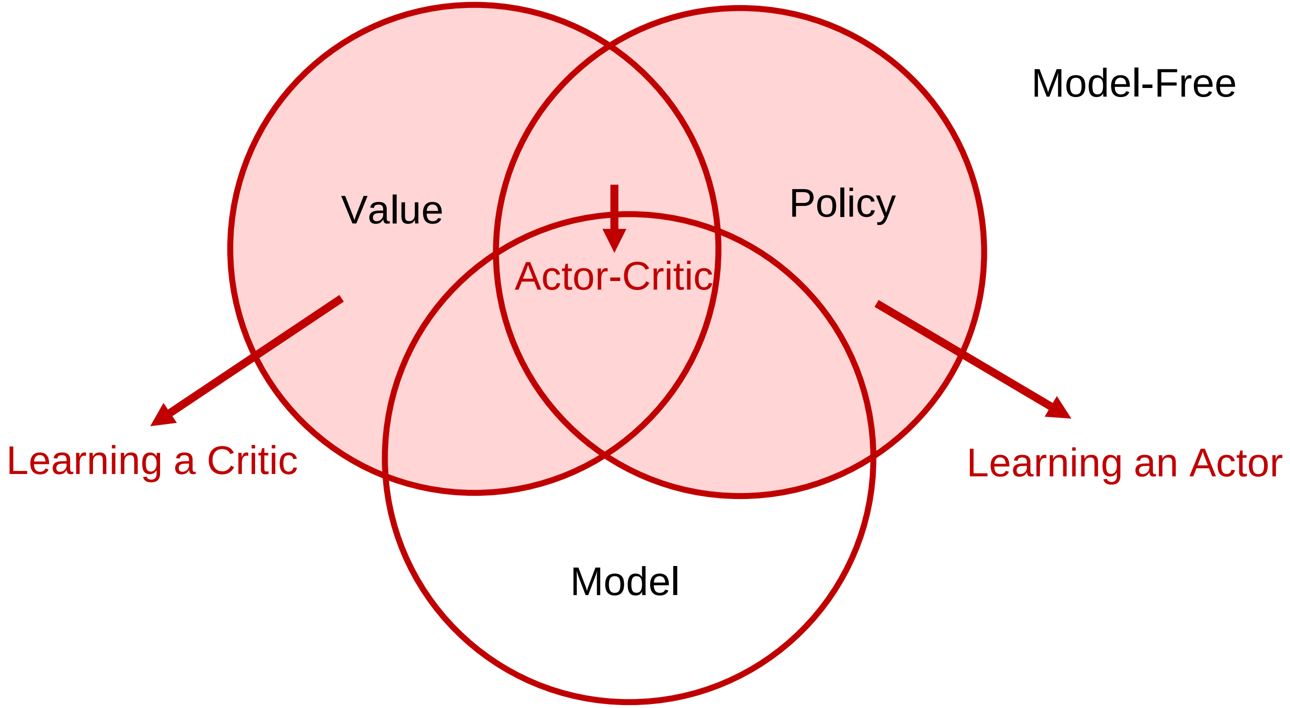
$Q^*(s, a)$ is maximum value achievable under any policy

- Policy-based RL
  - Search directly for optimal policy π^*

π^* is the policy achieving maximum future reward

- Model-based RL
  - Build a model of the environment
  - Plan (e.g. by lookahead) using model

RL Agent Taxonomy



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Policy-Based Approach

Learning an Actor

Policy

- A policy is the agent's behavior
- A policy maps from state to action
  - Deterministic policy: $a = \pi(s)$
  - Stochastic policy: $\pi(a) = P(a \mid s)$

6 Policy Networks

- Represent policy by a network with parameters θ

$$a = \pi(a \mid s, \theta) \qquad\qquad a = \pi(s, \theta)$$

stochastic policy — deterministic policy

- Objective is to maximize total discounted reward by SGD

$$O(\theta) = \mathrm{E}[r_1 + \gamma r_2 + \gamma^2 r_3 + \cdots \mid \pi(\cdot, \theta)]$$

On-Policy v.s. Off-Policy

- On-policy: The agent learned and the agent interacting with the environment is the same
- Off-policy: The agent learned and the agent interacting with the environment is different

Goodness of Actor

- An episode is considered as a trajectory τ
  - $\tau = \{s_1, a_1, r_1, s_2, a_2, r_2, \cdots, s_T, a_T, r_T\}$
  - Reward: $R(\tau) = \sum_{t=1}^{T} \gamma^{t-1} r_t$

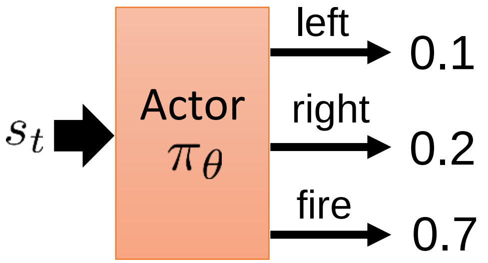
$$P(\tau \mid \theta) =$$

$$p(s_1)p(a_1 \mid s_1, \theta)p(r_1, s_2 \mid s_1, a_1)p(a_2 \mid s_2, \theta)p(r_2, s_3 \mid s_2, a_2)\cdots$$

$$= p(s_1) \prod_{t=1}^{T} p(a_t \mid s_t, \theta) p(r_t, s_{t+1} \mid s_t, a_t)$$

not related to your actor

control by your actor



$$p(a_t = \text{fire} \mid s_t, \theta) = 0.7$$

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Goodness of Actor

- An episode is considered as a trajectory τ
  - $\tau = \{s_1, a_1, r_1, s_2, a_2, r_2, \cdots, s_T, a_T, r_T\}$
  - Reward: $R(\tau) = \sum_{t=1}^{T} \gamma^{t-1} r_t$

Goodness of Actor

- An episode is considered as a trajectory τ
  - $\tau = \{s_1, a_1, r_1, s_2, a_2, r_2, \cdots, s_T, a_T, r_T\}$
  - Reward: $R(\tau) = \sum_{t=1}^{T} \gamma^{t-1} r_t$
- We define $\mathcal{R}(\theta)$ as the *expected value* of reward
  - If you use an actor to play game, each τ has $P(\tau|\theta)$ to be sampled

$$\mathcal{R}(\theta) = \sum_{\tau} R(\tau) P(\tau \mid \theta) \approx \frac{1}{N} \sum_{n=1}^{N} R(\tau^n)$$

- Use π_θ to play the game N times, obtain $\{\tau^1, \tau^2, \cdots, \tau^N\}$
- Sampling τ from $P(\tau|\theta)$ N times

sum over all possible trajectory

Deep Policy Networks

- Represent policy by deep network with weights
- Objective is to maximize total discounted reward by SGD

$$\mathcal{R}(\theta) = \mathbb{E}\left[r_1 + \gamma r_2 + \gamma^2 r_3 + \cdots \mid \pi(\cdot, \theta) \right]$$

- Update the model parameters iteratively

$$\theta^* = \arg\max_{\theta} \mathcal{R}(\theta)$$

$$\theta' \leftarrow \theta + \eta \nabla \mathcal{R}(\theta)$$

Policy Gradient $\mathcal{R}(\theta) = \sum_\tau R(\tau)P(\tau \mid \theta)$

- Gradient assent to maximize the expected reward

$$\nabla \mathcal{R}(\theta) = \sum_\tau R(\tau) \nabla P(\tau \mid \theta) = \sum_\tau R(\tau) P(\tau \mid \theta) \frac{\nabla P(\tau \mid \theta)}{P(\tau \mid \theta)}$$

do not have to be differentiable
can even be a black box

$$= \sum_\tau R(\tau) P(\tau \mid \theta) \nabla \log P(\tau \mid \theta)$$

$$\frac{d \log f(x)}{dx} = \frac{1}{f(x)} \frac{df(x)}{dx}$$

use π_θ to play the game N times, obtain $\{\tau^1, \tau^2, \cdots, \tau^N\}$

$$\approx \frac{1}{N} \sum_{n=1}^{N} R(\tau^n) \underline{\nabla \log P(\tau^n \mid \theta)}$$

Policy Gradient $\nabla \log P(\tau \mid \theta)$

- An episode trajectory $\tau = \{s_1, a_1, r_1, s_2, a_2, r_2, \cdots, s_T, a_T, r_T\}$

$$P(\tau \mid \theta) = p(s_1) \prod_{t=1}^{T} p(a_t \mid s_t, \theta) p(r_t, s_{t+1} \mid s_t, a_t)$$

$$\log P(\tau \mid \theta) = \log p(s_1) \sum_{t=1}^{T} \log p(a_t \mid s_t, \theta) + \log p(r_t, s_{t+1} \mid s_t, a_t)$$

$$\nabla \log P(\tau \mid \theta) = \sum_{t=1}^{T} \nabla \log p(a_t \mid s_t, \theta)$$ ignore the terms not related to θ

Policy Gradient

- Gradient assent for iteratively updating the parameters

$$\theta' \leftarrow \theta + \eta \nabla \bar{R}(\theta)$$

$$\nabla \bar{R}(\theta) \approx \frac{1}{N} \sum_{n=1}^{N} R(\tau^n) \nabla \log P(\tau^n \mid \theta)$$

$$= \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla \log p(a_t^n \mid s_t^n, \theta)$$

  - If τ^n machine takes a_t^n when seeing s_t^n

$R(\tau^n) > 0$ ⇒ Tuning θ to increase $p(a_t^n \mid s_t^n)$

$R(\tau^n) < 0$ ⇒ Tuning θ to decrease $p(a_t^n \mid s_t^n)$

Important: use *cumulative* reward $R(\tau^n)$ of the whole trajectory τ^n instead of *immediate* reward r_t^n

Policy Gradient

Given actor parameter θ

$$\begin{array}{llll} \tau^1\!: (s_1^1, a_1^1) & R(\tau^1) & \tau^2\!: (s_1^2, a_1^2) & R(\tau^2) \\ \quad\;\; (s_2^1, a_2^1) & R(\tau^1) & \quad\;\; (s_2^2, a_2^2) & R(\tau^2) \\ \quad\;\; \vdots & \vdots & \quad\;\; \vdots & \vdots \end{array}$$

data collection

model update

$$\theta' \leftarrow \theta + \eta \nabla \mathcal{R}(\theta)$$

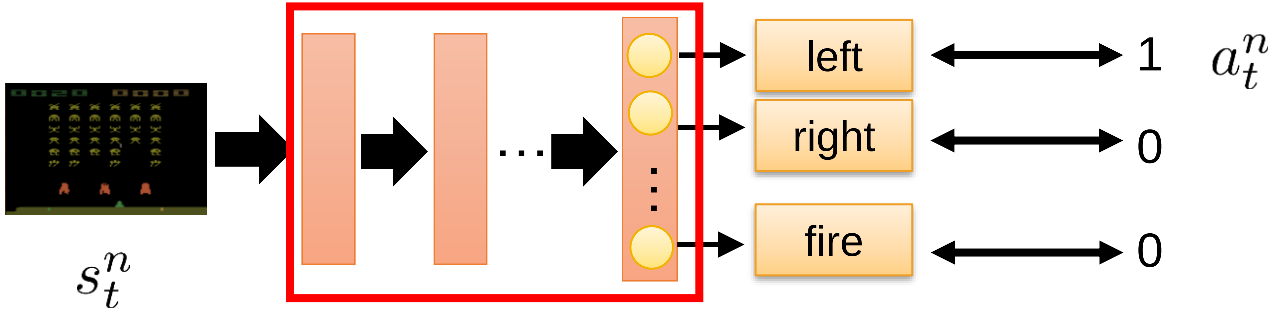
$$\nabla \mathcal{R}(\theta) = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla \log p(a_t^n \mid s_t^n, \theta)$$

Implementation

$$\theta' \leftarrow \theta + \eta \nabla \mathcal{R}(\theta)$$

$$\nabla \mathcal{R}(\theta) = \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} R(\tau^n) \nabla \log p(a_t^n \mid s_t^n, \theta)$$

- Treat it as a classification problem



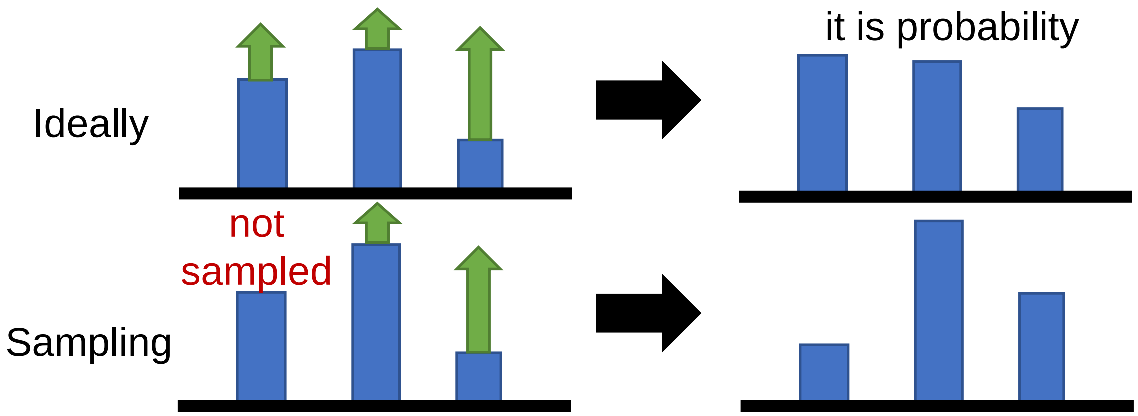
$$\frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} \log p(a_t^n \mid s_t^n) \xrightarrow{\text{TF, PyTorch …}} \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} \nabla \log p(a_t^n \mid s_t^n)$$

$$\frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} R(\tau^n) \log p(a_t^n \mid s_t^n) \longrightarrow \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} R(\tau^n) \nabla \log p(a_t^n \mid s_t^n)$$

Improvement: Adding Baseline

$$\theta' \leftarrow \theta + \eta \nabla \mathcal{R}(\theta)$$

$$\nabla \mathcal{R}(\theta) = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla \log p(a_t^n \mid s_t^n, \theta)$$



Issue: the probability of the actions not sampled will decrease

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Actor-Critic Approach

Learning an Actor & A Critic

Actor-Critic (Value-Based + Policy-Based)

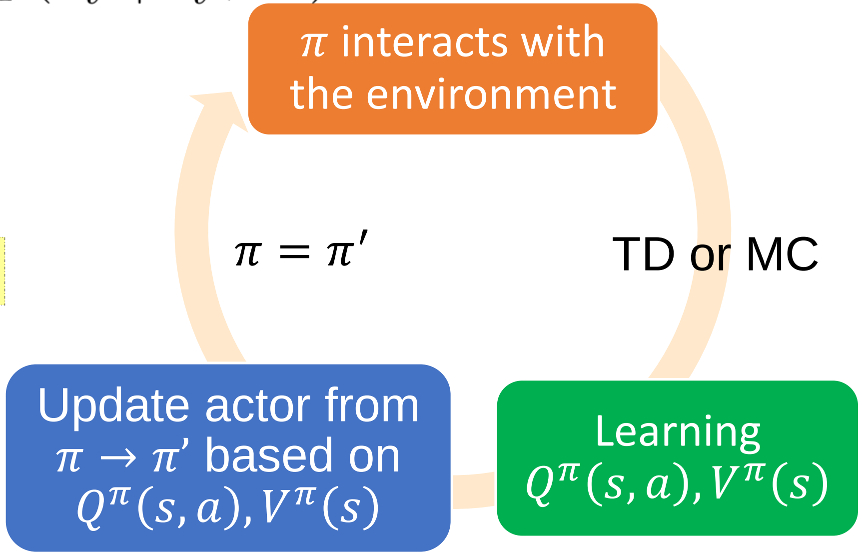
- Estimate value function $Q^{\pi}(s,a), V^{\pi}(s)$
- Update policy based on the value function evaluation π

$$\nabla \mathcal{R}(\theta^{\pi}) = \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} R(\tau^{n}) \nabla \log p(a_t^n \mid s_t^n, \theta^{\pi})$$

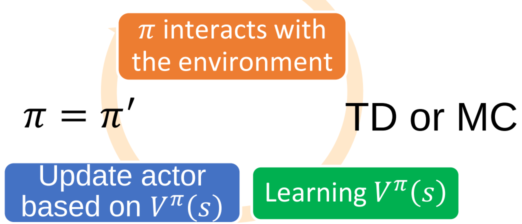
$$Q^{\pi}(s_t^n, a_t^n)$$

π is an actual function that maximizes the value

may work for continuous action



Advantage Actor-Critic



- Learning the policy (actor) using the value evaluated by critic

$$\theta^{\pi'} \leftarrow \theta^{\pi} + \eta \nabla \mathcal{R}(\theta^{\pi})$$

$$\nabla \mathcal{R}(\theta^{\pi}) = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla \log p(a_t^n \mid s_t^n, \theta^{\pi})$$

baseline is added

evaluated by critic

Advantage function: $r_t^n - \left(V^{\pi}(s_t^n) - V^{\pi}(s_{t+1}^n) \right)$

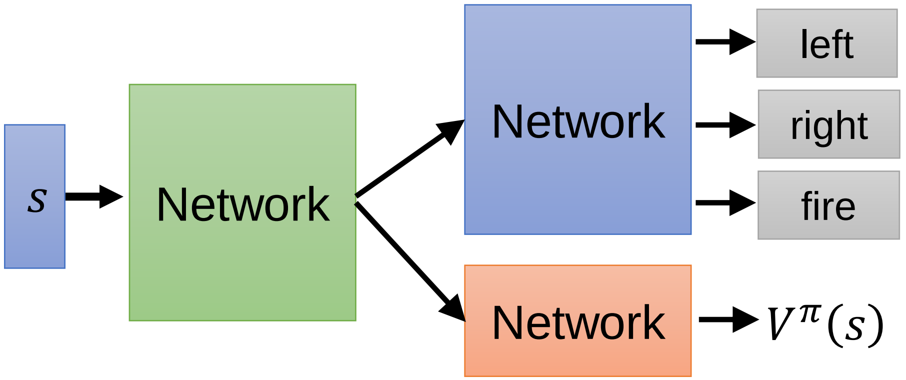
the reward r_t^n we truly obtain when taking action a_t^n

expected reward r_t^n we obtain if we use actor π

- Positive advantage function $\leftrightarrow$ increasing the prob. of action a_t^n
- Negative advantage function $\leftrightarrow$ decreasing the prob. of action a_t^n

Advantage Actor-Critic

- Tips
  - The parameters of actor $\pi(s)$ and critic $V^\pi(s)$ can be shared



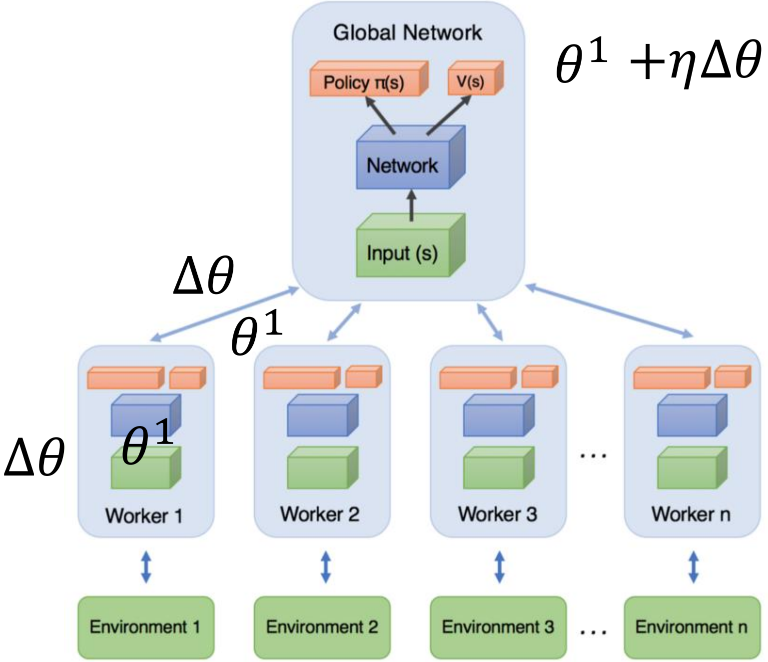
  - Use output entropy as regularization for $\pi(s)$
    - exploration

Asynchronous Advantage Actor-Critic (A3C)

- Asynchronous

1. Copy global parameters
2. Sampling some data
3. Compute gradients
4. Update global models

   (other workers also update models)



Mnih et al., "Asynchronous Methods for Deep Reinforcement Learning," in JMLR, 2016.

Pathwise Derivative Policy Gradient

- Original actor-critic tells that a given action is good or bad
- Pathwise derivative policy gradient tells that which action is good

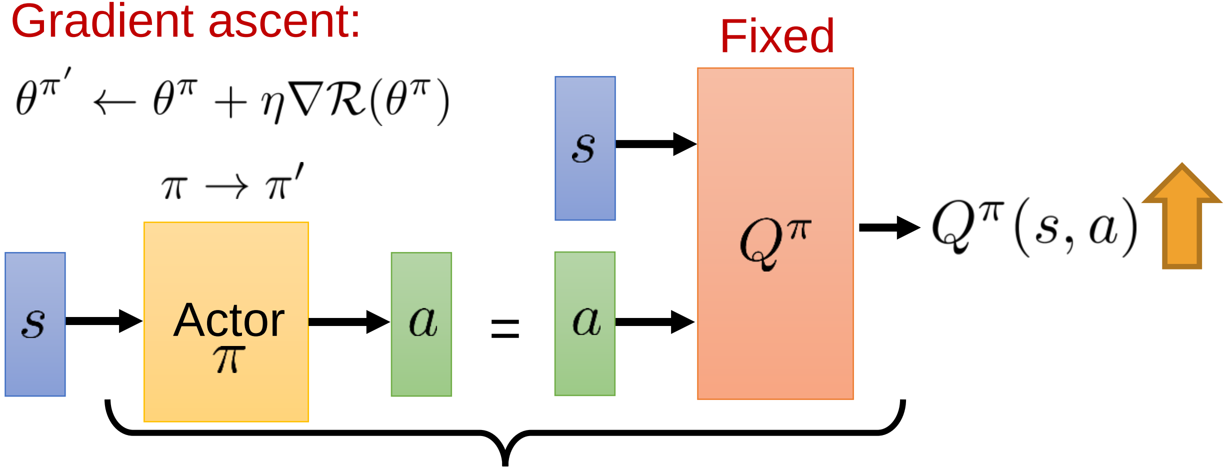
Pathwise Derivative Policy Gradient

$$\pi'(s) = \arg\max_a Q^\pi(s,a)$$ ← an actor's output

Gradient ascent:

$$\theta^{\pi'} \leftarrow \theta^\pi + \eta \nabla \mathcal{R}(\theta^\pi)$$

$$\pi \rightarrow \pi'$$

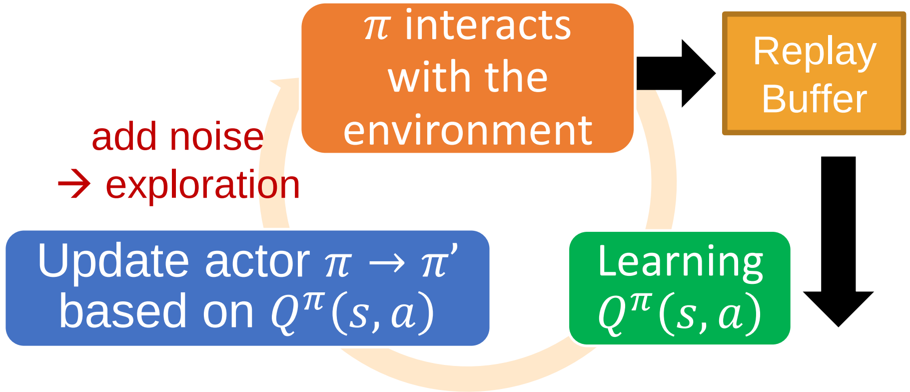


This is a large network

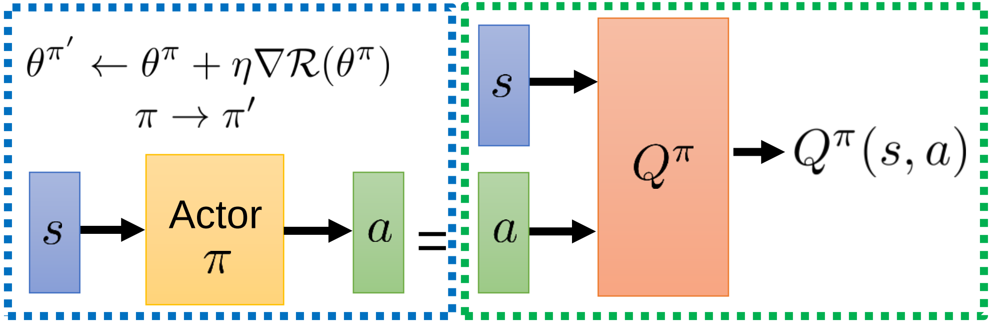
Silver et al., "Deterministic Policy Gradient Algorithms", ICML, 2014.
Lillicrap et al., "Continuous Control with Deep Reinforcement Learning", ICLR, 2016.

Deep Deterministic Policy Gradient (DDPG)

- Idea
  - Critic estimates value of current policy by DQN
  - Actor updates policy in direction that improves Q



Critic provides loss function for actor



Lillicrap et al., "Continuous Control with Deep Reinforcement Learning," ICLR, 2016.

DDPG Algorithm

- Initialize critic network θ^Q and actor network θ^π
- Initialize target critic network $\theta^{Q'} = \theta^Q$ and target actor network $\theta^{\pi'} = \theta^\pi$
- Initialize replay buffer R
- In each iteration
  - Use $\pi(s) + \text{noise}$ to interact with the environment, collect a set of $\{s_t, a_t, r_t, s_{t+1}\}$, put them in R
  - Sample N examples $\{s_n, a_n, r_n, s_{n+1}\}$ from R
  - Update critic Q to minimize $\sum_n (\hat{y}_n - Q(s_n, a_n))^2$

    $\hat{y}_n = r_n + Q'(s_{n+1}, \pi'(s_{n+1}))$ using target networks
  - Update actor π to maximize $\sum_n Q(s_n, \pi(s_n))$
  - Update target networks:

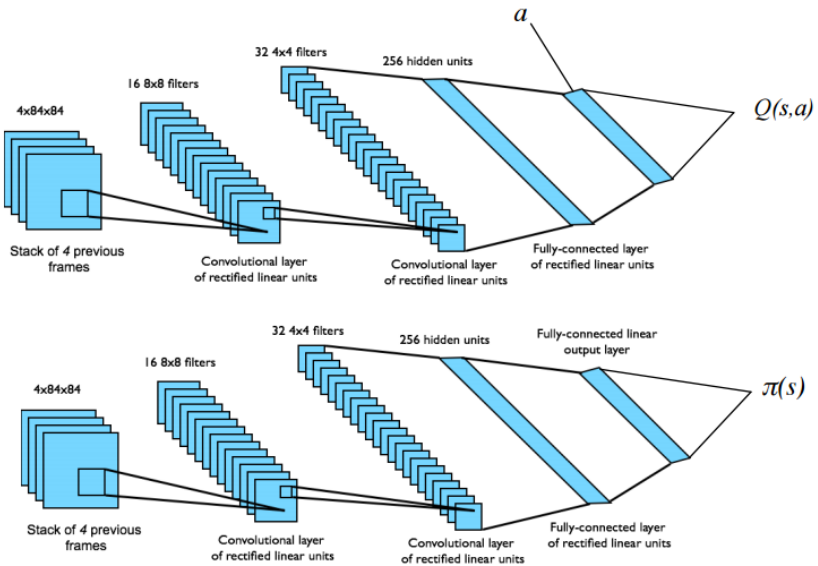
$$\theta^{\pi'} \leftarrow m\theta^\pi + (1-m)\theta^{\pi'}$$
$$\theta^{Q'} \leftarrow m\theta^Q + (1-m)\theta^{Q'}$$

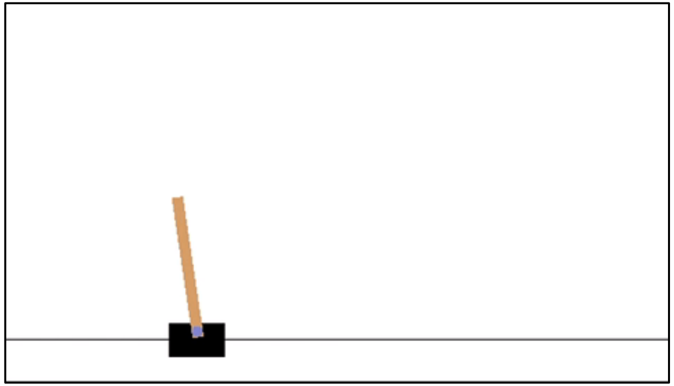
the target networks update slower

Lillicrap et al., “Continuous Control with Deep Reinforcement Learning,” ICLR, 2016.

DDPG in Simulated Physics

- Goal: end-to-end learning of control policy from pixels
  - Input: state is stack of raw pixels from last 4 frames
  - Output: two separate CNNs for Q and π





Lillicrap et al., “Continuous Control with Deep Reinforcement Learning,” ICLR, 2016.

Concluding Remarks

- RL is a general purpose framework for **decision making** under interactions between agent and environment
- Policy gradient
  - learns a policy that maps from state to action
- Actor-critic
  - estimates value function $Q^{\pi}(s,a), V^{\pi}(s)$
  - updates policy based on the value function evaluation π