Applied Deep Learning

Word Embeddings

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Review

Word Representation

Meaning Representations in Computers

- Knowledge-based representation
- Corpus-based representation
  - Atomic symbol
  - Neighbors
    - High-dimensional sparse word vector
    - Low-dimensional dense word vector
      - Method 1 – dimension reduction
      - Method 2 – direct learning

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Corpus-based representation

- Atomic symbols: ***one-hot*** representation

car $[0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ \ldots\ 0]$

car

Issues: difficult to compute the similarity (i.e. comparing “car” and “motorcycle”)

$$\underset{\text{car}}{[0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ \ldots\ 0]} \text{ AND } \underset{\text{motorcycle}}{[0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ \ldots\ 0]} = 0$$

Idea: words with similar meanings often have similar neighbors

Meaning Representations in Computers

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Window-based Co-occurrence Matrix

- Example
  - Window length=1
  - Left or right context
  - Corpus:

I love NTU.
I love deep learning.
I enjoy learning.

similarity > 0

Counts	I	love	enjoy	NTU	deep	learning
I	0	2	1	0	0	0
love	2	0	0	1	1	0
enjoy	1	0	0	0	0	1
NTU	0	1	0	0	0	0
deep	0	1	0	0	0	1
learning	0	0	1	0	1	0

Issues:
- matrix size increases with vocabulary
- high dimensional
- sparsity → poor robustness

Idea: low dimensional word vector

Meaning Representations in Computers

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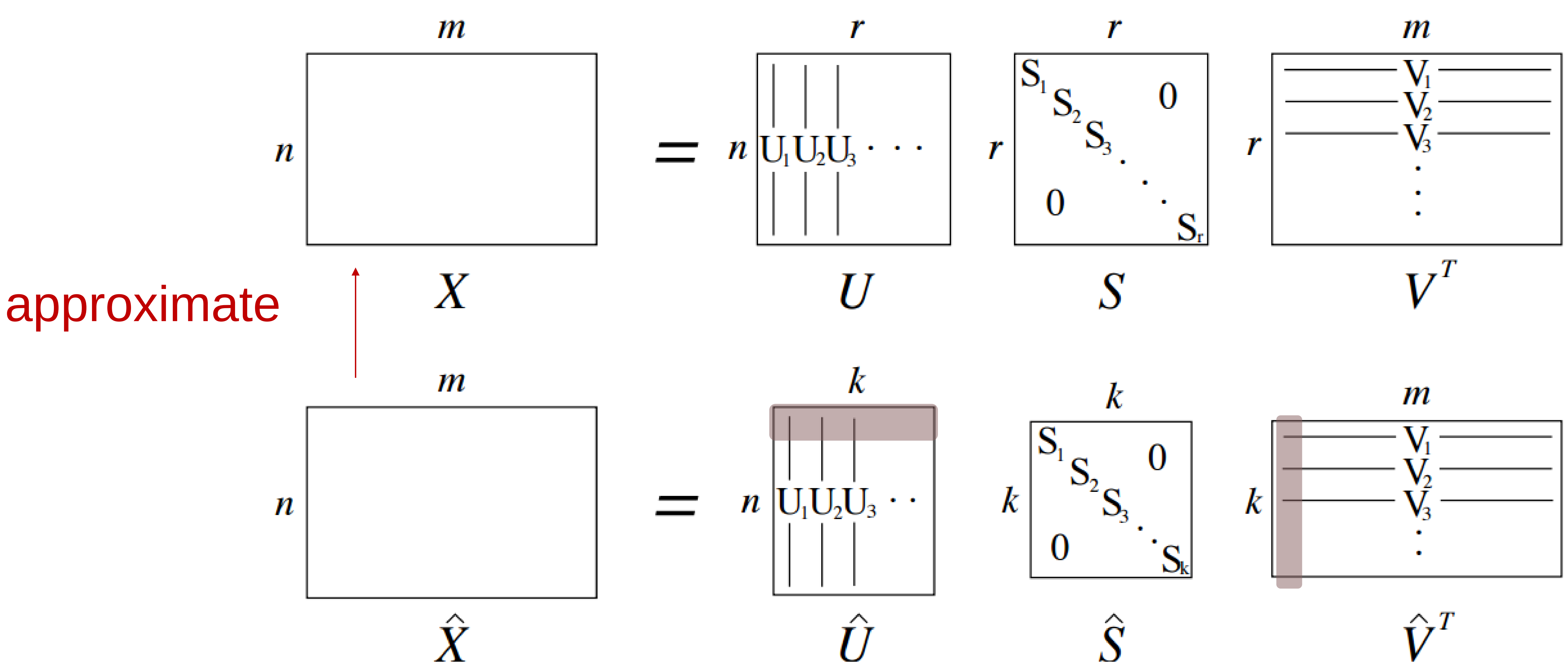
Low-Dimensional Dense Word Vector

- Method 1: dimension reduction on the matrix
- Singular Value Decomposition (SVD) of co-occurrence matrix X



Low-Dimensional Dense Word Vector

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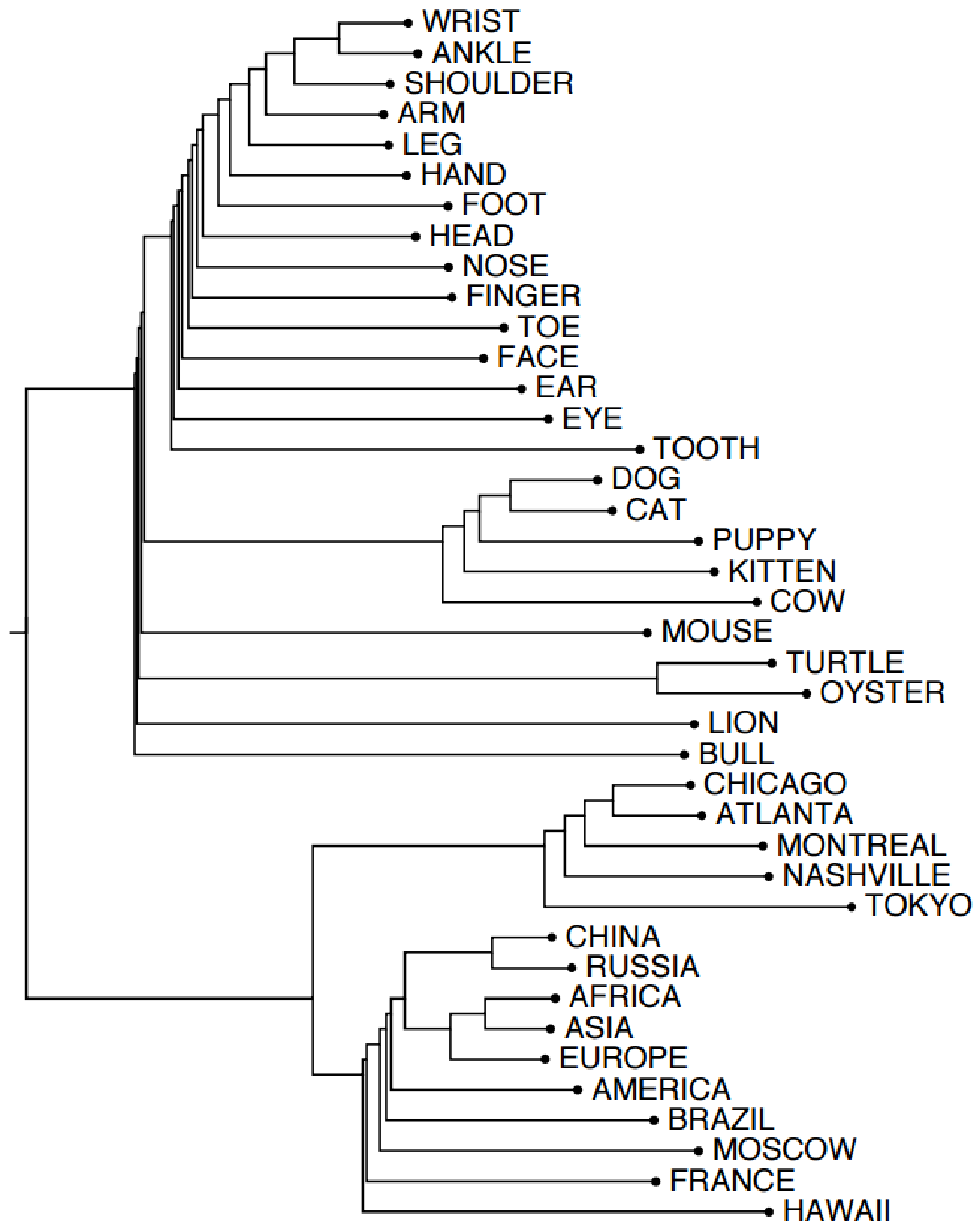
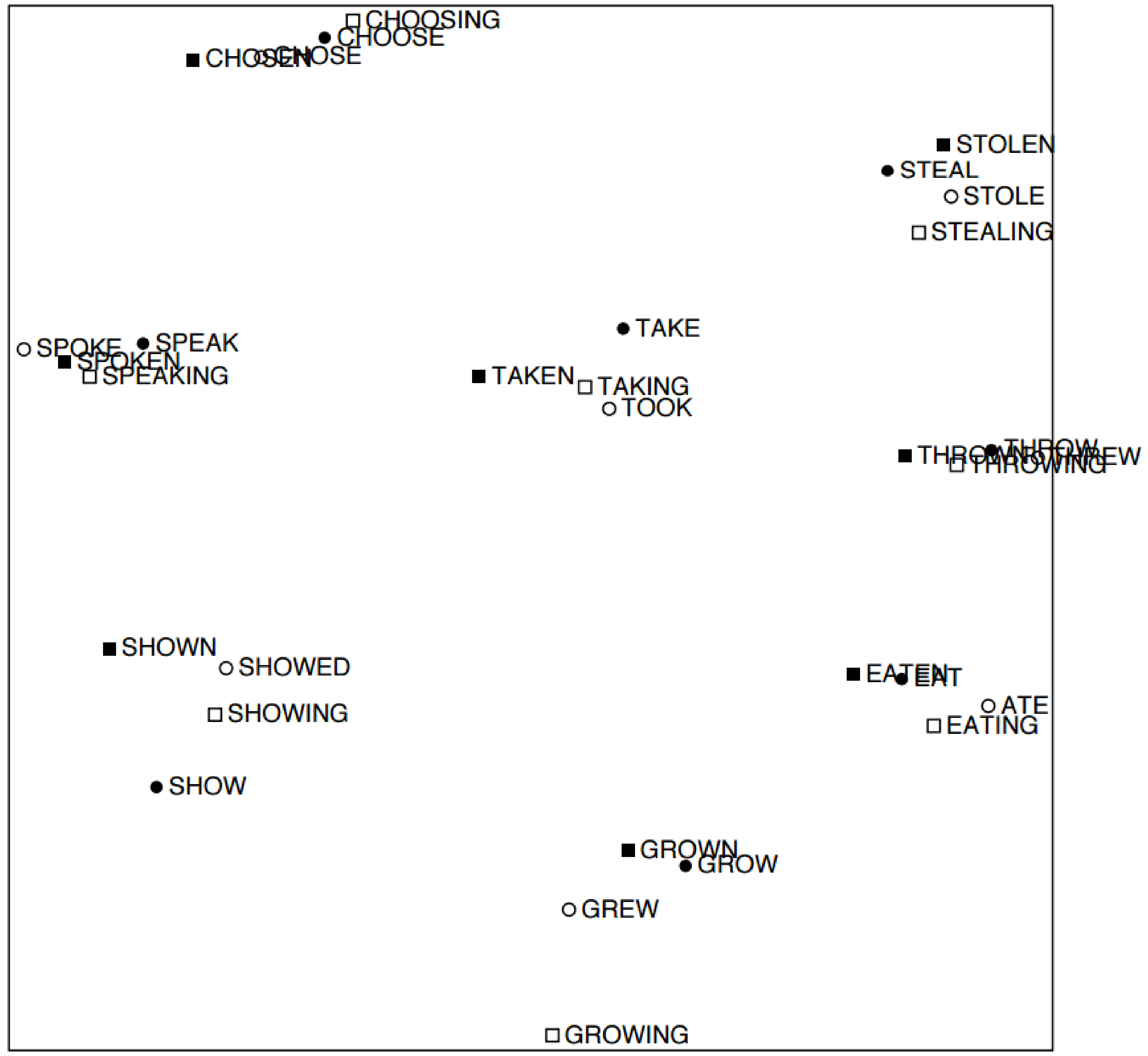


semantic relations



syntactic relations

Issues:

- computationally expensive: $O(mn^2)$ when n<m for n x m matrix
- difficult to add new words

Idea: directly learn low-dimensional word vectors

Word Representation

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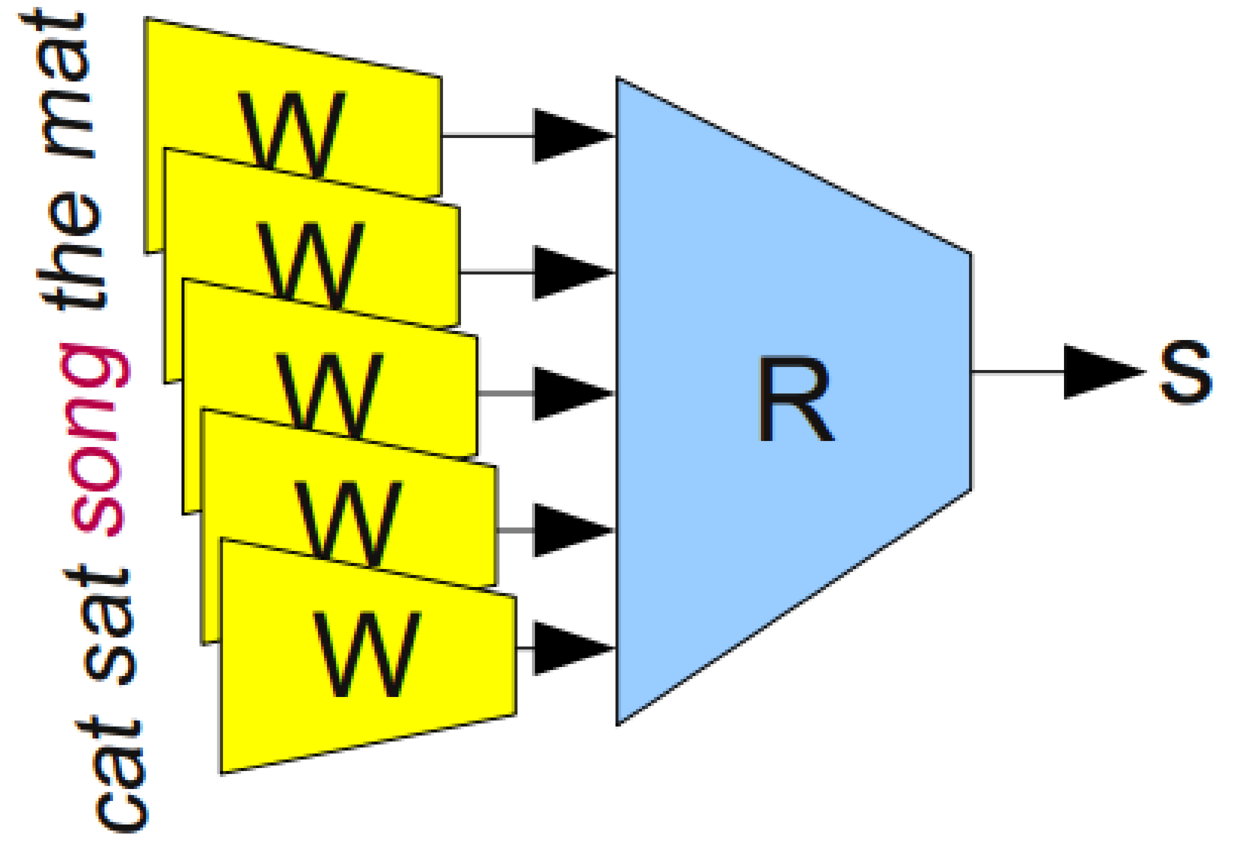
    - High-dimensional sparse word vector
    - Low-dimensional dense word vector
      - Method 1 – dimension reduction
      - Method 2 – direct learning → word embedding

Word Embedding

- Method 2: directly learn low-dimensional word vectors
  - Learning representations by back-propagation. (Rumelhart et al., 1986)
  - A neural probabilistic language model (Bengio et al., 2003)
  - NLP (almost) from Scratch (Collobert & Weston, 2008)
  - Recent and most popular models: word2vec (Mikolov et al. 2013) and Glove (Pennington et al., 2014)

Word Embedding Benefit

- Given an unlabeled training corpus, produce a vector for each word that encodes its semantic information. These vectors are useful because:
  1) semantic similarity between two words can be calculated as the cosine similarity between their corresponding word vectors
  2) word vectors as powerful features for various supervised NLP tasks since the vectors contain semantic information
  3) propagate any information into them via neural networks and update during training



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Word Embeddings

Word2Vec

Word2Vec – Skip-Gram Model

- Goal: predict surrounding words within a window of each word
- Objective function: maximize the probability of any context word given the current center word

$$w_1, w_2, \cdots, \underbrace{w_{t-m}, \cdots, w_{t-1}, w_t, w_{t+1}, \cdots, w_{t+m}}_{\text{context window}}, \cdots, w_{T-1}, w_T$$

w_I w_O

$$p(w_{O,1}, w_{O,2}, \cdots, w_{O,C} \mid w_I) = \prod_{c=1}^{C} p(w_{O,c} \mid w_I)$$

target word vector

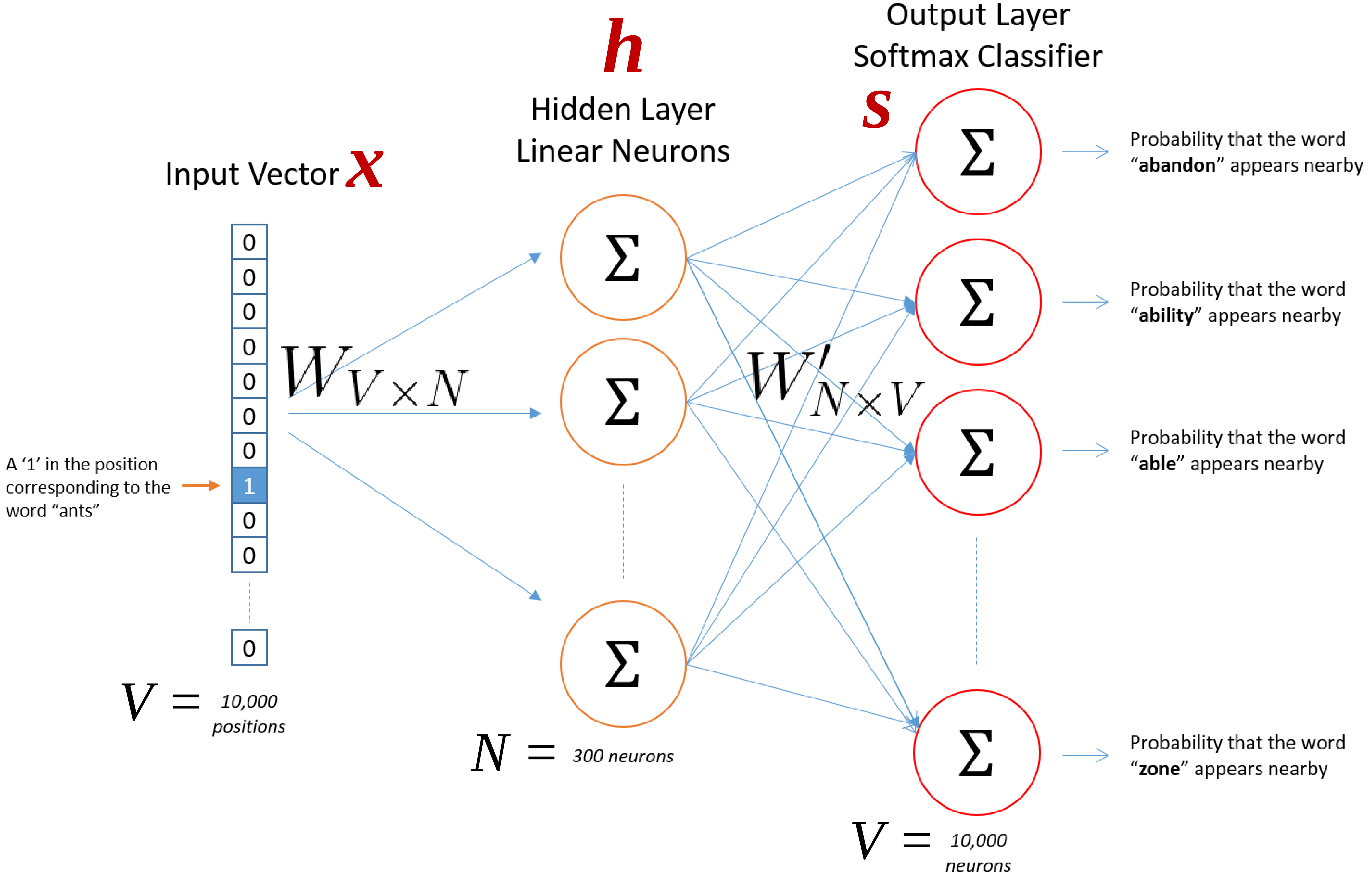
$$C(\theta) = -\sum_{w_I} \sum_{c=1}^{C} \log p(w_{O,c} \mid w_I) \qquad p(w_O \mid w_I) = \frac{\exp(v'^T_{w_O} v_{w_I})}{\sum_j \exp(v'^T_{w_j} v_{w_I})}$$

outside target word

Benefit: faster, easily incorporate a new sentence/document or add a word to vocab

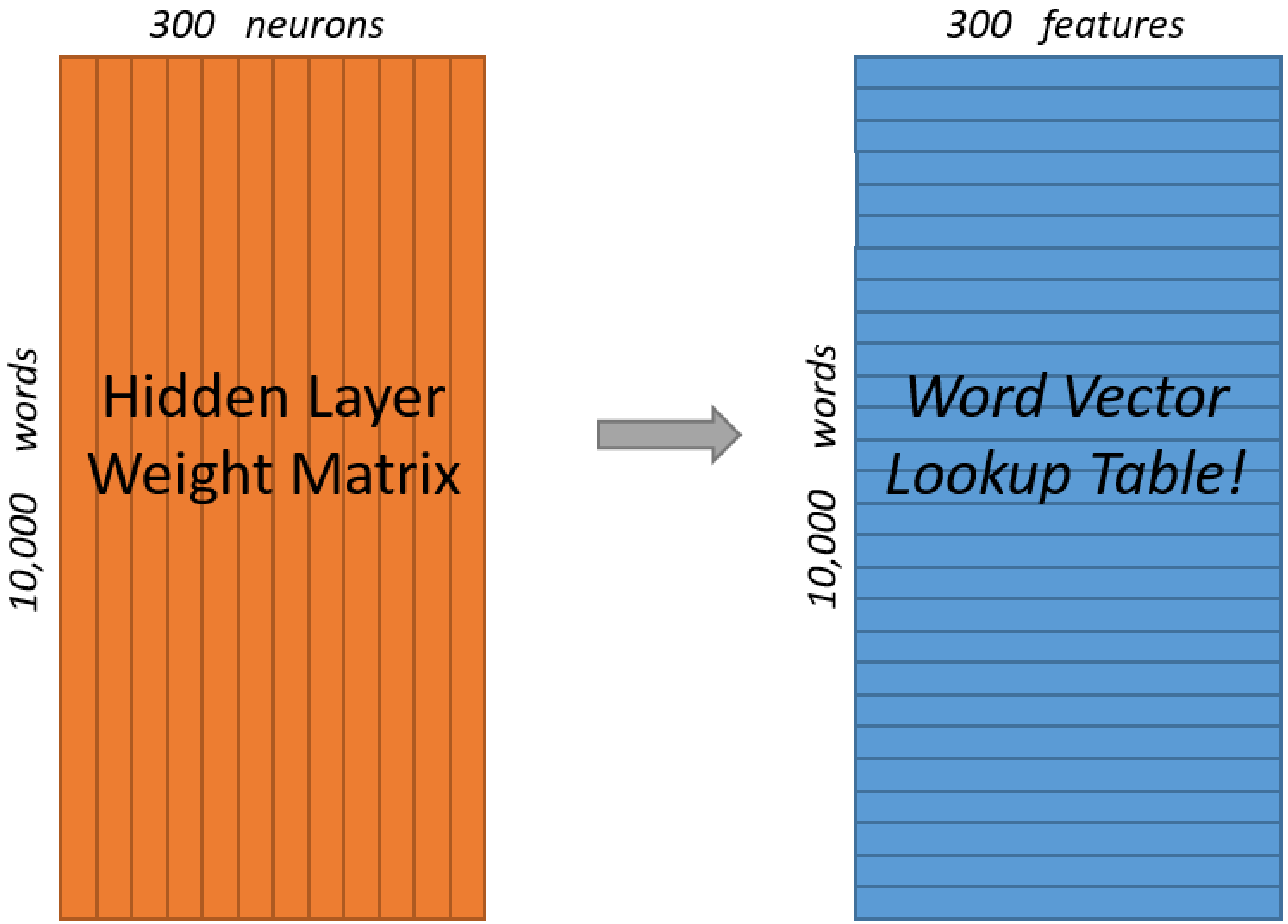
Word2Vec Skip-Gram Illustration

- Goal: predict surrounding words within a window of each word



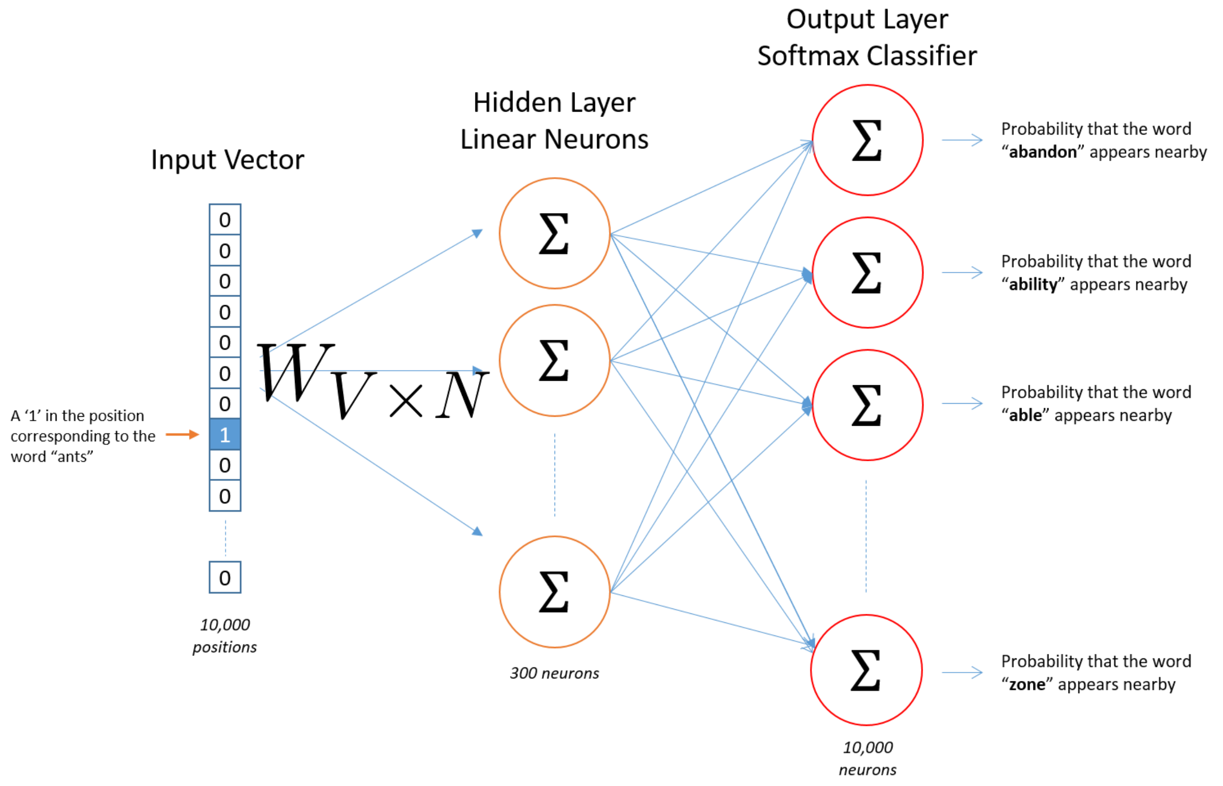
Hidden Layer Matrix → Word Embedding Matrix

$$W_{V \times N}$$



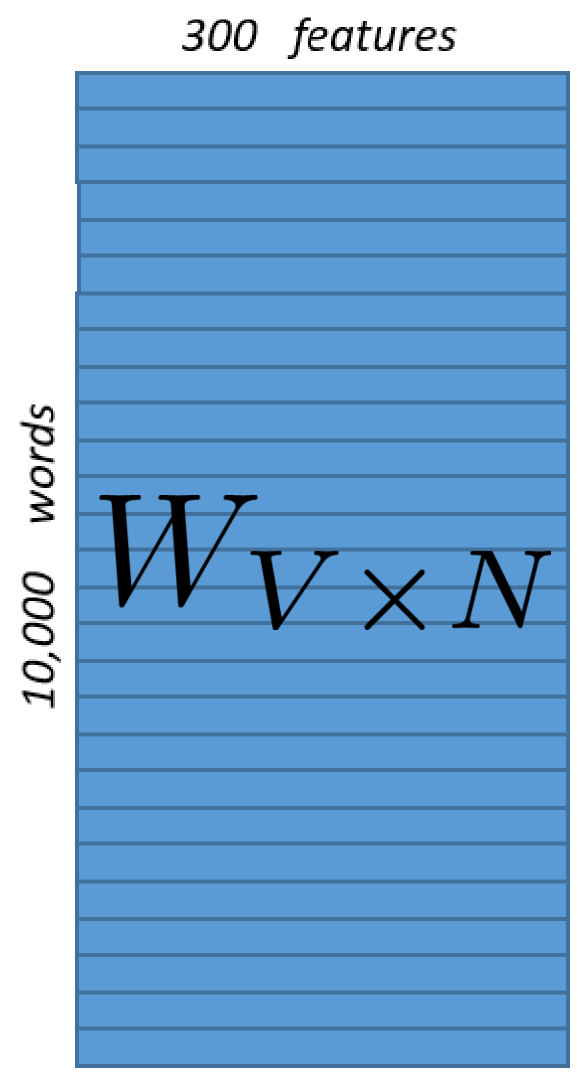
Weight Matrix Relation

- Hidden layer weight matrix = word vector lookup



$$h = x^T W = W_{(k,.)} := v_{w_I}$$

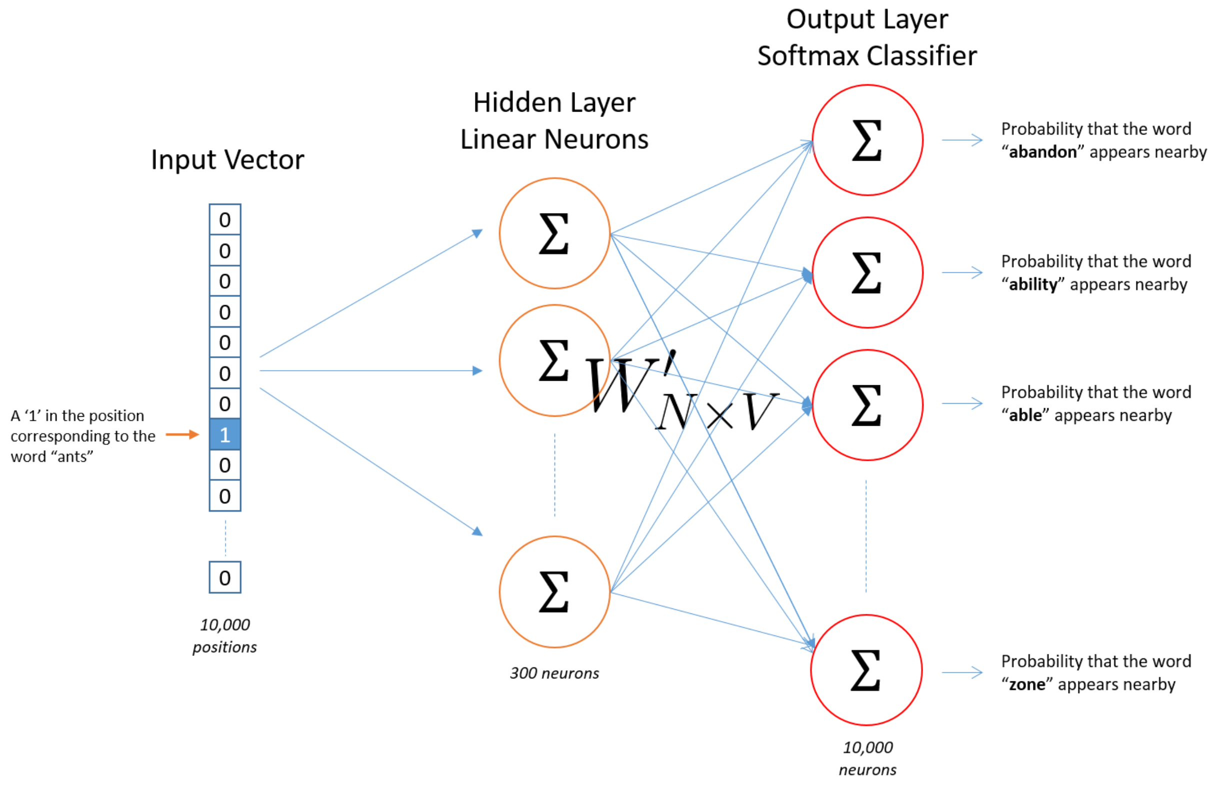
$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 17 & 24 & 1 \\ 23 & 5 & 7 \\ 4 & 6 & 13 \\ 10 & 12 & 19 \\ 11 & 18 & 25 \end{bmatrix} = \begin{bmatrix} 10 & 12 & 19 \end{bmatrix}$$

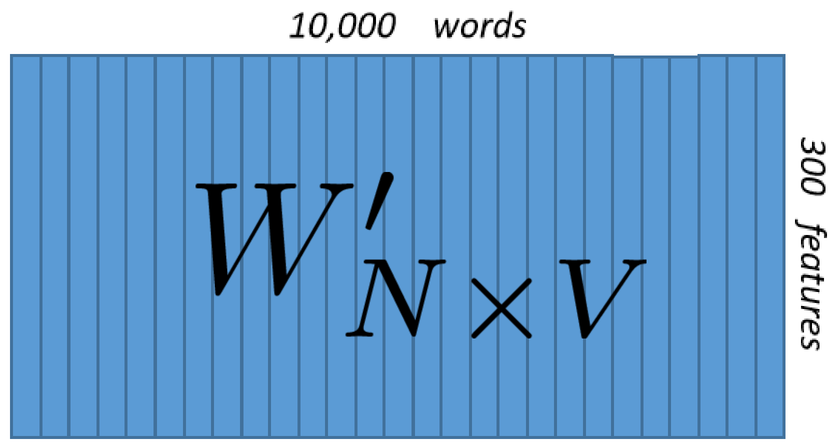


Each vocabulary entry has two vectors: as a **target** word and as a **context** word

Weight Matrix Relation

- Output layer weight matrix = weighted sum as final score



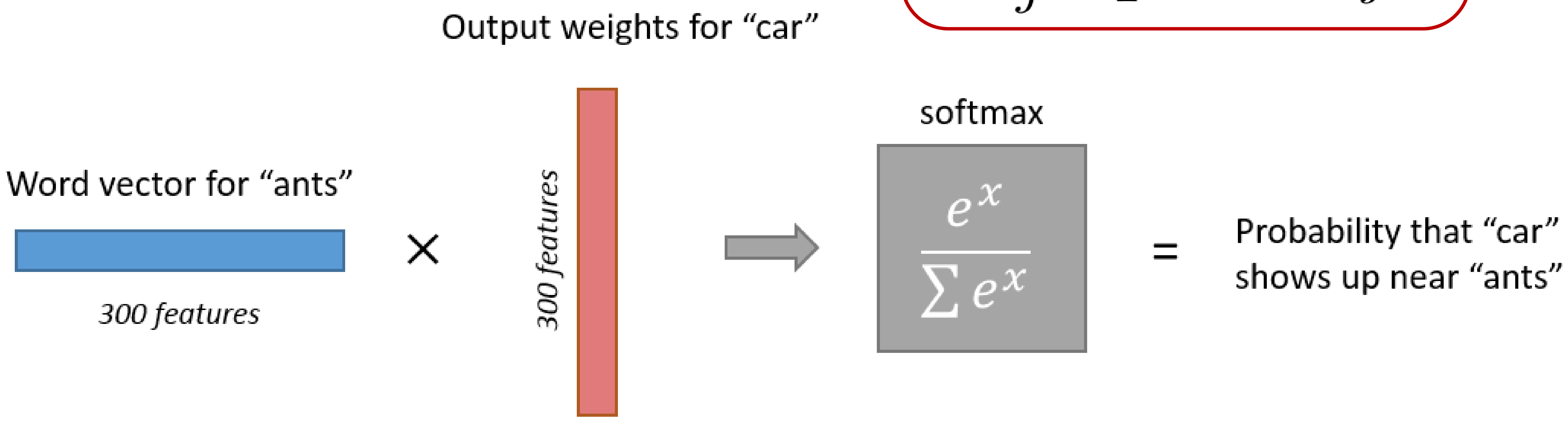


$$s_j = h v'_{w_j}$$

$$p(w_j = w_{O,c} \mid w_I) = y_{jc} = \boxed{\frac{\exp(s_{jc})}{\sum_{j'=1}^{V} \exp(s_{j'})}}$$

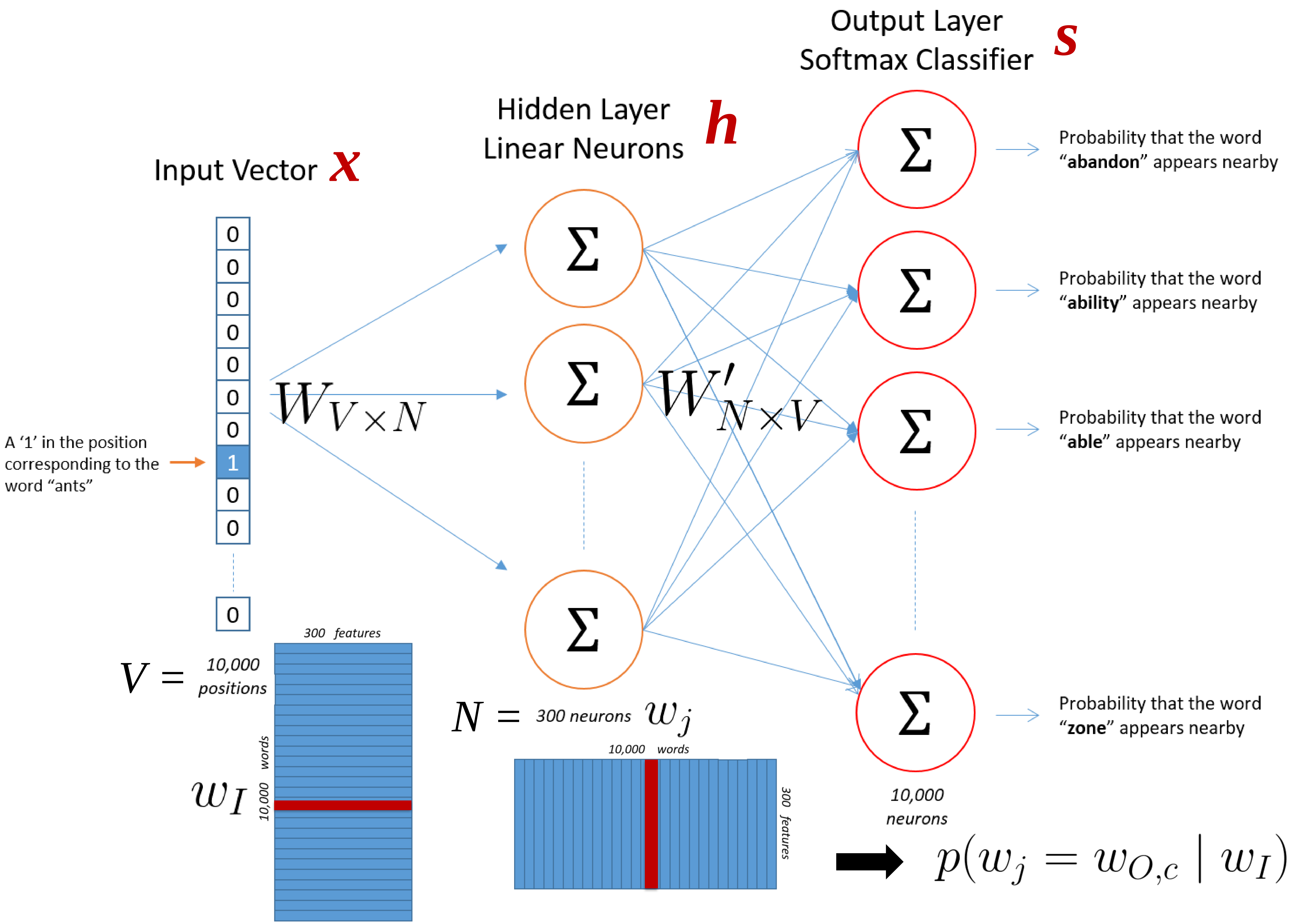
within the context window

softmax



Each vocabulary entry has two vectors: as a **target** word and as a **context** word

Word2Vec Skip-Gram Illustration



$$p(w_j = w_{O,c} \mid w_I)$$

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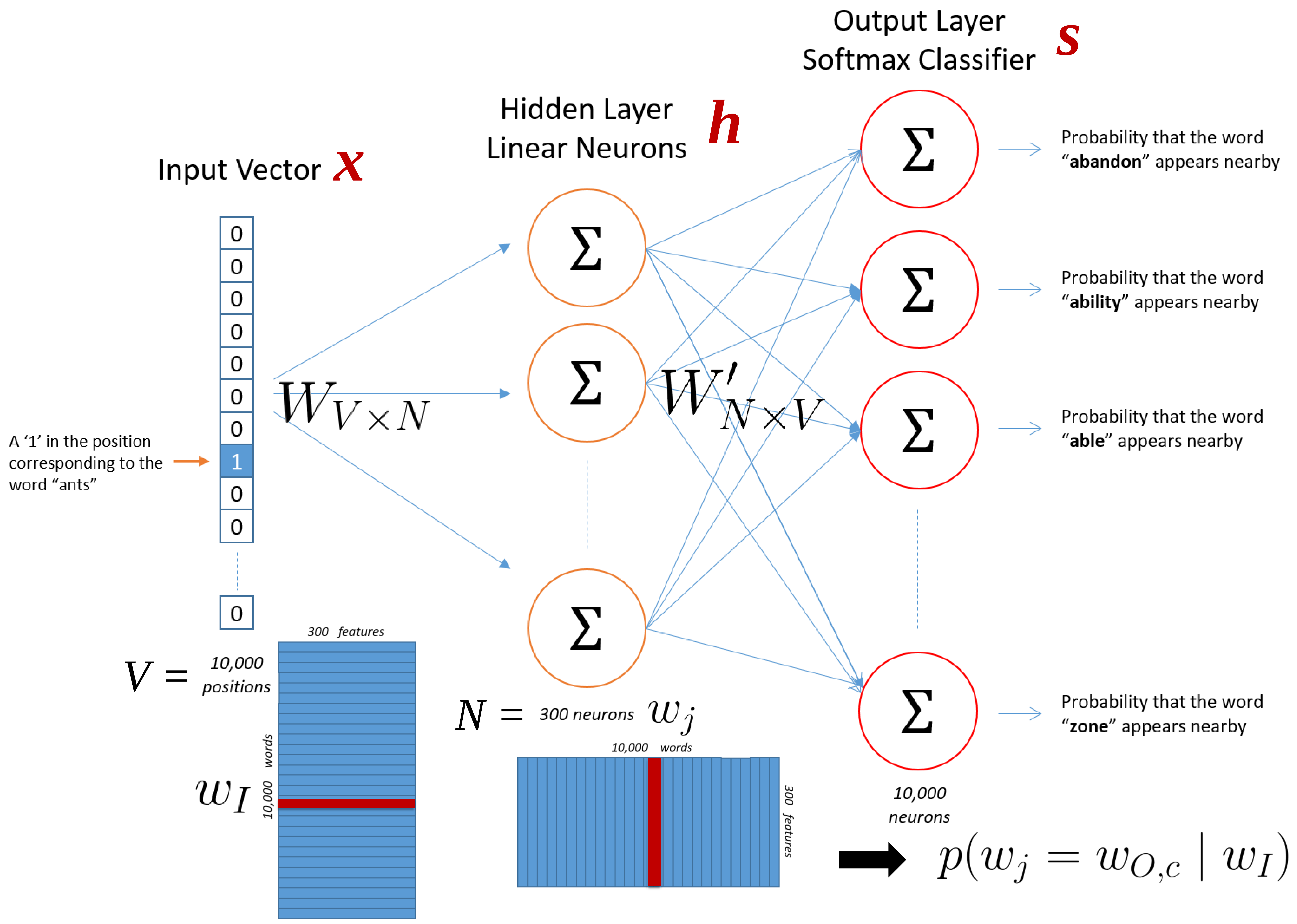
Word Embeddings

Word2Vec Training

Word2Vec Skip-Gram Illustration



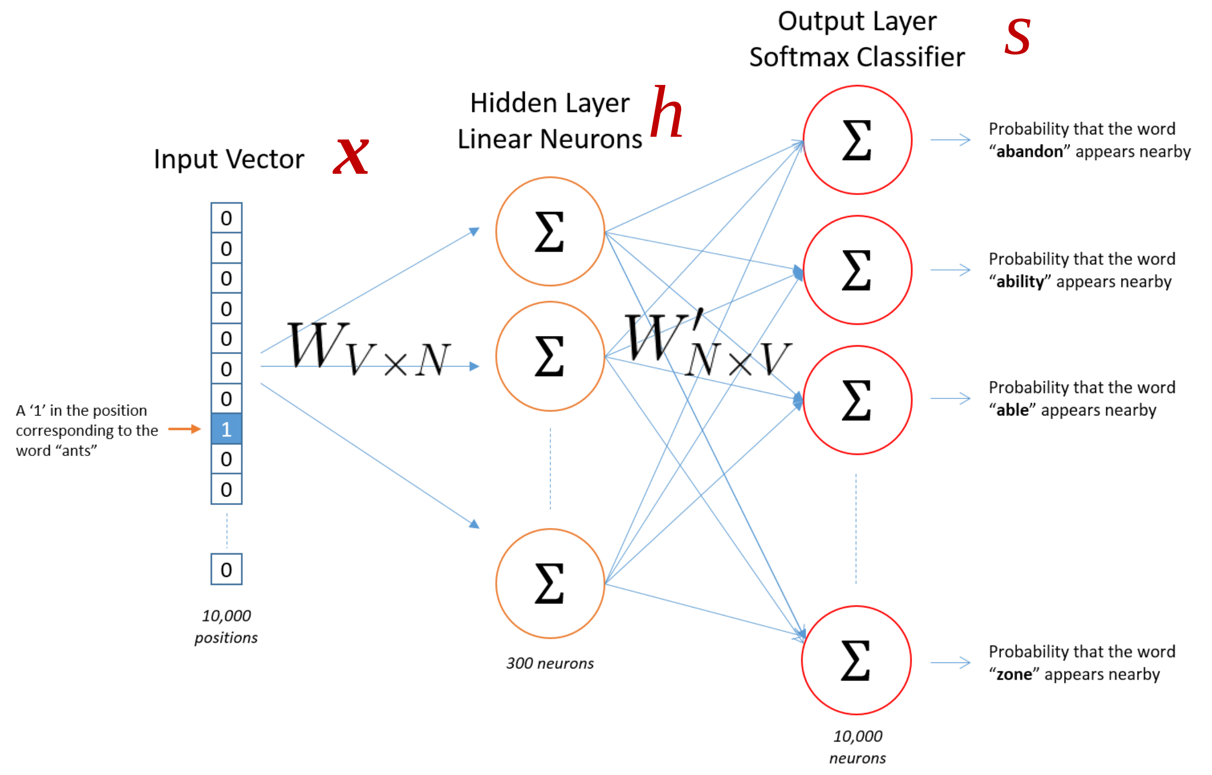
Loss Function

- Given a target word (w_I)

$$
\begin{aligned}
C(\theta) &= -\log p(w_{O,1}, w_{O,2}, \cdots, w_{O,C} \mid w_I) \\
&= -\log \prod_{c=1}^{C} \frac{\exp(s_{j_c})}{\sum_{j'=1}^{V} \exp(s_{j'})} \\
&= -\sum_{c=1}^{C} s_{j_c} + C \log \sum_{j'=1}^{V} \exp(s_{j'})
\end{aligned}
$$

SGD Update for *W'*

- Given a target word (w_I)



$$\frac{\partial C(\theta)}{\partial w'_{ij}} = \sum_{c=1}^{C} \frac{\partial C(\theta)}{\partial s_{j_c}} \frac{\partial s_{j_c}}{\partial w'_{ij}} = \sum_{c=1}^{C} (y_{j_c} - t_{j_c}) \cdot h_i$$

$$s_j = {v'_{w_j}}^T \cdot h$$

$$\frac{\partial C(\theta)}{\partial s_{j_c}} = y_{j_c} - t_{j_c} := e_{j_c}$$

error term

=1, when w_{jc} is within the context window
=0, otherwise

$$\boxed{{w'_{ij}}^{(t+1)} = {w'_{ij}}^{(t)} - \eta \cdot \sum_{c=1}^{C} (y_{j_c} - t_{j_c}) \cdot h_i}$$

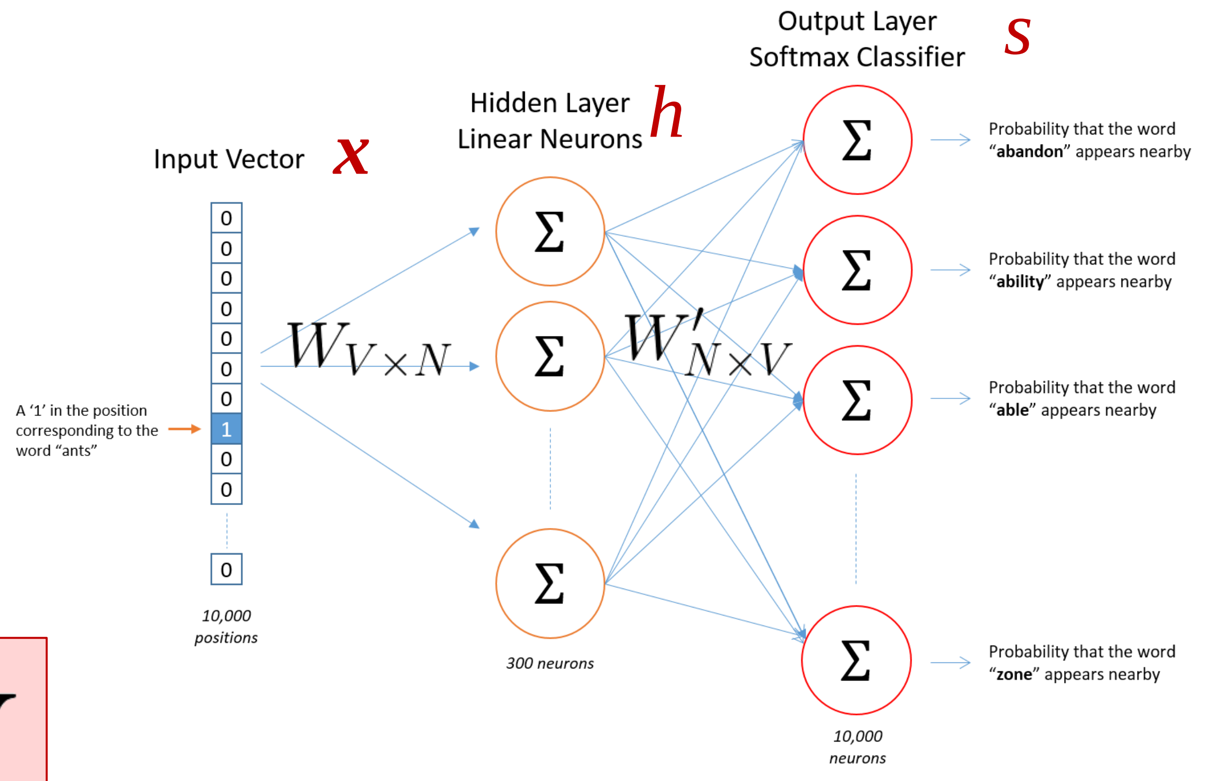
SGD Update for *W*

$$\frac{\partial C(\theta)}{\partial w_{ki}} = \frac{\partial C(\theta)}{\partial h_i}\frac{\partial h_i}{\partial w_{ki}} = \sum_{j=1}^{V}\sum_{c=1}^{C}(y_{jc} - t_{jc}) \cdot w'_{ij} \cdot x_k$$

$$\boxed{h = x^T W}$$

$$\frac{\partial C(\theta)}{\partial h_i} = \sum_{j=1}^{V}\frac{\partial C(\theta)}{\partial s_j}\frac{\partial s_j}{\partial h_i} = \sum_{j=1}^{V}\sum_{c=1}^{C}(y_{jc} - t_{jc}) \cdot w'_{ij}$$

$$\boxed{s_j = {v'_{w_j}}^T \cdot h}$$



$$w_{ij}^{(t+1)} = w_{ij}^{(t)} - \eta \cdot \sum_{j=1}^{V}\sum_{c=1}^{C}(y_{jc} - t_{jc}) \cdot w'_{ij} \cdot x_j$$

SGD Update

$$w'^{(t+1)}_{ij} = w'^{(t)}_{ij} - \eta \cdot \sum_{c=1}^{C} (y_{j_c} - t_{j_c}) \cdot h_i$$

$$EI_j = \sum_{c=1}^{C} (y_{j_c} - t_{j_c})$$

$$v'^{(t+1)}_{w_j} = v'^{(t)}_{w_j} - \eta \cdot EI_j \cdot h$$

$$w^{(t+1)}_{ij} = w^{(t)}_{ij} - \eta \cdot \sum_{j=1}^{V} \sum_{c=1}^{C} (y_{j_c} - t_{j_c}) \cdot w'_{ij} \cdot x_j$$

$$v^{(t+1)}_{w_I} = v^{(t)}_{w_I} - \eta \cdot EH^T$$

$$EH_i = \sum_{j=1}^{V} EI_j \cdot w'_{ij} \cdot x_j$$

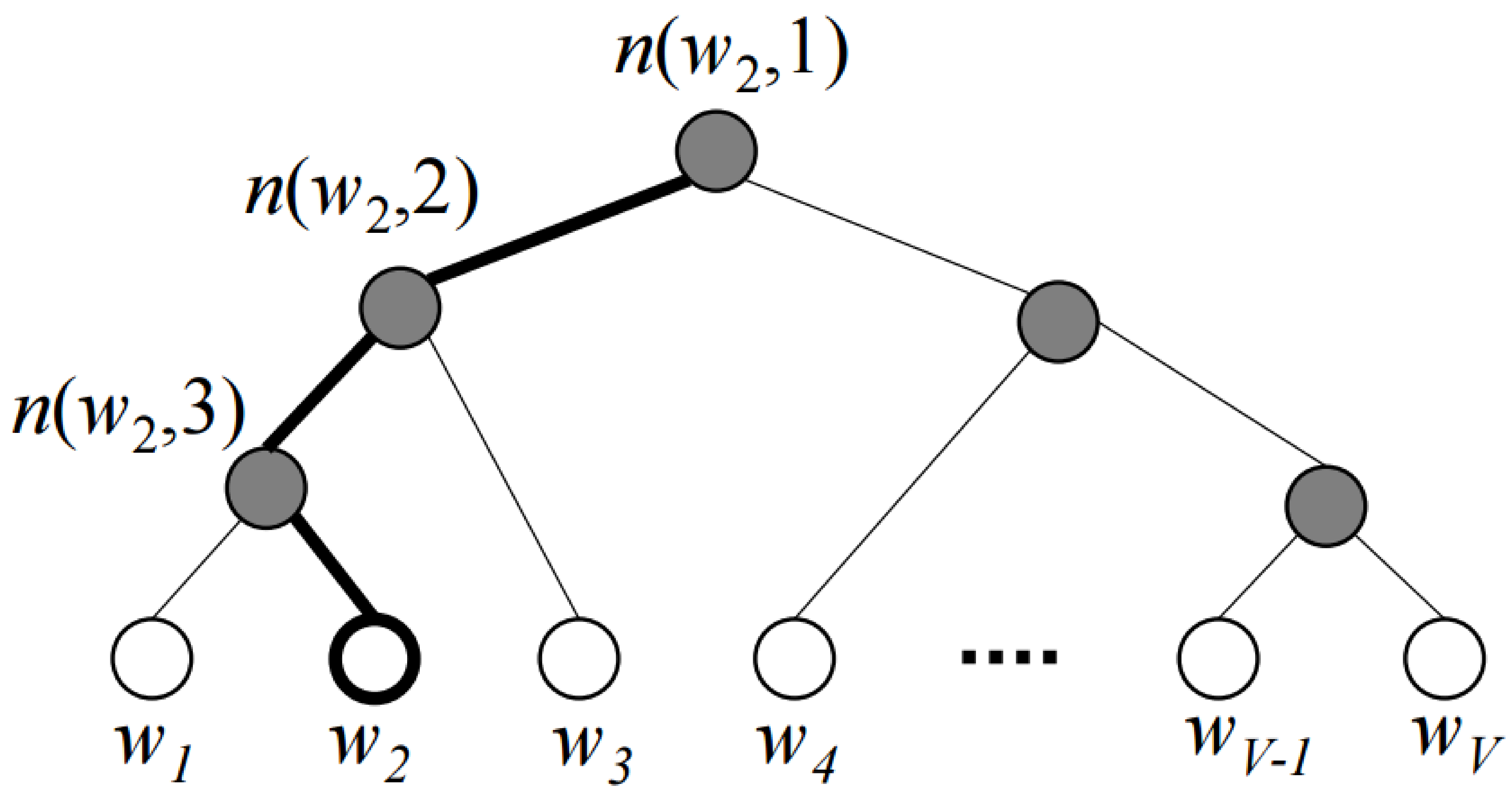
large vocabularies or large training corpora → expensive computations

limit the number of output vectors that must be updated per training instance
→ hierarchical softmax, sampling

Word Embeddings
Negative Sampling

Hierarchical Softmax

- Idea: compute the probability of leaf nodes using the paths



$$O(N) \rightarrow O(\log N)$$

Mikolov et al., “Distributed representations of words and phrases and their compositionality,” in NIPS, 2013.

Negative Sampling

- Idea: only update a sample of output vectors

$$C(\theta) = -\log \sigma({v'_{w_O}}^T v_{w_I}) + \sum_{w_j \in \mathcal{W}_{\text{neg}}} \log \sigma({v'_{w_j}}^T v_{w_I})$$

$$\boxed{{v'_{w_j}}^{(t+1)} = {v'_{w_j}}^{(t)} - \eta \cdot EI_j \cdot h}$$

$$\boxed{EI_j = \sigma({v'_{w_j}}^T v_{w_I}) - t_j}$$

$$\boxed{v_{w_I}^{(t+1)} = v_{w_I}^{(t)} - \eta \cdot EH^T}$$

$$\boxed{EH = \sum_{w_j \in \{w_O\} \cup \mathcal{W}_{\text{neg}}} EI_j \cdot v'_{w_j}}$$

$$w_j \in \{w_O\} \cup \mathcal{W}_{\text{neg}}$$

Mikolov et al., “Distributed representations of words and phrases and their compositionality,” in NIPS, 2013.

Negative Sampling

- Sampling methods
  - Random sampling $w_j \in \{w_O\} \cup \mathcal{W}_{\text{neg}}$
  - Distribution sampling: w_j is sampled from $P(w)$

What is a good $P(w)$?

Idea: less frequent words sampled more often

- Empirical setting: unigram model raised to the power of 3/4

Word	Probability to be sampled for "neg"
is	$0.9^{3/4} = 0.92$
constitution	$0.09^{3/4} = 0.16$
bombastic	$0.01^{3/4} = 0.032$

Mikolov et al., "Distributed representations of words and phrases and their compositionality," in NIPS, 2013.

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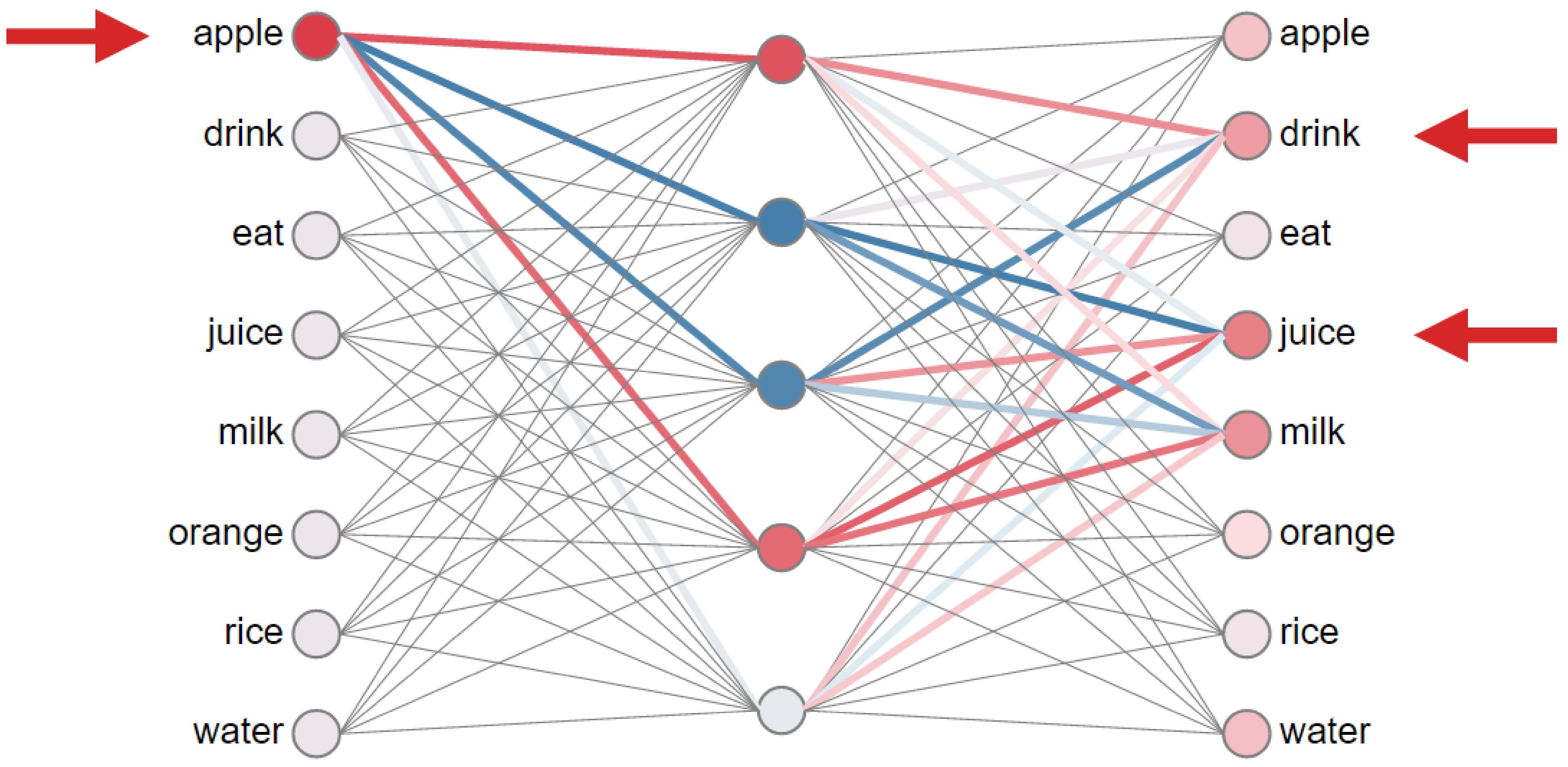
Word Embeddings
Word2Vec Variants

Word2Vec Skip-Gram Visualization https://ronxin.github.io/wevi/

- Skip-gram training data: apple|drink^juice,orange|eat^apple,rice|drink^juice,juice|drink^milk,milk|drink^rice,water|drink^milk,juice|orange^apple,juice|apple^drink,milk|rice^drink,drink|milk^water,drink|water^juice,drink|juice^water



Word2Vec Variants

- **Skip-gram**: predicting surrounding words given the target word (Mikolov+, 2013)

better

$$p(w_{t-m}, \cdots w_{t-1}, w_{t+1}, \cdots , w_{t+m} \mid w_t)$$

- **CBOW (continuous bag-of-words)**: predicting the target word given the surrounding words (Mikolov+, 2013)

$$p(w_t \mid w_{t-m}, \cdots w_{t-1}, w_{t+1}, \cdots , w_{t+m})$$

- **LM (Language modeling)**: predicting the next words given the proceeding contexts (Mikolov+, 2013)

first

$$p(w_{t+1} \mid w_t)$$

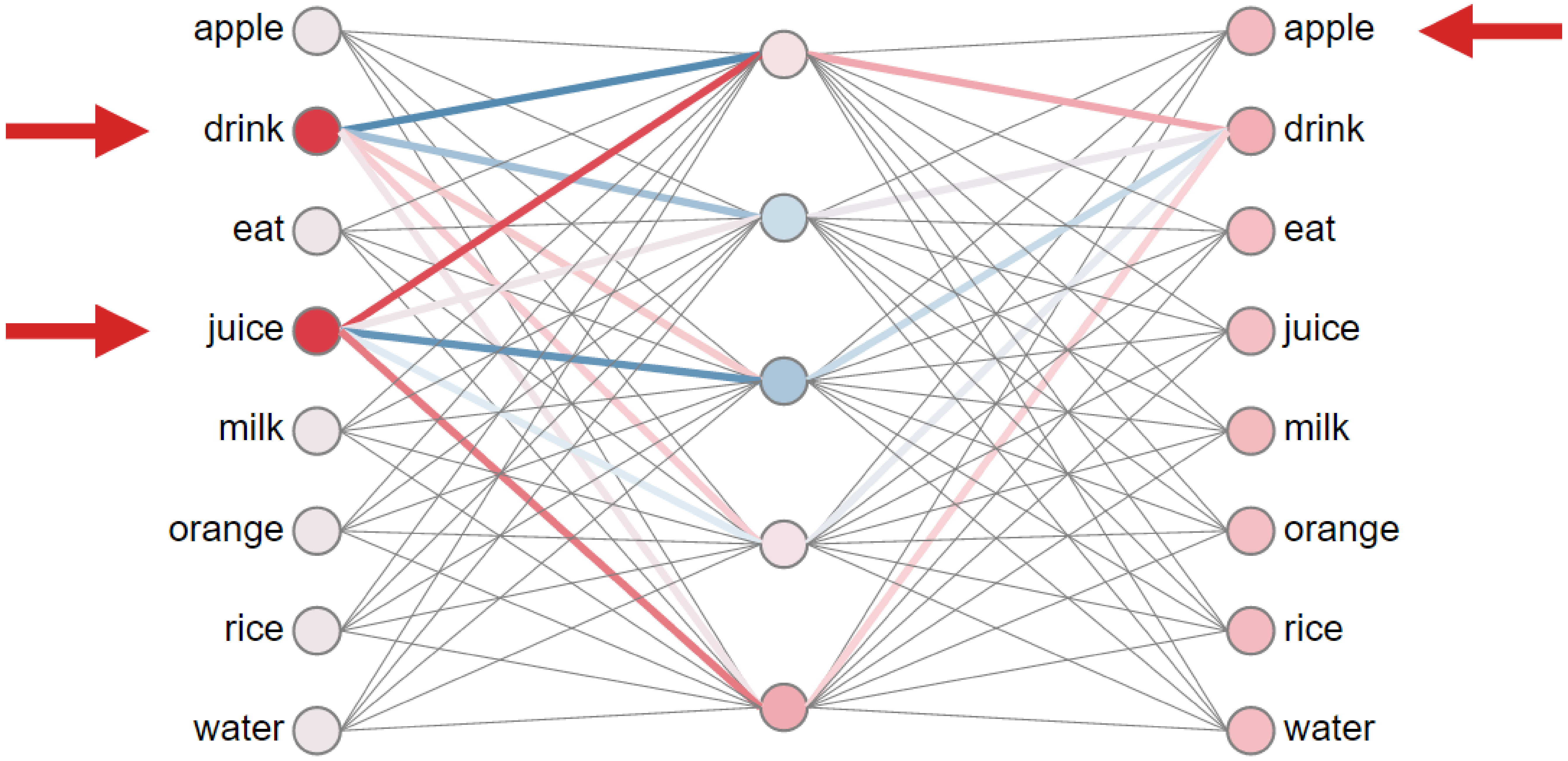
Practice the derivation by yourself!!

Mikolov et al., "Efficient estimation of word representations in vector space," in *ICLR Workshop*, 2013.
Mikolov et al., "Linguistic regularities in continuous space word representations," in *NAACL HLT*, 2013.

Word2Vec CBOW

- Goal: predicting the target word given the surrounding words

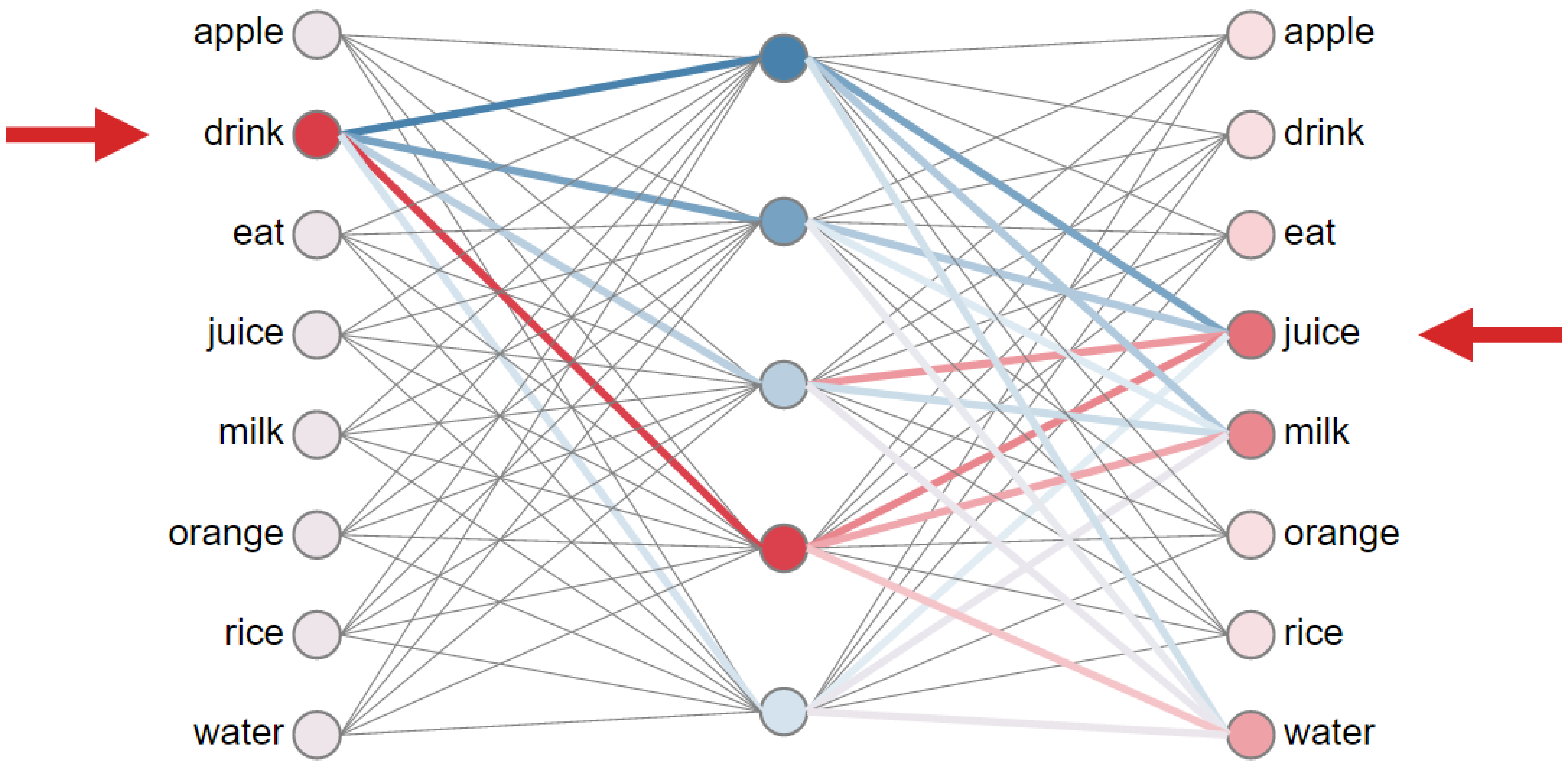
$$p(w_t \mid w_{t-m}, \cdots w_{t-1}, w_{t+1}, \cdots , w_{t+m})$$



Word2Vec LM

- Goal: predicting the next words given the proceeding contexts

$$p(w_{t+1} \mid w_t)$$



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Word Embeddings

GloVe

Comparison

- **Count-based**
  - LSA, HAL (Lund & Burgess), COALS (Rohde et al), Hellinger-PCA (Lebret & Collobert)
  - Pros
    - ✓ Fast training
    - ✓ Efficient usage of statistics
  - Cons
    - ✓ Primarily used to capture word similarity
    - ✓ Disproportionate importance given to large counts

- **Direct prediction**
  - NNLM, HLBL, RNN, Skipgram/CBOW (Bengio et al; Collobert & Weston; Huang et al; Mnih & Hinton; Mikolov et al; Mnih & Kavukcuoglu)
  - Pros
    - ✓ Generate improved performance on other tasks
    - ✓ Capture complex patterns beyond word similarity
  - Cons
    - ✓ Benefits mainly from large corpus
    - ✓ Inefficient usage of statistics

Combining the benefits from both worlds → GloVe

GloVe

- Idea: ratio of co-occurrence probability can encode meaning
- P_{ij} is the probability that word w_j appears in the context of word w_i

$$P_{ij} = P(w_j \mid w_i) = X_{ij}/X_i$$

- Relationship between the words w_i and w_j

	x = solid	x = gas	x = water	x = random
$P(x \mid \text{ice})$	large	small	large	small
$P(x \mid \text{stream})$	small	large	large	small
$\dfrac{P(x \mid \text{ice})}{P(x \mid \text{stream})}$	large	small	~ 1	~ 1

Pennington et al., "GloVe: Global Vectors for Word Representation," in EMNLP, 2014.

GloVe

- The relationship of w_i and w_j approximates the ratio of their co-occurrence probabilities with various w_k

$$F(w_i, w_j, \tilde{w}_k) = \frac{P_{ik}}{P_{jk}}$$

$$F(w_i - w_j, \tilde{w}_k) = \frac{P_{ik}}{P_{jk}}$$

$$F((v_{w_i} - v_{w_j})^T v'_{\tilde{w}_k}) = \frac{P_{ik}}{P_{jk}} \qquad F(\cdot) = \exp(\cdot)$$

$$v_{w_i} \cdot v'_{\tilde{w}_k} = v_{w_i}^T v'_{\tilde{w}_k} = \log P(w_k \mid w_i)$$

Pennington et al., "GloVe: Global Vectors for Word Representation," in EMNLP, 2014.

GloVe

$$
\begin{aligned}
v_{w_i} \cdot v'_{\tilde{w}_j} &= v_{w_i}^T v'_{\tilde{w}_j} = \log P(w_j \mid w_i) \\
&= \log P_{ij} = \log(X_{ij}) - \log(X_i)
\end{aligned}
$$

$$\boxed{P_{ij} = X_{ij} / X_i}$$

$$v_{w_i}^T v'_{\tilde{w}_j} + b_i + \tilde{b}_j = \log(X_{ij})$$

$$C(\theta) = \sum_{i,j=1}^{V} f(P_{ij})(v_{w_i} \cdot v'_{\tilde{w}_j} - \log P_{ij})^2$$

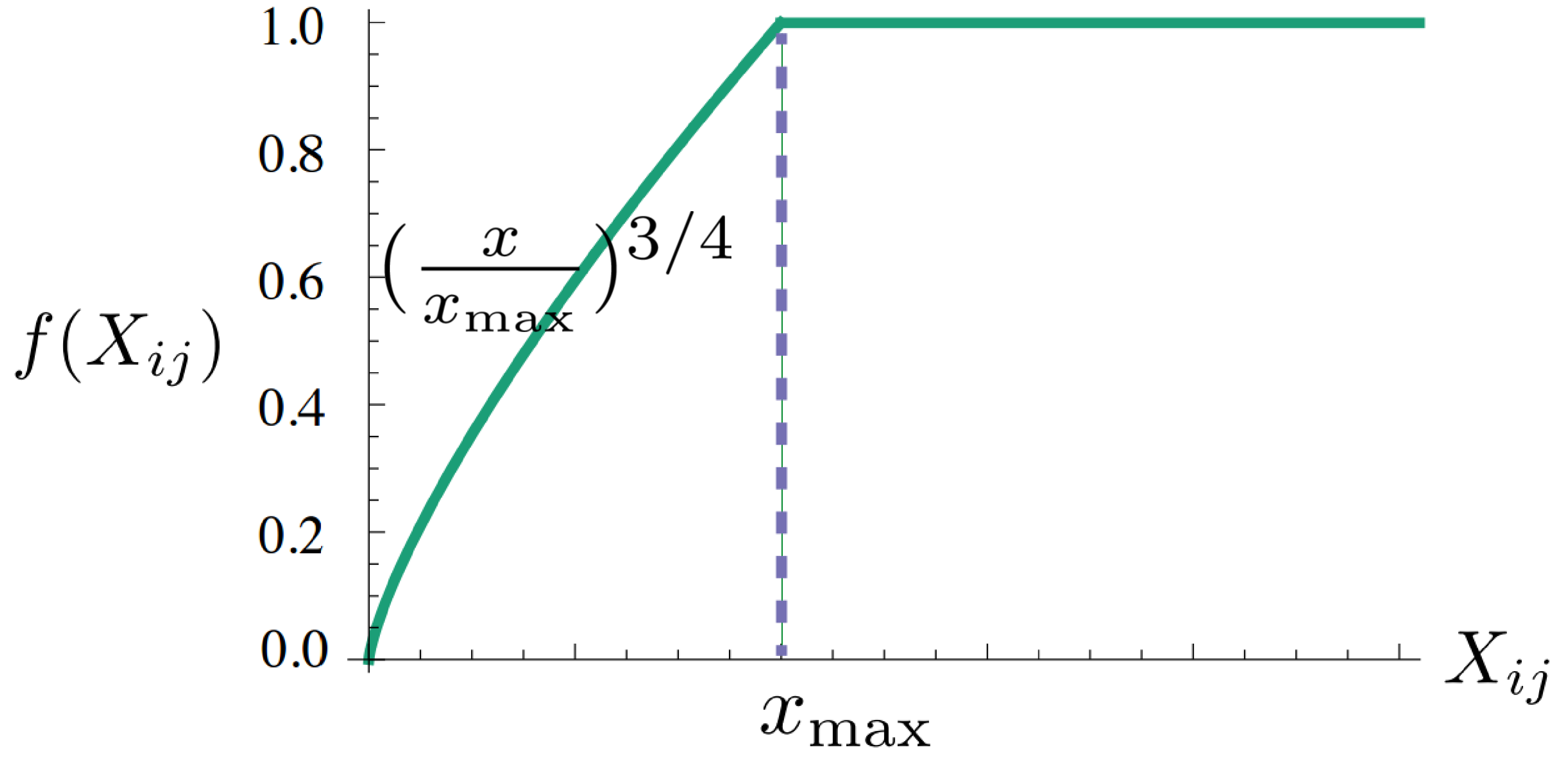
$$C(\theta) = \sum_{i,j=1}^{V} f(X_{ij})(v_{w_i}^T v'_{\tilde{w}_j} + b_i + \tilde{b}_j - \log X_{ij})^2$$

Pennington et al., ”GloVe: Global Vectors for Word Representation,” in EMNLP, 2014.

GloVe – Weighted Least Squares Regression Model

$$C(\theta) = \sum_{i,j=1}^{V} f(X_{ij})(v_{w_i}^T v'_{\tilde{w}_j} + b_i + \tilde{b}_j - \log X_{ij})^2$$

- Weighting function should obey
  - $f(0) = 0$
  - $f(x)$ should be non-decreasing so that *rare co-occurrences* are not overweighted
  - $f(x)$ should be relatively small for large values of x, so that *frequent co-occurrences* are not overweighted



fast training, scalable, good performance even with small corpus, and small vectors

Pennington et al., "GloVe: Global Vectors for Word Representation," in EMNLP, 2014.

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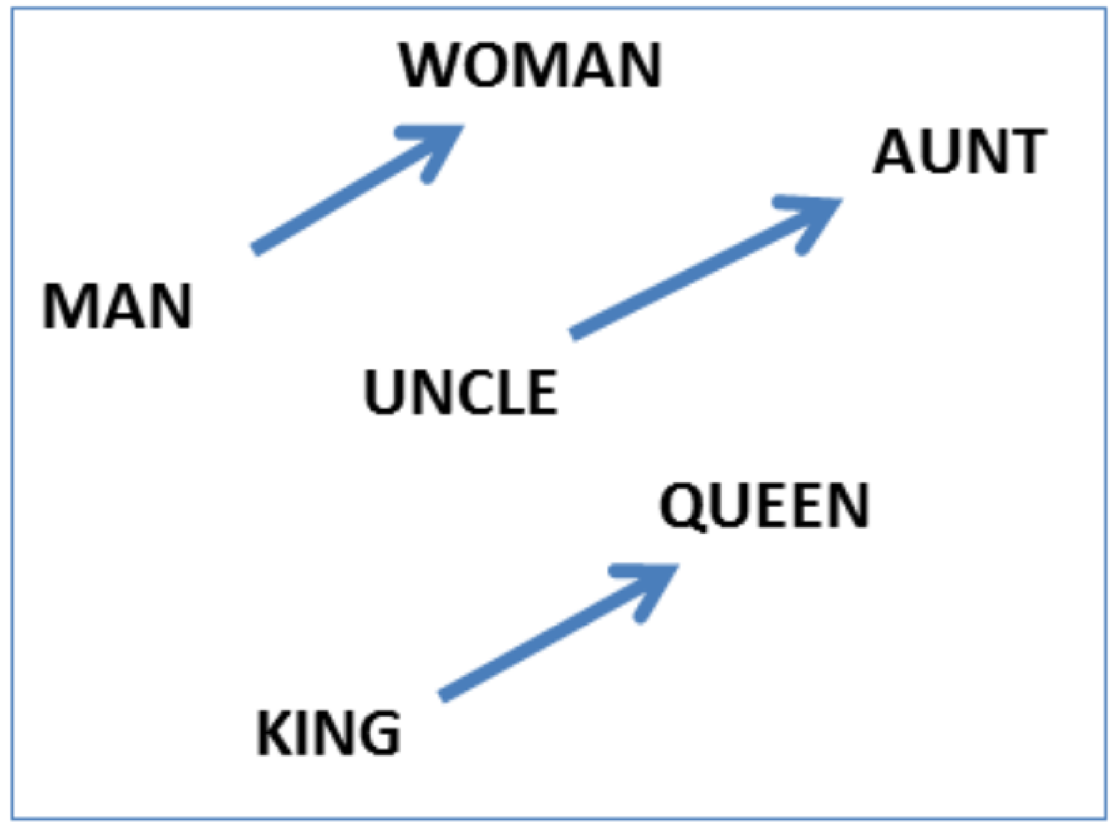
Word Vector Evaluation

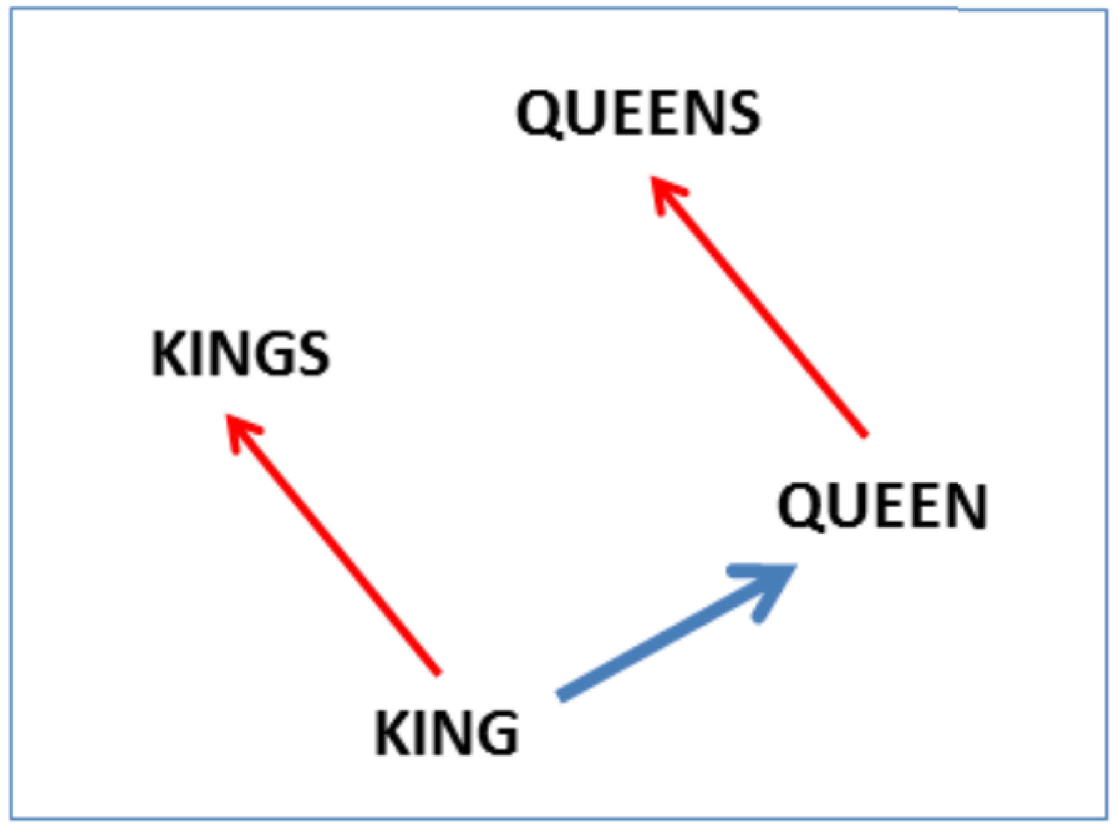
Intrinsic Evaluation – Word Analogies

- Word linear relationship $w_A : w_B = w_C : w_x$

$$x = \arg\max_x \frac{(v_{w_B} - v_{w_A} + v_{w_C})^T v_{w_x}}{\left\| v_{w_B} - v_{w_A} + v_{w_C} \right\|}$$

- **Syntactic** and **Semantic** example questions [link]





Issue: what if the information is there but not linear

Intrinsic Evaluation – Word Analogies

- Word linear relationship $w_A : w_B = w_C : w_x$
- Syntactic and **Semantic** example questions [link]

city---in---state
Chicago : Illinois = Houston : Texas
Chicago : Illinois = Philadelphia : Pennsylvania
Chicago : Illinois = Phoenix : Arizona
Chicago : Illinois = Dallas : Texas
Chicago : Illinois = Jacksonville : Florida
Chicago : Illinois = Indianapolis : Indiana
Chicago : Illinois = Aus8n : Texas
Chicago : Illinois = Detroit : Michigan
Chicago : Illinois = Memphis : Tennessee
Chicago : Illinois = Boston : Massachusetts

Issue: different cities may have same name

capital---country
Abuja : Nigeria = Accra : Ghana
Abuja : Nigeria = Algiers : Algeria
Abuja : Nigeria = Amman : Jordan
Abuja : Nigeria = Ankara : Turkey
Abuja : Nigeria = Antananarivo : Madagascar
Abuja : Nigeria = Apia : Samoa
Abuja : Nigeria = Ashgabat : Turkmenistan
Abuja : Nigeria = Asmara : Eritrea
Abuja : Nigeria = Astana : Kazakhstan

Issue: can change with time

Intrinsic Evaluation – Word Analogies

- Word linear relationship $w_A : w_B = w_C : w_x$
- **Syntactic** and Semantic example questions [link]

superlative
bad : worst = big : biggest
bad : worst = bright : brightest
bad : worst = cold : coldest
bad : worst = cool : coolest
bad : worst = dark : darkest
bad : worst = easy : easiest
bad : worst = fast : fastest
bad : worst = good : best
bad : worst = great : greatest

past tense
dancing : danced = decreasing : decreased
dancing : danced = describing : described
dancing : danced = enhancing : enhanced
dancing : danced = falling : fell
dancing : danced = feeding : fed
dancing : danced = flying : flew
dancing : danced = generating : generated
dancing : danced = going : went
dancing : danced = hiding : hid
dancing : danced = hiding : hit

Intrinsic Evaluation – Word Correlation

- Comparing word correlation with human-judged scores
- Human-judged word correlation [link]

Word 1	Word 2	Human-Judged Score
tiger	cat	7.35
tiger	tiger	10.00
book	paper	7.46
computer	internet	7.58
plane	car	5.77
professor	doctor	6.62
stock	phone	1.62

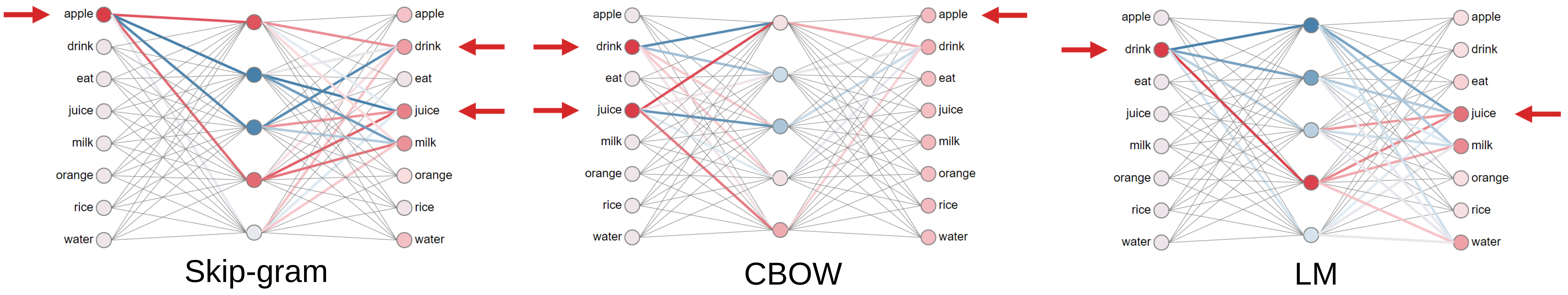
Ambiguity: synonym or same word with different POSs

Extrinsic Evaluation – Subsequent Task

- Goal: use word vectors in neural net models built for subsequent tasks
- Benefit
  - Ability to also classify words accurately
    - Ex. countries cluster together a classifying location words should be possible with word vectors
  - Incorporate any information into them other tasks
    - Ex. project sentiment into words to find most positive/negative words in corpus

Concluding Remarks

- Low dimensional word vector
  - word2vec



  - GloVe: combining count-based and direct learning
- Word vector evaluation
  - Intrinsic: word analogy, word correlation
  - Extrinsic: subsequent task