Pricing CDOs with the Fourier Transform Method

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ABSTRACT

In pricing CDOs, the correlation between assets is a major issue. A multi-asset joint distribution function is too complicated to transform to a loss distribution. Chen and Zang developed a method to price a large credit portfolio. This method is composed of two elements: Factor model and Fourier inversion. This thesis generalizes their method. We assume that there are two common factors, and all assets have their own correlations with the common factors. Since the assets in a pool are not affected by only one common factor, and each asset has different degrees of influence over that common factor, we generalize the one-factor model with more accurate performance.

Chapter 1

Introduction

1.1 Introduction

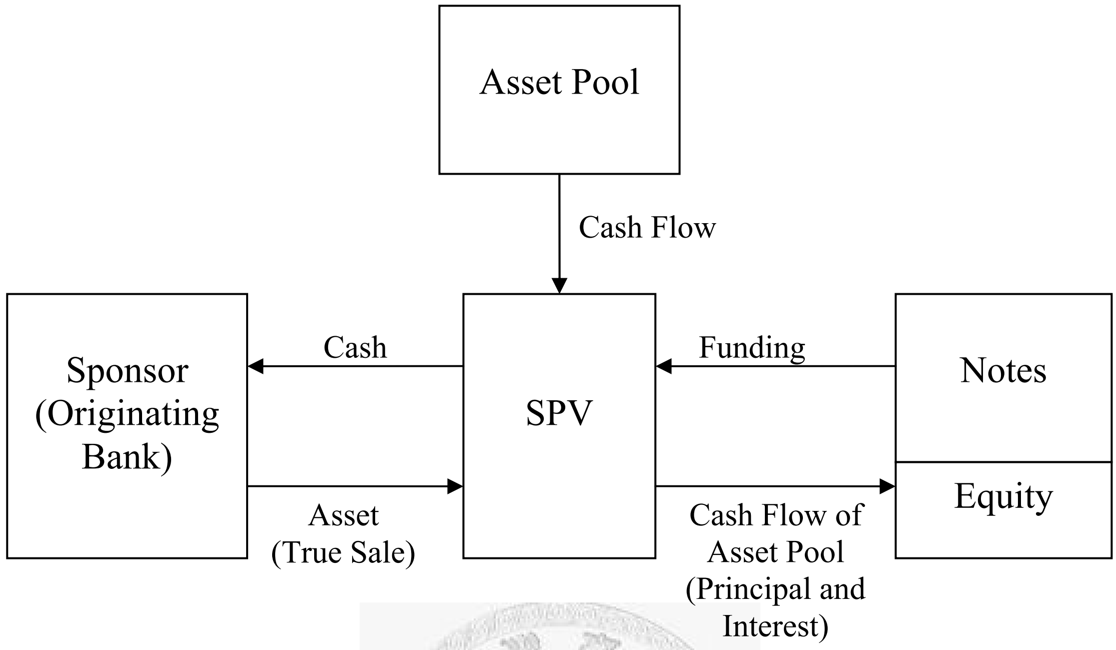
A collateralized debt obligation (CDO) is an asset-backed security which is backed by a diversified pool of one or more classes of debts. All CDO investors receive the cash flow of the collateralized asset. However, they share the credit risk of the assets too. CDO investors are usually divided into several tranches by senior/subordinate of loss bearing. Roughly speaking, notes investors could be divided into four parts: senior, mezzanine, subordinate, and equity tranches. When loss occurs, the lowest tranche (i.e., the equity tranche) has to absorb loss first. If equity tranche is exhausted, then the higher tranche (i.e., the subordinate tranche) has to absorb the following loss. Each CDO tranche has different expected return and risk. Investors can choose the tranche to match their own risk preferences.

To be a sponsor of CDO, banks have to establish a special purpose vehicle (SPV), which is in charge of issuing securities and holding collateral. This mechanism ensures the operation of the CDO even if the originating bank goes bankrupt. Furthermore, the sponsor always retains the equity tranche of a CDO. A typical CDO structure is shown in Figure 1.1.1. The primary advantages of CDO products are as following. First, it can remove the credit risk and interest rate risk from the originating bank. Second, issuing CDO is a comparatively cheaper way of funding for a bank.

Generally speaking, CDOs are usually divided as: Cash flow CDOs, Market value CDOs, and Synthetic CDOs.

A cash flow CDO is the simplest one. All cash flows of collateral assets are directly paid to investors. For a market value CDO, CDO manager actively trades the assets in the collateral pool. The payment CDO investors receive depends on rate of return of the collateral pool. The return is calculated by mark to market, and it is apparently determined by the performance of the CDO manager.

Figure 1.1.1: A typical structure of a CDO.

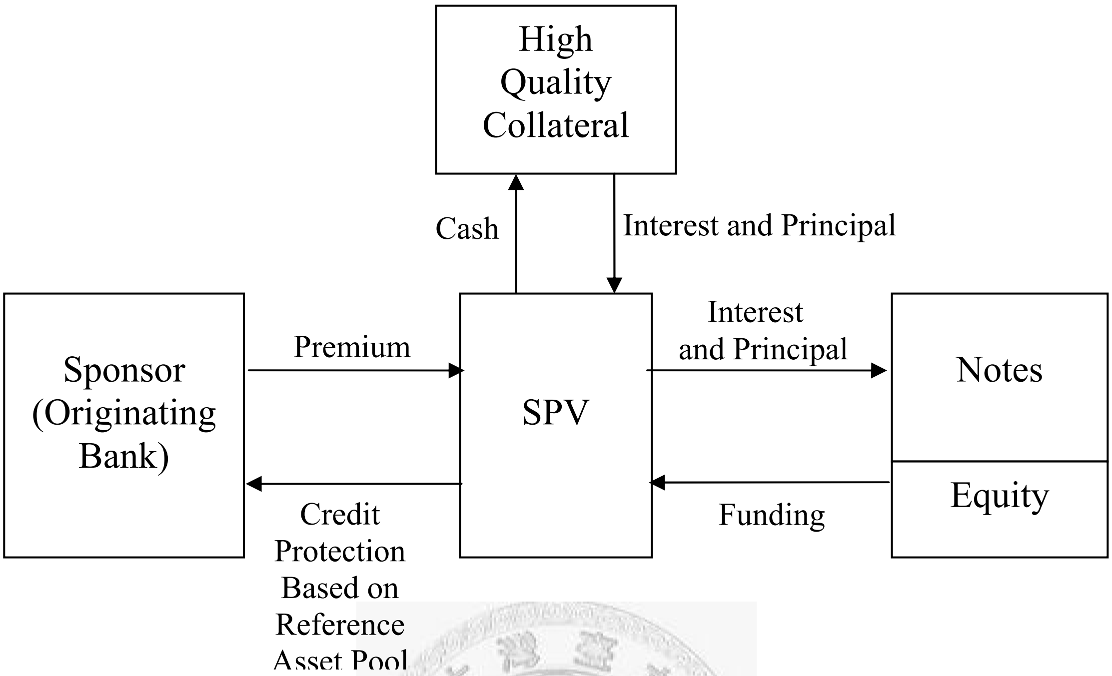


The major difference between a synthetic CDO and others is that the notes of a synthetic CDO are synthetic. It means the underlying asset pool is still held by the originating bank, and notes investors sign a contract with the originating bank. The contract claims that the received cash flow of investors is decided by the collateral held by the bank. To ensure that operation of this CDO from the bankruptcy of the bank, the SPV has the duty of buying another asset pool. The pool has to be composed of good quality asset, such as government bonds or triple A rating assets. When assets in original pool default, the SPV sells assets in new pool and pays the amount of loss to the originating bank.

In practice, price of a tranche in a CDO is equal to its principal when the CDO is issued. Therefore, we adjust the coupon rate of a tranche to let the expected loss equal to the expected revenue when pricing CDO tranches. The derived coupon rate is called fair credit spread of this tranche.

For investors of a synthetic CDO, their cash flow is from two ways. First, the interest and principal of the asset pool held by the SPV. Second, the premium paid by the originating bank. The structure of a synthetic CDO is shown in Figure 1.1.2.

Figure 1.1.2: A typical structure of a synthetic CDO.



The advantage of a synthetic CDO is that true sale of the underlying assets is not necessary since banks have some reasons to avoid the transfer of the ownership of debts. The primary reason is that if a loan is transferred into a SPV, borrower notification is always required. And it may influence the customer relationships. Through a synthetic CDO, banks can remove the credit risk and interest rate risk from their balance sheet without the transfer of assets.

The CDO market is fast-growing. According to "Global CDO Market Issuance Data" (2007) from SIFMA, the amount of CDO issued grows from 157 billion US dollars in 2004 to 550 billion US dollars in 2006. The goal of the thesis is to find a method to evaluate the CDO tranches accurately.

There are two main types of modeling credit risk. The structured form model is based on the asset value, which assumes that the asset value follows a stochastic process and firms default when asset value is less then a specified level. In contrast, the reduced form model directly models the default event of a firm as a stochastic process with jumps.

This thesis follows the first and best-known structured form model of Merton (1974). We assume the assets' values follow a joint-lognormal distribution. In modeling the credit risk, the biggest obstacle is in handling the correlation between assets. To tackle this problem, we introduce factor models, which simplify this problem.

Even if all the asset values are independent, the transformation from assets' joint distribution to the loss distribution of a large credit portfolio remains complicated computationally. We will follow Chen and Zang (2004) in introducing the Fourier transform method, which has been applied to value CDOs recently. The reason is that simulation is too slow and time consuming, whereas through the Fourier inversion, a CDO can be analyzed in much shorter time.

1.2 Organization of This Thesis

There are six chapters in this thesis. In chapter 1, a brief introduction is presented. In chapter 2, we review the literature on pricing credit products. Chapter 3 introduces the generalization of Chen and Zang model. In chapter 4, we show a numerical example. Chapter 5 compares the results from the Fourier transform method and the simulation method. Finally, conclusions are given in chapter 6.

Chapter 2

Literature Review

2.1 The Merton Model

This chapter reviews three papers on default risk. The first one is the Merton model. In Merton (1974), the value of a firm is assumed to follow a geometric Brownian motion: $dA = \mu A_0 dt + \sigma_A A_0 dz$,where A is the asset value of the firm, A_0 is the initial asset value, μ is the expected rate of return, t is time, σ_A is the volatility, and dz is a standard Wiener process. The asset value at time T is given by:

$$A_T = A_0 \exp\left[\left(\mu - \frac{\sigma_A^{\ 2}}{2}\right)T + \sigma_A Z(T)\right]$$

There are two other assumptions: A firm has only one zero coupon bond, and the default event happens only on the maturity time T of the bond. If A_T is less than the principal of the bond at time T, then the firm defaults.

First, define the probability density function of A as $f(A)$. By the definition of default, we define another random variable L as:

$$L = \begin{cases} B - A, & A \le B \\ 0, & \text{otherwise} \end{cases}$$

where L represents the loss of default, and B is the bond's principal. The equation above means that if the asset defaults (i.e., $A \le B$), the loss of default L is equal to $B - A$. If the asset does not default, then L (loss) equals zero. Through f ,we can derive the probability density function of L:

$$g(L) = \begin{cases} f(B - L), & A \le B \\ 0, & \text{otherwise} \end{cases}$$

The default probability can also be derived, as follows:

$$\Pr(A \le B) = N(d_1),$$

where $d_1 = \dfrac{\ln A_T - \ln B + \left(\mu - \dfrac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}$.

2.2 The Chen-Zang Model

2.2.1 One-Factor Model

This thesis draws upon two methods: factor model and Fourier inversion. Similar to the Merton model, the one-factor model assumes the asset value of a firm follows the following stochastic process:

$$d\ln A_j = \left(\mu_j - \frac{\sigma_j^2}{2}\right)dt + \sigma_j d\ln z_j(t),$$

where $d\ln z_j(t) = \sqrt{\rho}dW_M(t) + \sqrt{1-\rho}dW_j(t)$, A_j is the asset value of asset *j*, σ_j is the volatility, μ_j is the expected rate of return, W_j represents the idiosyncratic risk of firm *j*, W_M is the common factor, and ρ is the correlation everybody has with the common factor. In this model, all asset values are assumed to be independent of each other conditional on W_M.

Note that $W_M/\sqrt{t}$ is a common factor, which affects all assets. For example, it could be the annualized return of the market portfolio. The default probability is then:

$$\Pr\left[A_j(t) \le K_j\right] = \Pr\left[\ln A_j(t) \le \ln K_j\right]$$

$$= \Pr\left[\ln A_j(0) + \left(\mu_j - \frac{\sigma_j^{\ 2}}{2}\right)t + \sigma_j \ln z_j(t) \le \ln K_j\right]$$

$$= \Pr\left\{\ln z_j(t) \le \left[\ln K_j - \ln A_j(0) - \left(\mu_j - \frac{\sigma_j^{\ 2}}{2}\right)t\right] \times \frac{1}{\sigma_j}\right\}$$

$$= \Pr\left[\sqrt{\rho}W_M(t) + \sqrt{1-\rho}W_j(t) \le \left(\ln K_j - \mu_j^*\right) \times \frac{1}{\sigma_j}\right]$$

$$= \Pr\left[\frac{W_j(t)}{\sqrt{t}} \le \frac{\frac{\ln K_j - \mu_j^*}{\sigma_j} - \sqrt{\rho}W_M(t)}{\sqrt{(1-\rho)t}}\right]$$

$$= N\left[\frac{\frac{\ln K_j - \mu_j^*}{\sigma_j} - \sqrt{\rho}W_M(t)}{\sqrt{(1-\rho)t}}\right]$$

Here, K_j is the default barrier of asset *j*.

Conditional on $W_M(t)$ being a real number m, the above probability becomes:

$$P_{j|m}$$

$$\equiv \Pr\left[A_j(t) \le K_j \,|\, W_M(t) = m\right]$$

$$= N\left(\frac{\frac{\ln K_j - \mu_j^*}{\sigma_j} - \sqrt{\rho}m}{\sqrt{(1-\rho)t}}\right)$$

2.2.2 Fourier Transform

Since we know the probability density function of every firm, we have the probability density function of each firm's losses. Therefore, we can derive the conditional characteristic function for the loss of all the assets:

$$\phi_{j|m}(u)$$

$$\equiv E\left[e^{-iuL_j} \,|\, W_M = m\right] ,$$

$$= 1 - P_{j|m} + P_{j|m}E\left[e^{-iuL_j} \,|\, m, L_j > 0\right]$$

where L_j is the loss of asset *j*.

Let *Z* be the total loss. Then *Z* can be represented as: $Z = \sum_{j=1}^{N} L_j$. Conditional on $W_M(t) = m$, L_i, L_j are independent for all $i \neq j$. The characteristic function of *Z* is:

$$\begin{aligned}
&\phi_{Z|m}(u) \\
&\equiv E\left[e^{-iuZ} \mid W_M = m\right] \\
&= E\left[e^{-iu(L_1+L_2+L_3+\cdots+L_N)} \mid W_M = m\right] \\
&= E\left[e^{-iuL_1} \mid m\right] E\left[e^{-iuL_2} \mid m\right] \cdots E\left[e^{-iuL_N} \mid m\right] \\
&= \prod_{j=1}^{N} \phi_{j|m}(u)
\end{aligned}$$

Finally, we can derive the unconditional characteristic function of *Z* as follows: $\phi_Z(u) = \int_{-\infty}^{\infty} \prod_{j=1}^{N} \phi_{j|m}(u) f(m)\, dm$, where $f(m)$ is a standard normal probability density function. Thus, we can use Fourier inversion to obtain the unconditional probability density function of the total loss *Z*:

$$h(Z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iuZ} \phi_Z(u)\, du$$

Finally, we can use this probability density function to calculate the value of each tranche in a CDO structure. Since the tranches differ by their orders of suffering loss, we can easily calculate the expected loss of each tranche from $h(Z)$.

2.3 The Yeh-Liao-Tao Model

This thesis also adapts the Fourier transform method to evaluate tranches of a CDO. It defined N indicator functions X_i and a random variable *PDR* (portfolio default rate):

$$X_i = \begin{cases} 1, & \text{if asset } i \text{ default} \\ 0, & \text{otherwise} \end{cases}$$

$$PDR = \frac{\sum_{i=1}^{N} S_i X_i}{\sum_{i=1}^{N} S_i}$$

Through the Fourier transform method, the expected *PDR* is derived. Since PDR means the percentage of assets defaults, there must be an exogenous recovery rate to calculate the total loss.

The advantage of adapting an exogenous recovery rate is that the rating information could be considered. The two disadvantages are: A global recovery rate is not reasonable, and estimating errors may exist.

Chapter 3

Generalization of the Chen-Zang Model

3.1 Generalizations

As described in the previous chapter, the Chen-Zang model assumes that there is only one common factor influencing all assets. But often assets in a CDO pool are so diversified that one common factor is not sufficient to describe the correlations between assets. Therefore, we use a two-factor model to describe the correlations more accurately. Besides, the correlation coefficients between different assets and common factors are surely distinct from each other in the real world. Therefore, we want to generalize the Chen-Zang model by improving it in two respects: From one-factor model to two-factor model, and from the single correlation coefficient to multiple ones.

We assume:

$$d \ln A_j = \left(\mu_j - \frac{\sigma_j^{\ 2}}{2} \right) dt + \sigma_j d \ln z_j(t),$$

where

$$d \ln z_j(t) = r_{1j} dM_1(t) + r_{2j} dM_2(t) + \sqrt{1 - r_{1j}^{\ 2} - r_{2j}^{\ 2}}\, dW_j(t).$$

The parameters assume the same meanings as in the last chapter, M_1, M_2 are the common factors, and r_{1j}, r_{2j} are the correlation between asset j and two common factors, respectively. And we assume that $W_j(t)$ is independent of $M_1(t)$ and $M_2(t)$.

3.2 Structure of the Model

The goal of this section is to derive the probability density function of the total loss. For the *j*th asset, the asset value follows:

$$d\ln A_j = \left(\mu_j - \frac{\sigma_j^{\ 2}}{2}\right)dt + \sigma_j d\ln z_j(t),$$

where $d\ln z_j(t) = r_{1j}dM_1(t) + r_{2j}dM_2(t) + \sqrt{1 - r_{1j}^{\ 2} - r_{2j}^{\ 2}}\,dW_j(t)$. The default probability is:

$$
\begin{aligned}
&\Pr\left[A_j(t) \le K_j\right] = \Pr\left[\ln A_j(t) \le \ln K_j\right] \\
&= \Pr\left[\ln A_j(0) + \left(\mu_j - \frac{\sigma_j^{\ 2}}{2}\right)t + \sigma_j \ln z_j(t) \le \ln K_j\right] \\
&= \Pr\left\{\ln z_j(t) \le \left[\ln K_j - \ln A_j(0) - \left(\mu_j - \frac{\sigma_j^{\ 2}}{2}\right)t\right] \times \frac{1}{\sigma_j}\right\} \\
&= \Pr\left[r_{1j}M_1(t) + r_{2j}M_2(t) + \sqrt{1 - r_{1j}^{\ 2} - r_{2j}^{\ 2}}\,W_j(t) \le \left(\ln K_j - \mu_j^*\right) \times \frac{1}{\sigma_j}\right] \\
&= \Pr\left[\frac{W_j(t)}{\sqrt{t}} \le \frac{\dfrac{\ln K_j - \mu_j^*}{\sigma_j} - \sqrt{r_{1j}}M_1(t) - \sqrt{r_{2j}}M_2(t)}{\sqrt{\left(1 - r_{1j}^{\ 2} - r_{2j}^{\ 2}\right)t}}\right] \\
&= N\left[\frac{\dfrac{\ln K_j - \mu_j^*}{\sigma_j} - r_{1j}M_1(t) - r_{2j}M_2(t)}{\sqrt{\left(1 - r_{1j}^{\ 2} - r_{2j}^{\ 2}\right)t}}\right]
\end{aligned}
$$

Conditional on $M_1(t) = m_1$ and $M_2(t) = m_2$ we have the distributions of all A_j. We find that asset values follow a joint lognormal distribution. Recall that a lognormal random variable *x* is defined as follows:

$x \sim \text{lognormal}(\mu, \sigma)$ if and only if $\ln x \sim \text{normal}(\mu, \sigma)$.

The distribution of the asset value is:

$$A_j \sim \text{lognormal}\left(\mu_j^* + \sigma_j r_{1j} m_1 + \sigma_j r_{2j} m_2, \sigma_j \sqrt{\left(1 - r_{1j}^{\ 2} - r_{2j}^{\ 2}\right)t}\right).$$

Now,

$$P_{j|m_1,m_2}$$
$$\equiv \Pr\left[A_j(t) \le K_j \mid M_1(t) = m_1, M_2(t) = m_2\right]$$
$$= N\left[\frac{\left(\ln K_j - \mu_j^*\right) - \sigma_j r_{1j} m_1 - \sigma_j r_{2j} m_2}{\sigma_j \sqrt{\left(1 - {r_{1j}}^2 - {r_{2j}}^2\right)t}}\right]$$

Substituting the mean and variance into the probability density function of the lognormal random variable, we have:

$$A_j \sim \text{lognormal}\left(\mu_j^* + \sigma_j r_{1j} m_1 + \sigma_j r_{2j} m_2, \sigma_j \sqrt{\left(1 - {r_{1j}}^2 - {r_{2j}}^2\right)t}\right)$$

$$f\left(A_j \mid m_1, m_2\right) = \frac{\exp\left[\dfrac{-\left(\ln A_j - \mu_j^* - \sigma_j r_{1j} m_1 - \sigma_j r_{2j} m_2\right)^2}{2{\sigma_j}^2\left(1 - {r_{1j}}^2 - {r_{2j}}^2\right)t}\right]}{A_j \sigma_j \sqrt{\left(1 - {r_{1j}}^2 - {r_{2j}}^2\right)2\pi t}}$$

Note that f is the probability density function of A. Let L_j be the loss from the jth asset. Then we derive L_j's probability density function from f as follows:

$$g\left(L_j \mid m_1, m_2\right) = \begin{cases} f\left(K_j - L_j \mid m_1, m_2, L_j \ge 0\right), & \text{if } K_j \ge L_j \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

Since the Fourier transform is also the characteristic function of L_j and we have the conditional probability density function of L_j, we can derive the conditional characteristic function of L_j as follows:

$$\phi_{j|m_1,m_2}(u) = E\left[e^{-iuL_j} \mid m_1, m_2\right]$$
$$= 1 - P_{j|m_1,m_2} + P_{j|m_1,m_2} E\left[e^{-iuL_j} \mid m_1, m_2, L_j > 0\right]$$

Let Z be the total loss of a portfolio. Then we have: $Z = \sum_{j=1}^{N} L_j$. Conditional on $M_1(t) = m_1$, and $M_2(t) = m_2$, L_i, L_j are independent for all $i \ne j$. The characteristic function of Z is:

$$
\begin{aligned}
&\phi_{Z|m_1,m_2}(u) \\
&= \mathrm{E}\left[e^{-iuZ} \mid M_1 = m_1, M_2 = m_2 \right] \\
&= \mathrm{E}\left[e^{-iu\left(L_1+L_2+L_3+\cdots+L_N\right)} \mid M_1 = m_1, M_2 = m_2 \right] \\
&= \mathrm{E}\left[e^{-iuL_1} \mid m_1, m_2 \right] \mathrm{E}\left[e^{-iuL_2} \mid m_1, m_2 \right] \cdots \mathrm{E}\left[e^{-iuL_N} \mid m_1, m_2 \right] \\
&= \prod_{j=1}^{N} \phi_{j|m_1,m_2}(u)
\end{aligned}
$$

Finally, we can derive the unconditional characteristic function of Z:

$\phi_Z(u) = \int_{-\infty,-\infty}^{\infty,\infty} \prod_{j=1}^{N} \phi_{j|m_1,m_2}(u) f\left(m_1, m_2\right) d\left(m_1, m_2\right)$, where $f\left(m_1, m_2\right)$ is a two-dimensional standard normal probability density function. Thus, we can use Fourier inversion to obtain the unconditional probability density function of the total loss Z:

$$h\left(Z\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iuZ} \phi_Z(u) du$$

Our original goal has been achieved.

Chapter 4

A Numerical Example

4.1 Environment Settings and Results

In order to test if this model is workable, we fix a group of parameters for the study of a CDO pool. We assume that the CDO pool has twenty underlying assets. Parameters of each asset are listed in table 4.1.1. We also assume this CDO has 5 tranches: 0–3, 3–7, 7–15, 15–30, 30–100. It means that tranche 1 bears the first three percent loss, tranche 2 bears the next four percent loss, and so on.

We calculate the expected loss of each tranche by the probability density function with the following formulas:

$$\text{loss of tranche } 1 = L^1 = \int_0^{2000\times 3\%} h(z)zdz$$

$$\text{loss of tranche } 2 = L^2 = \int_0^{2000\times 7\%} h(z)zdz - L^1$$

$$\text{loss of tranche } 3 = L^3 = \int_0^{2000\times 15\%} h(z)zdz - \sum_{i=1}^{2} L^i$$

$$\text{loss of tranche } 4 = L^4 = \int_0^{2000\times 30\%} h(z)zdz - \sum_{i=1}^{3} L^i$$

$$\text{loss of tranche } 5 = L^5 = \int_0^{2000\times 100\%} h(z)zdz - \sum_{i=1}^{4} L^i$$

The derived probability density function is shown in Figure 4.1.1. We can see that the probability is highest when the loss is about three percent. The probability of loss exceeding forty percent of amount is almost zero.

We can also calculate the expected loss of each tranche. The result is shown in Table 4.1.2.

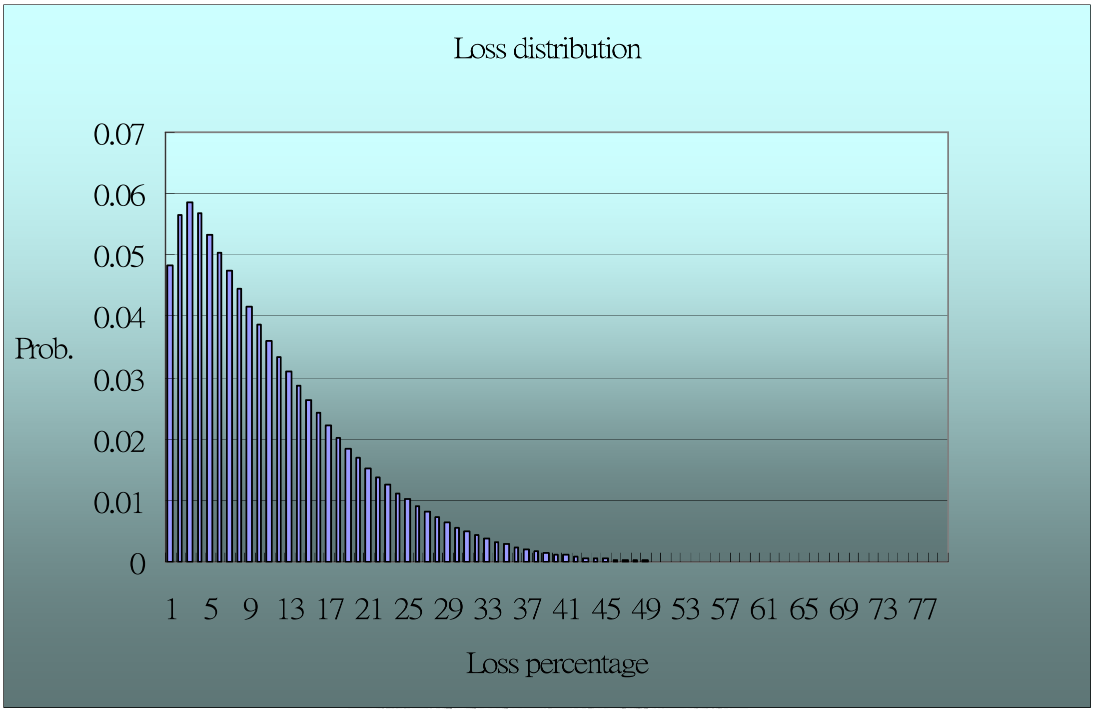
Table 4.1.1: Parameters of assets.

	μ	Maturity	σ	r_1	r_2	Asset value	K
asset_1	7%	2	40%	20%	15%	150	100
asset_2	10%	2	40%	31%	12%	150	100
asset_3	4%	2	30%	24%	5%	130	100
asset_4	10%	2	50%	20%	10%	150	100
asset_5	10%	2	60%	20%	10%	132	100
asset_6	5%	2	20%	30%	13%	144	100
asset_7	7%	2	70%	41%	12%	132	100
asset_8	8%	2	40%	30%	25%	134	100
asset_9	9%	2	30%	9%	30%	164	100
asset_10	5%	2	60%	30%	24%	132	100
asset_11	7%	2	40%	20%	15%	140	100
asset_12	10%	2	40%	31%	12%	130	100
asset_13	4%	2	30%	24%	5%	150	100
asset_14	10%	2	50%	20%	10%	144	100
asset_15	10%	2	60%	20%	10%	162	100
asset_16	5%	2	20%	30%	13%	134	100
asset_17	7%	2	70%	41%	12%	132	100
asset_18	8%	2	40%	30%	25%	154	100
asset_19	9%	2	30%	9%	30%	144	100
asset_20	5%	2	60%	30%	24%	152	100

Table 4.1.2: Comparison of scenarios.

tranche	expected loss	percentage loss
1	50.13	83.55%
2	51.28	64.10%
3	60.55	37.84%
4	34.77	11.59%
5	23.69	1.69%

Figure 4.1.1: Probability density function of example.



4.2 Comparison with the Original Model

To measure the influence of the generalizations (from one factor to two factors and from a single correlation to multiple correlations), we devise a numerical test to compare these models.

For comparison, we continue to use the parameters in the previous section. Then we calculate the probability density function of the total loss under five scenarios. The factor loading of the second common factor (r_2) decreases from scenario 1 to scenario 5. (In scenario 1, $r_2^* = r_2$. In scenario 2, $r_2^* = \frac{r_2}{2}$. In scenario 3, $r_2^* = \frac{r_2}{4}$. In scenario 4, $r_2^* = \frac{r_2}{8}$. In scenario 5, $r_2^* = 0$.)

Figure 4.2.1 shows the probability density functions for the five scenarios. Table 4.2.1 shows the expected loss in percentage of all the tranches in each scenario.

The shapes of scenario 1 and scenario 5 in Figure 4.2.1 are very similar to each other. But the probability density function of scenario 5 has less kurtosis than scenario 1. Furthermore, the kurtosis of the probability density function is decreasing as r_2^* increases. Therefore, there are some increasing or decreasing trends of the expected loss in Table 4.2.1.

With an increasing r_2^*, the losses of tranche 4 and tranche 5 increase. This result is caused by probability density functions in Figure 4.2.1. Between 21% and 45%, the probability increases with an increasing r_2^*. This interval of loss is mostly borne by tranche 4 and tranche 5.

This implication of this result is that equity tranche and very senior tranche are under-valued in the original model.

Figure 4.2.1: Comparison of five scenarios.

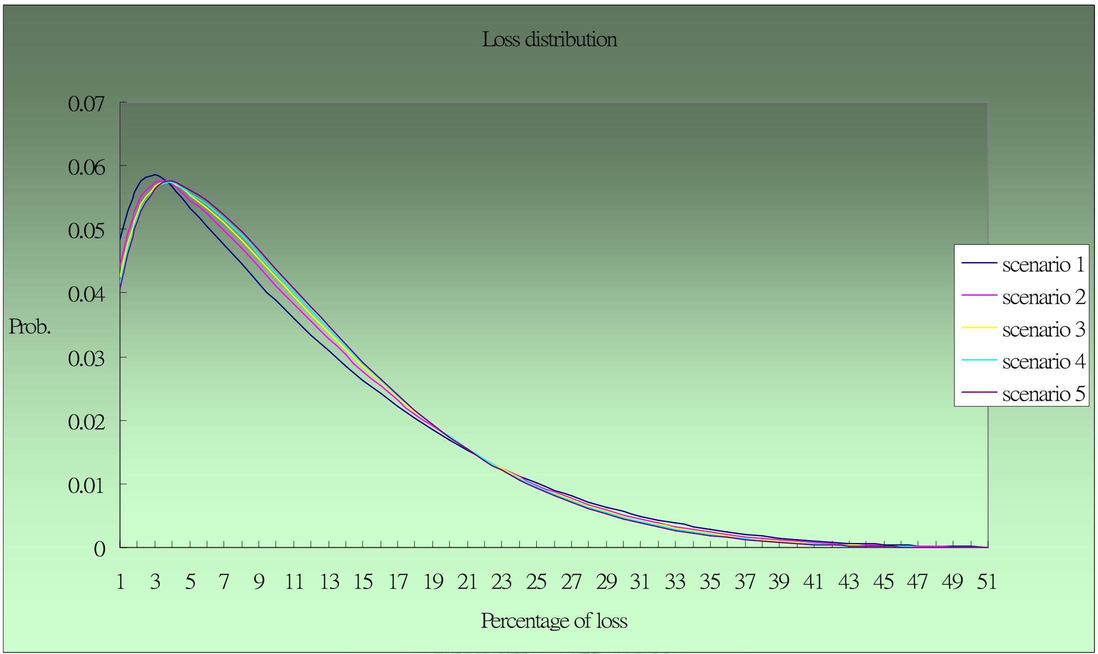


Table 4.2.1: Expected loss of each tranches in all scenarios.

tranche	scenario 1	scenario 2	scenario 3	scenario 4	scenario 5
1	83.55%	84.90%	85.55%	85.86%	86.12%
2	64.10%	65.76%	66.58%	66.96%	67.22%
3	37.84%	38.25%	38.40%	38.44%	38.33%
4	11.59%	10.84%	10.42%	10.19%	9.88%
5	1.69%	1.36%	1.19%	1.11%	1.02%

Chapter 5
Conclusion

This thesis introduces a general Fourier method to evaluate large credit portfolios and CDO tranches. This model is more flexible and closer to the real world. Especially when dealing with a well diversified portfolio, it can analyze asset returns more accurately and generate more results. Surprisingly, this generalization costs little time beyond the original model. However, some problems remain to be solved, such as parameter estimation and asset value determination.

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