Pricing European and American Options with Extrapolation

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Abstract

This thesis deals with European and American options with tree methods via extrapolation and provides an efficient methodology. Binomial and trinomial trees are widely used in numerical methods for derivatives pricing and applicable across a wide range of option types. However, convergence to the correct option price is oscillatory and nonmonotonic. This situation makes the tree method inaccurate and unsuitable for extrapolation. We fix the problem by pegging the strike price in the CRR method and make it applicable for extrapolation.

Keywords: Option pricing, extrapolation, binomial tree, sawtooth effect.

Chapter 1

Introduction

1.1 Setting the Ground

Options are contingent claims or financial derivatives because their value depends on the underlying asset. Options give their holder the right to buy or sell the underlying asset. With the rapid growth and deregulation of the economy, a variety of derivatives are designed by the financial institutions to satisfy the needs of their customers. Although financial derivatives are getting more and more complicated, options are still one of the most important financial instruments and have wide applications in the market.

There are two basic types of options: calls and puts. A call option gives the holder the right to buy a specific number of the underlying asset at a certain price on a certain date or during a period. A put option gives the holder the right to sell a specific number of the underlying asset at a certain price on a certain date or during a period. The price is known as the strike price or exercise price. The date is known as the expiration date or maturity. The underlying asset may be stocks, stock indices, options, foreign currencies, futures contracts, interest rates, and so on. When an option is embedded, it has to be traded along with the underlying asset.

American options can be exercised at any time up to the expiration date. An option can be exercise before the maturity, which is called early exercise. European options can be exercised only on the expiration date. Early exercise makes American options differ from European options.

What's more, a call is said to be *in the money* if $S > X$, *at the money* if $S = X$, and *out of the money* if $S < X$. A put is said to be *in the money* if $S < X$, *at the money* if $S = X$, and *out of the money* if $S > X$.

This thesis deals with European and American options with tree methods via extrapolation and provides an efficient methodology. Binomial and trinomial trees are widely used in numerical methods for derivatives pricing and applicable across a wide range of option types. However, their convergence to the correct option price is oscillatory and nonmonotonic. This situation makes the tree method inaccurate and unsuitable for extrapolation. We fix the problem and make it applicable for extrapolation.

Figure 1.1 illustrates the well-known nonmonotonicity or sawtooth effect when using the CRR method[1] to value a European call option. The characteristic sawtooth pattern, with high-frequency ringing and low-frequency oscillations (the shrinking envelope or general shape of the plots as the number of time steps in a tree type is increased), is due to the position of the nodes at expiration date in relation to the strike price. In order to improve the sawtooth pattern, we use the method of pegging the strike price to smooth the oscillation; therefore, we are able to price the options with extrapolation.

[1] CRR method is a binomial tree method in the paper published by Cox, Ross, and Rubinstein in 1979.

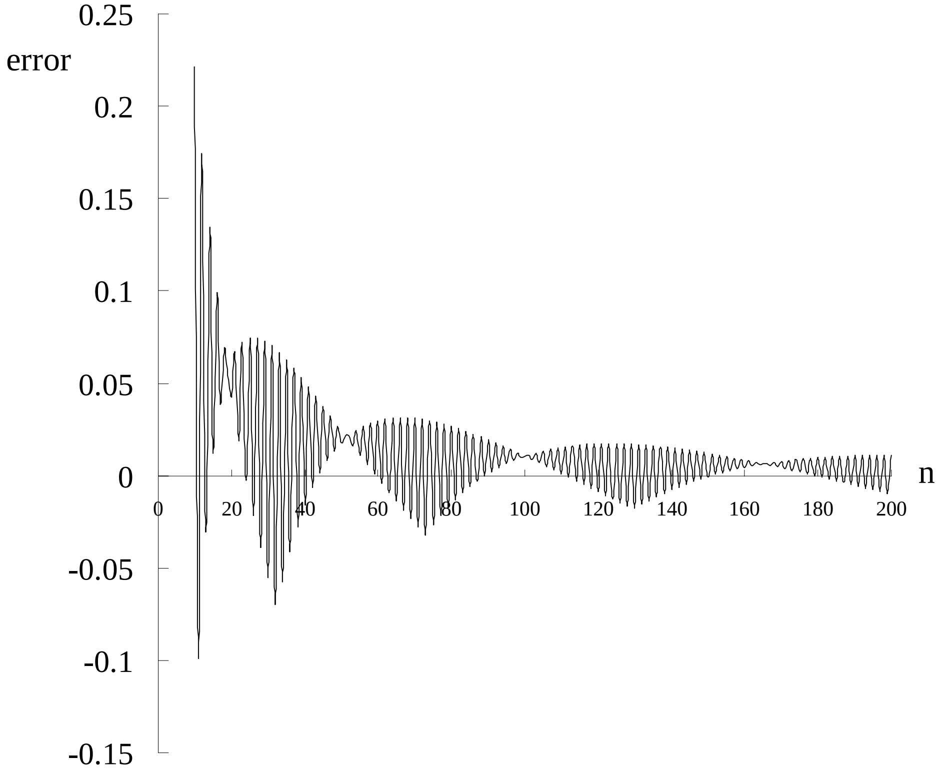


Figure 1.1: SAWTOOTH EFFECT.
A graph of error against the number of periods, n, for a European call, using the CRR binomial model.
$S = 100$, $X = 90$, $\sigma = 0.3$, $r = 0.1$, $q = 0.05$, and $T = 1$.

1.2 Survey of Literature

The importance of the placement of the final nodes in the tree methods cannot be overemphasized. A few tree methods take it into account to device techniques to overcome the oscillation problem.

Modifying Ritchken's (1995) model for trinomial lattices, Tian (1999) made use of an extra degree of freedom in the CRR model to tilt the tree so that one of the nodes at the expiration date was just on the strike price. Leisen and Reimer (1996) applied inversion methods to transform from a lognormal distribution to a binomial one and set up the lattice such that the exercise price was at the center of the final nodes. Figlewski and Gao (1999) adopted the Adaptive Mesh Model (AMM), in which a finer mesh is incorporated into the structure in the final time step before the expiration date just around the strike price. Widdicks, Andricopoulos, Newton, and Duck (2002) introduced the concept of Λ, a normalized distance between the strike price and the node above. By keeping Λ constant, they got smooth monotonic results before applying extrapolation.

1.3 Structures of the Thesis

The thesis is organized as follows. In Chapter two, the underlying theory about financial derivatives, including the properties of options, pricing models and tree methods, is introduced. The concept of extrapolation will also be presented. In Chapter 3, we will describe how to implement a computer program to price the derivatives efficiently. Numerical results will be discussed in this chapter, too. Chapter 4 concludes.

Chapter 2

Methodologies

In this chapter, we begin with the payoffs of options. Then we introduce the pricing model and apply the tree methods. Last, but not least, the method of extrapolation is presented.

2.1 Payoffs of Options

There are two sides to every option contract: the long position and short position. The trader having bought the option has taken the long position. The trader having sold or written the option has taken the short position. The purchaser of the option pays a premium to gain the right to buy the underlying asset. The writer of the option receives the premium and then bears the obligation to sell the underlying asset. The writer of an option receives cash in the beginning but bears potential liabilities in the future. The writer's profit or loss is opposite to that for the purchaser of the option. It is a zero-sum game.

Assume S is the final price of the underlying asset, X is the strike price, and O is the premium. The payoff of a long position on a European call at the expiration date is $\max(S - X, 0)$ for the option will be exercised only when S is larger than X. In contrast, the payoff of a long position on a European put at expiration date is

$\max(X - S, 0)$.

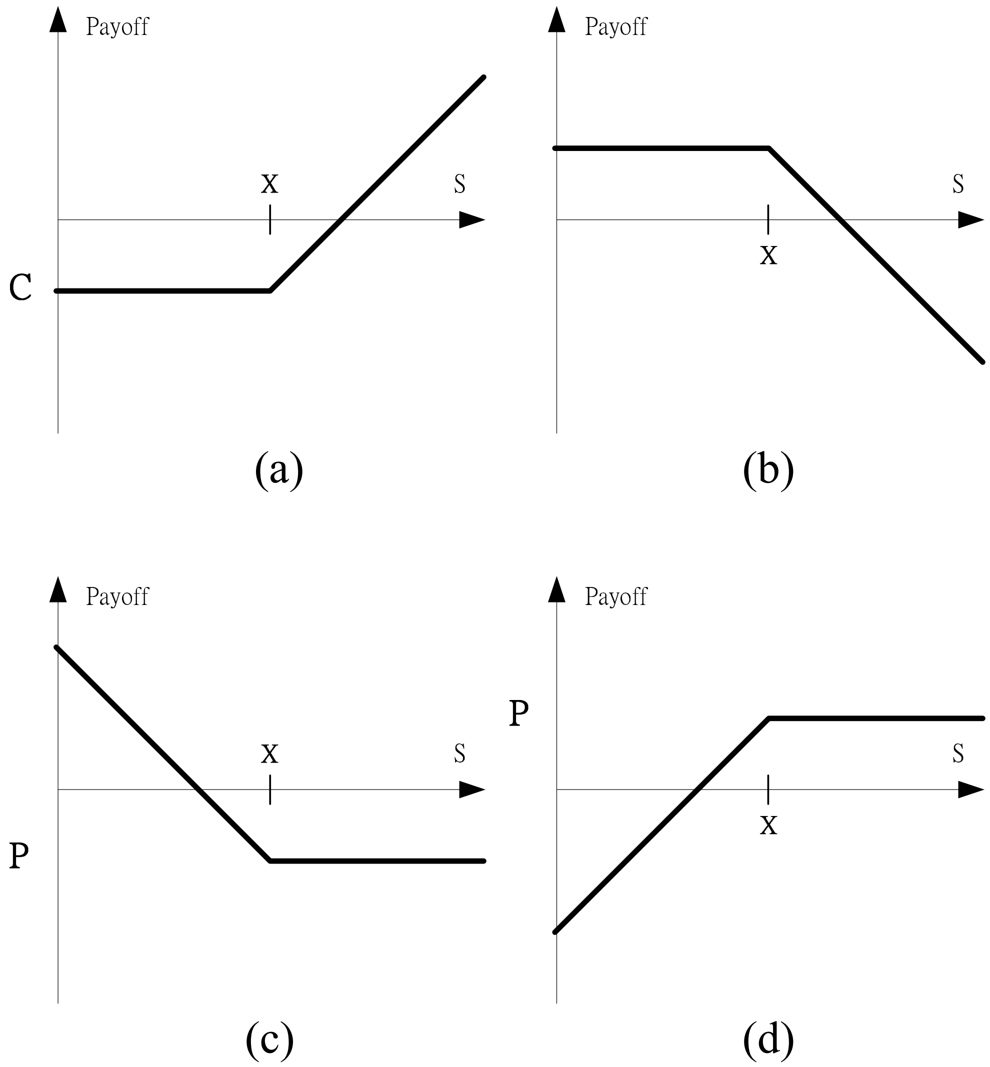


Figure 2.1: PROFIT/LOSS OF OPTIONS.
(a) Long a call. (b) Short a call. (c) Long a put. (d) Short a put.

The profit for a long position on a European call is

$$\max(S - X, 0) - O$$

So the profit for a short position on a European call is

$$-(\max(S - X, 0) - O) = \min(X - S, 0) + O$$

The profit for a long position on a European put is

$$\max(X - S, 0) - O$$

While the profit for a short position on a European put is

$$-(\max(X - S, 0) - O) = \min(S - X, 0) + O$$

Figure 2.1 illustrates the profit/loss patterns graphically.

2.2 Pricing Models

As we know the payoffs on options at the expiration date, we are able to price options in a backward manner. We take stock options for example in this thesis and apply tree methods to price them. In order to value stock options, we must figure out the process for the stock price under the continuous-time or the discrete-time pricing model. The Black-Scholes model stands out among continuous-time stock pricing models, whereas the tree method is a representative of the discrete-time pricing model. When the individual time period is small enough, that is to say, when the number of periods, n, approaches infinity, the price produced by tree methods will equal that under the Black-Scholes model.

2.2.1 Log-normal Model for Stock Price

The stock price is assumed to follow a log-normal distribution. Therefore, a log-normal model for the stock price is the standard model frequently used in finance. This is because its properties can satisfy reasonable assumptions about the stochastic behavior of stock prices. The stochastic log-normal model for a non-dividend-paying stock is

$$\frac{dS}{S} = \mu dt + \sigma dz. \tag{2.1}$$

By Ito's lemma,

$$d \ln S = (\mu - \frac{\sigma^2}{2})dt + \sigma dz. \tag{2.2}$$

Equation (2.1) is also known as *geometric Brownian motion*, where S is the value of stock. The variables μ and σ are viewed as the expected return and volatility, respectively.

Process z follows a Wiener process if the following properties hold:

Property 1

$$\triangle z = \epsilon \sqrt{\triangle t}$$

where ϵ is a random drawn from the standardized normal distribution.

Property 2

The value of $\triangle z$ for any two disjoint time intervals are independent.

Thus $\triangle z$ is a normal distribution with zero mean and a standard deviation equal to $\sqrt{\triangle t}$ by property 1. Property 2 implies that z follows a *Markov process*. A Markov process is a particular type of stochastic process where only the present value of a variable is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant.

Obviously, it is the return rate of the stock not the stock price itself that is a random variable with a normal distribution. That is why we call the stock price *log-normal*. The stock price in this model will never be negative, and the percent changes of S are independent and identically distributed. These properties make it a good model for the stock price.

2.2.2 The Black-Scholes Model

Speaking of the continuous-time model to price stock options, few can ignore the fundamental contribution that Fischer Black and Myron Scholes made in the early 1970s. They made a major breakthrough in pricing stock options by developing the well-known Black-Scholes model for valuing European call and put options. They also derived the famous Black-Scholes differential equation that must be satisfied by the price of any derivative dependent on a non-dividend-paying stock. Of course, the Black-Scholes model can be extended to deal with European call and put options on dividend-paying stocks as will be presented later.

2.2.2.1 Assumptions

The assumptions used to derive the Black-Scholes differential equation are:

1. The stock price follows the geometric Brownian motion with μ and σ constant.
2. The short selling of securities with full use of proceeds is permitted.
3. There are no transactions costs or taxes; all securities are perfectly divisible.
4. There are no dividends during the life of the derivative.
5. There are no riskless arbitrage opportunities.
6. Security trading is continuous.
7. The risk-free rate of interest, r, is constant and the same for all maturities.

2.2.2.2 The Black-Scholes Differential Equation

Under the assumptions listed above, Black and Scholes form a riskless portfolio consisting of a position in the option and a position in the underlying stock. In the absence of arbitrage opportunities, the return from the portfolio must be the risk-free interest rate. The Black-Scholes differential equation is

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf, \tag{2.3}$$

where f is the price of a derivative security, S is the stock price, σ is the volatility of the stock price, and r is the continuously compounded risk-free rate.
Equation (2.3) can in principle be solved if the boundary conditions are given. In the case of a European call option, when $t = T$, the key boundary condition is

$$f = \max(S - X, 0).$$

In the case of a European put option, when $t = T$, the key boundary condition is

$$f = \max(X - S, 0).$$

2.2.2.3 Black-Scholes Pricing Formula

The major breakthrough in the pricing of financial derivatives is that Black and Scholes obtained the closed form formula for European options. The Black-Scholes formulas for the prices at time zero of a European call option and a European put option on a non-dividend-paying stock are derived by solving Equation (2.3),

$$C = SN(d_1) - Xe^{-rT}N(d_2) \tag{2.4}$$

and

$$P = Xe^{-rT}N(-d_2) - SN(-d_1), \tag{2.5}$$

where

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}},$$
$$d_2 = \frac{\ln(S/X) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

We may extend the Black-Scholes formula to price European options on a continuous dividend-paying stock as follows,

$$C = Se^{-qT}N(d_1) - Xe^{-rT}N(d_2) \tag{2.6}$$

and

$$P = Xe^{-rT}N(-d_2) - Se^{-qT}N(-d_1), \tag{2.7}$$

where

$$d_1 = \frac{\ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}},$$
$$d_2 = \frac{\ln(S/X) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

The notations for the above equations are described below.

C = price of a European call.

P = price of a European put.

r = continuously compounded risk-free rate.

q = continuous dividend yield.

T = the time to maturity.

$N(x)$ = probability distribution function of standard normal distribution.

σ^2 = annualized variance of the continuously compounded return on stocks.

2.2.3 Tree Methods

Constructing a binomial or trinomial tree to price stock options is a very useful and popular technique. We introduce a famous binomial tree method, the CRR model, and then modify it by using the method of pegging the strike price in a proper way so as to smooth the sawtooth effect.

2.2.3.1 Binomial Tree

The tree method represents possible paths that might be followed by the stock price over the life of the option. The generalized two-period binomial tree is illustrated in Figure 2.2.

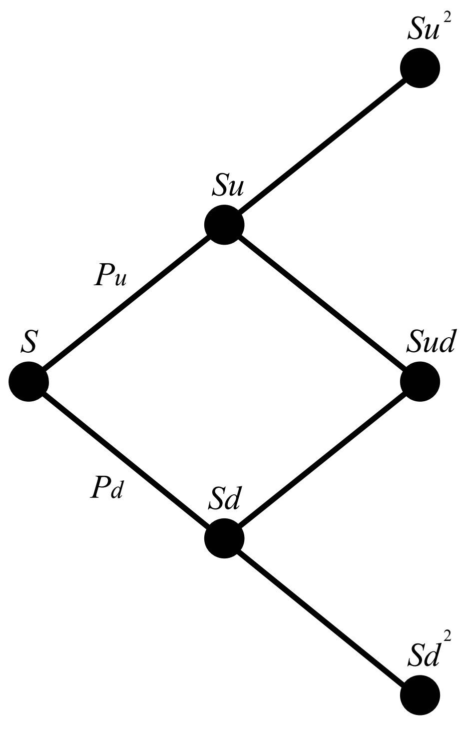


Figure 2.2: BINOMIAL MODEL FOR TWO PERIODS.
Stock price movements over two periods under the binomial model. S is the stock price at period 0, and u and d are constants indicating the upward and downward ratios of the stock movement. P_u is the probability of moving upwards and P_d is the probability of moving downwards.

Under the assumption that no arbitrage opportunities exist, we build a binomial tree for the log-normal distribution. Let P_u and P_d indicate the probability of upward and downward moving probabilities, respectively, and let u and d denote the constants indicating the upward and downward ratios of the stock movement. We therefore calibrate the first moment in Equation (2.8) and the second moment in Equation (2.9) as follows,

$$
\begin{aligned}
M &= (r - \frac{\sigma^2}{2})\Delta t \; = \; P_u \ln u + P_d \ln d, && (2.8) \\
V &= P_u[\ln u - (r - \frac{\sigma^2}{2})\Delta t]^2 + P_d[\ln d - (r - \frac{\sigma^2}{2})\Delta t]^2, && (2.9) \\
1 &= P_u + P_d, && (2.10)
\end{aligned}
$$

where M denotes the mean and V denotes the variance of the stock return.

However, these four unknown variables can not be determined uniquely by the three equations above, and one more constraint is needed. In the CRR model, the additional constraint is $ud = 1$. Until now, we have not considered dividend-paying stocks. The setting in Equation (2.8) makes the probability of an up movement, P_u, risk-neutral and is equals

$$
P_u = \frac{e^{r\Delta t} - d}{u - d}.
$$

Under the $ud = 1$ constraint, solutions to the above equations that are correct in the limit are

$$
\begin{aligned}
u &= e^{\sigma\sqrt{\Delta t}}, \\
d &= e^{-\sigma\sqrt{\Delta t}}.
\end{aligned}
$$

2.2.3.2 Pegging the Strike Price

There is one shortcoming, the sawtooth effect, when we use the CRR model. To modify the CRR model, Leisen and Reimer (1996) suggested the method of pegging the strike price and controlled the position of the node in relation to the strike price. They kept the strike price lying at a fixed proportional distance between the neighboring two nodes, independent of the periods, and let the strike price at the center of the final nodes for even n. The way we peg the strike price is to make it at the middle of the final nodes. The modified model which pegs the strike price and takes continuous dividend paying yield, q, into account is set as follows,

$$P_u = \frac{e^{(r-q)\Delta t} - d}{u - d}, \tag{2.11}$$

$$u = e^{K_n + V_n}, \tag{2.12}$$

$$d = e^{K_n - V_n}, \tag{2.13}$$

where

$$K_n = \frac{\ln(X/S)}{n},$$

$$V_n = \sigma\sqrt{\Delta t}.$$

The above parameter setting is derived by calibrating the first moment in Equation (2.8) and the second moment in Equation (2.9).

The method of pegging the strike price is illustrated as Figure 2.3. At the beginning, we peg the strike price along the dotted line, and then divide the distance between the $\ln X$ and $\ln S$ into two periods. This is how we derive K_n. In order to peg the strike price and make it exactly at the center of the final nodes, we only considered even number of periods. In fact, the parameter setting above is similar to that of the CRR method; therefore, we called it the modified CRR method in the present thesis.

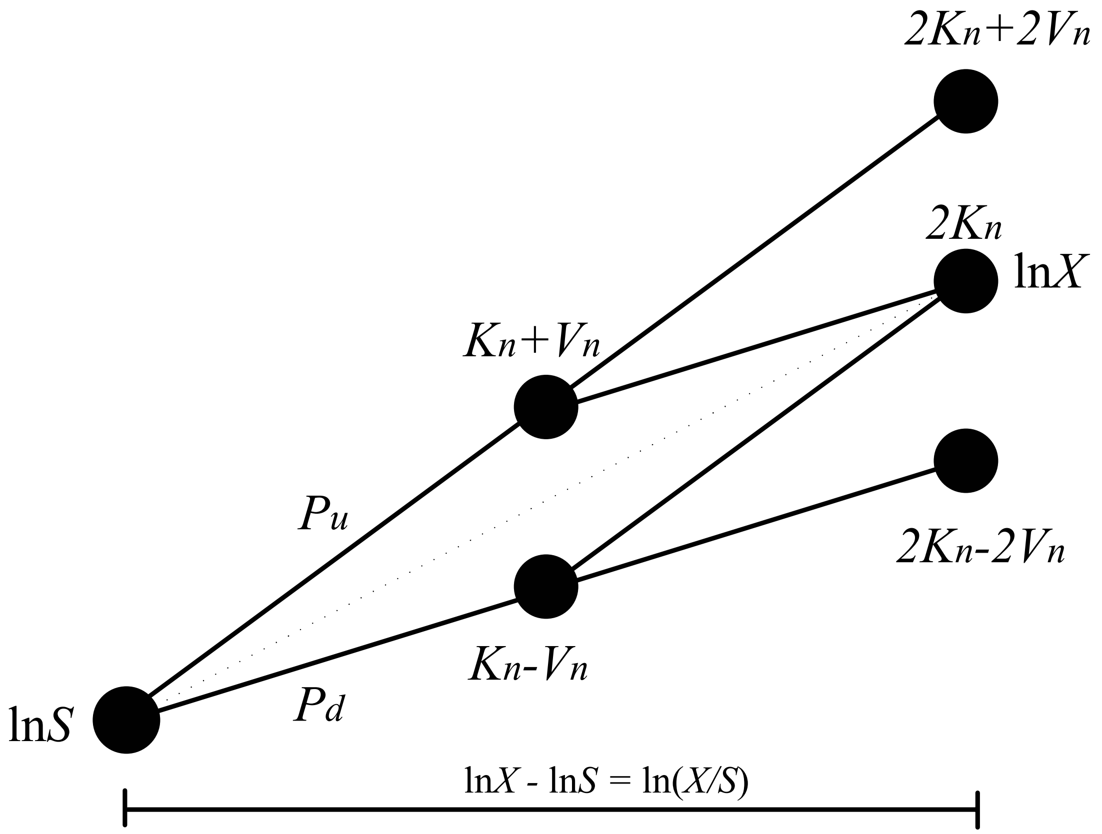


Figure 2.3: PEG THE STRIKE PRICE FOR TWO PERIODS.
Stock price movements over two periods using the method of pegging the strike price . $\ln S$ is the logarithmic stock price at period 0. $\ln X$ is the logarithmic strike price. The dotted line shows how we peg the strike price. The bar represents the distance between the $\ln X$ and $\ln S$. P_u is the probability of moving upwards and P_d is the probability of moving downwards.

2.3 Extrapolation

By pegging the strike price on the condition of even n, smooth monotonic results will be achieved and the price of European and American options converges as the periods increase. Once the phenomenon of smooth convergence exists, we may apply extrapolation to price options with even better accuracy. That is to say, we are able to use a smaller n to extrapolate a price that is as good as a larger n. Since Leisen and Reimer have proved European and American option prices typically converge with order one[1], the error e_n is as follows,

$$e_n = \frac{\alpha_1(n)}{n} + \frac{\alpha_2(n)}{n^2}.$$

They suppose that the function $\alpha_1(\cdot)$ is a constant, equal to α_1, and that the function $\alpha_2 \approx 0$. In other words, $P^e_n = \alpha_1/n + P^e_{(n_1,n_2)}$, where $P^e_{(n_1,n_2)}$ denotes the approximate P^e under this assumption. And then derive the following equations,

$$P^e_{n_1} = \frac{\alpha_1}{n_1} + P^e_{(n_1,n_2)}, \qquad (2.14)$$

$$P^e_{n_2} = \frac{\alpha_1}{n_2} + P^e_{(n_1,n_2)}, \qquad (2.15)$$

$$\Rightarrow P^e_{(n_1,n_2)} = \frac{n_2 P^e_{n_2} - n_1 P^e_{n_1}}{n_2 - n_1}. \qquad (2.16)$$

Equation (2.16) is called the extrapolation rule.

[1]Refer to "Pricing the American Put Option: a Detailed Convergence Analysis for Binomial Models."

Chapter 3

Numerical Results

In this chapter, we will begin with some numerical results about the prices of European and American options against the number of periods, n, under the method of CRR. We also present the prices of European and American options under the CRR method when n is even. Finally, we deal with the placement of the final nodes by pegging the strike price and modify the CRR method to price European and American options. We will compare the value of European options with Black-Scholes formula. Since true American values are unknown, we use the convergent binomial method[1] with n=15,000 as the basis for comparison.

3.1 The CRR Method

Figure 3.1 and Figure 3.2 illustrate the price of European and American options against n under the CRR method and the parameter setting is as follows, $S = 100$, $X = 90$, $\sigma = 0.3$, $r = 0.1$, $q = 0.05$, and $T = 1$. We find that the sawtooth effect exists. Because the sawtooth effect makes the price inaccurate and unstable, we try some tips to see if the convergence of price will occur.

[1] Amin and Khanna discussed the convergence of American option values from discrete- to continuous-time financial model in 1994

3.1.1 Even Number of Periods

First of all, we wonder whether the choice of n matters. To that purpose, we use the CRR method as before but select an even number of periods to see what will happen. In Figure 3.3 and Figure 3.4, we find that the volatility of price decreases but the convergence is not smooth. Although the sawtooth effect is not as serious as earlier, we are not satisfied with the results.

3.1.2 Modified CRR Method

After surveying the literature, we are convinced that the method of pegging strike price suggested in Leisen and Reimer (1996) is easier to implement compared with others, and the price will converge smoothly. We peg the strike price and use even number periods so as to keep the strike price at the center of the final nodes using the modified CRR method. Besides, we still use the same parameter setting[2] for comparison. We find the smooth convergence as expected in Figure 3.5 and Figure 3.6.

3.2 Extrapolation

When we use the modified CRR method to price European and American options, the price converges smoothly. Due to the smooth convergence, we are able to price European and American options with extrapolation. Here, we use n and $n+40$ to extrapolate. The relative error, e, is computed as

$$e = \frac{A - B}{B}.$$

In the above equation, A denotes the European or American options whereas B denotes the price computed by Black-Scholes formula when A is a European option

[2] $S = 100$, $X = 90$, $\sigma = 0.3$, $r = 0.1$, $q = 0.05$, and $T = 1$.

or the price computed by the convergent binomial method with n=15,000 when A is an American option.

Figure 3.7 and Figure 3.18 illustrate the results of the relative errors of European and American options against an even number period, n. The solid line represents the relative error of unextrapolated values, whereas the gray line represents the relative error of extrapolated values. As the figures show, the relative errors of extrapolated value is smaller than that of unextrapolated one and almost equal to zero from the beginning. We compute the relative error of options when options are out of the money, at the money, and in the money. The results hold whether the option is out of the money, at the money, or in the money.

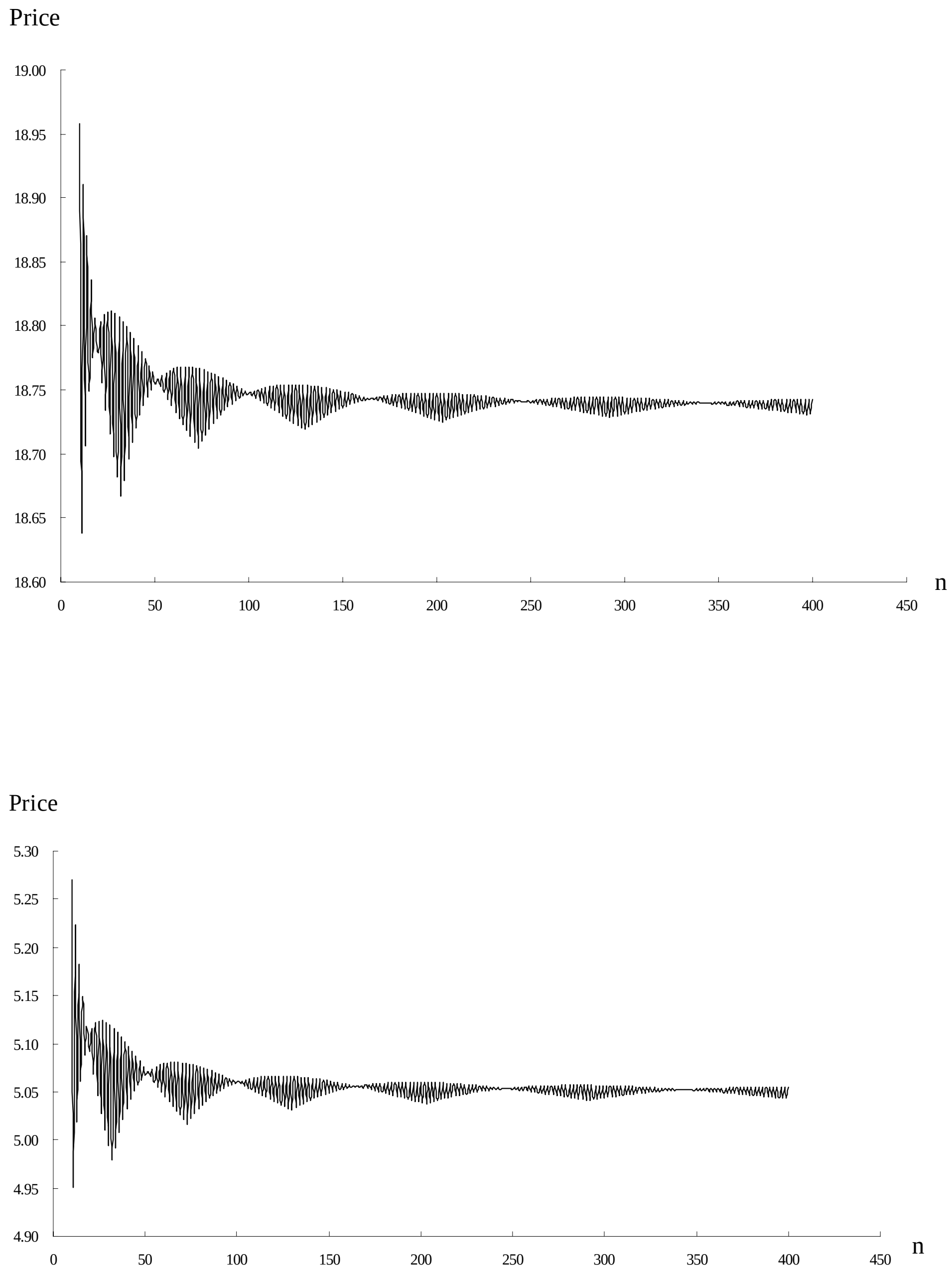


Figure 3.1: EUROPEAN CALL AND PUT.
A graph of price against the number of periods, n, for a European call (up) and put (down), under the CRR method.

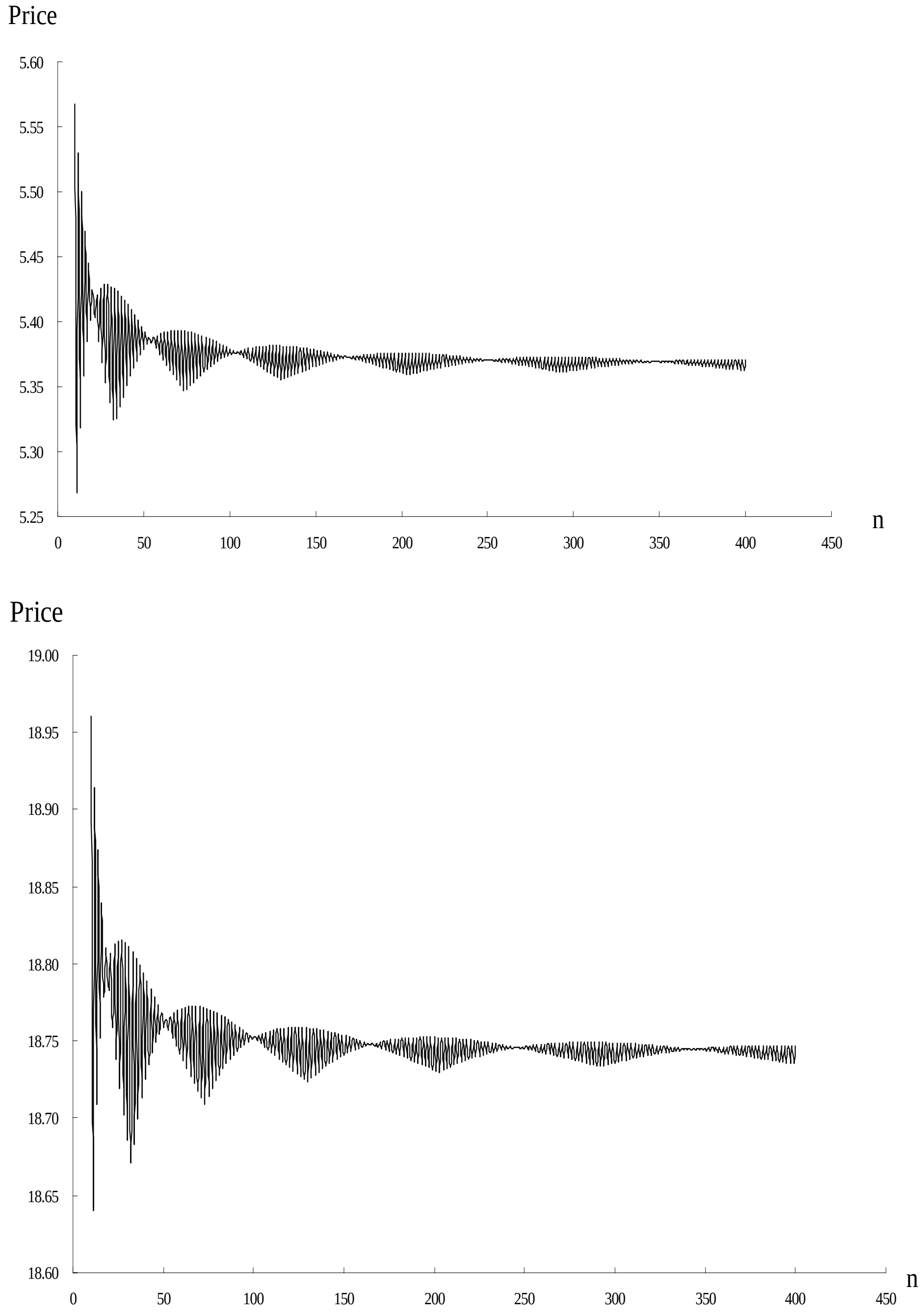
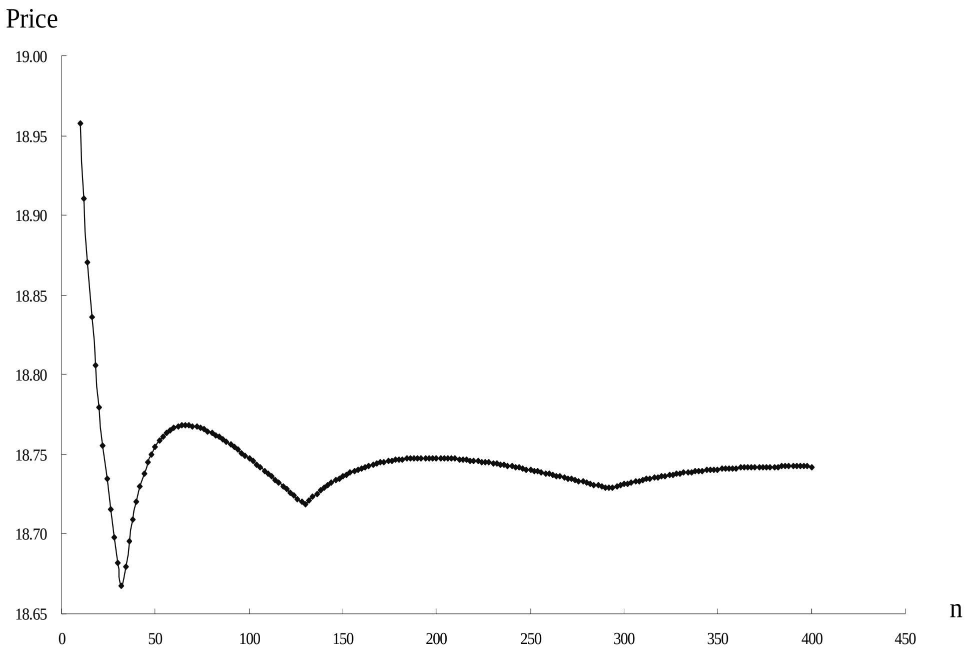


Figure 3.2: American put and call.
A graph of price against the number of periods, n, for an American put (up) and call (down), under the CRR method.



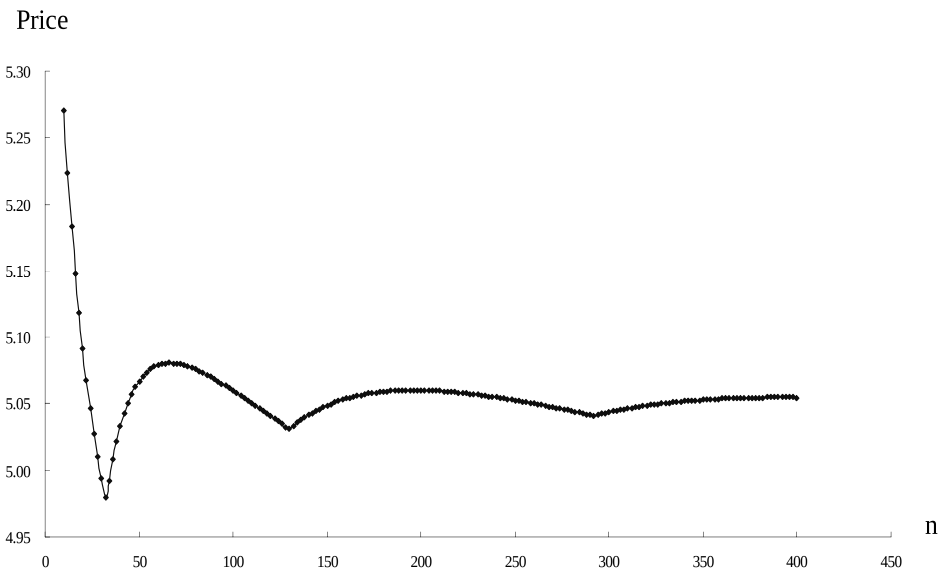


Figure 3.3: EUROPEAN CALL AND PUT UNDER EVEN NUMBER OF PERIODS.
A graph of price against even number of periods, n, for a European call (up) and put (down), under the CRR method.

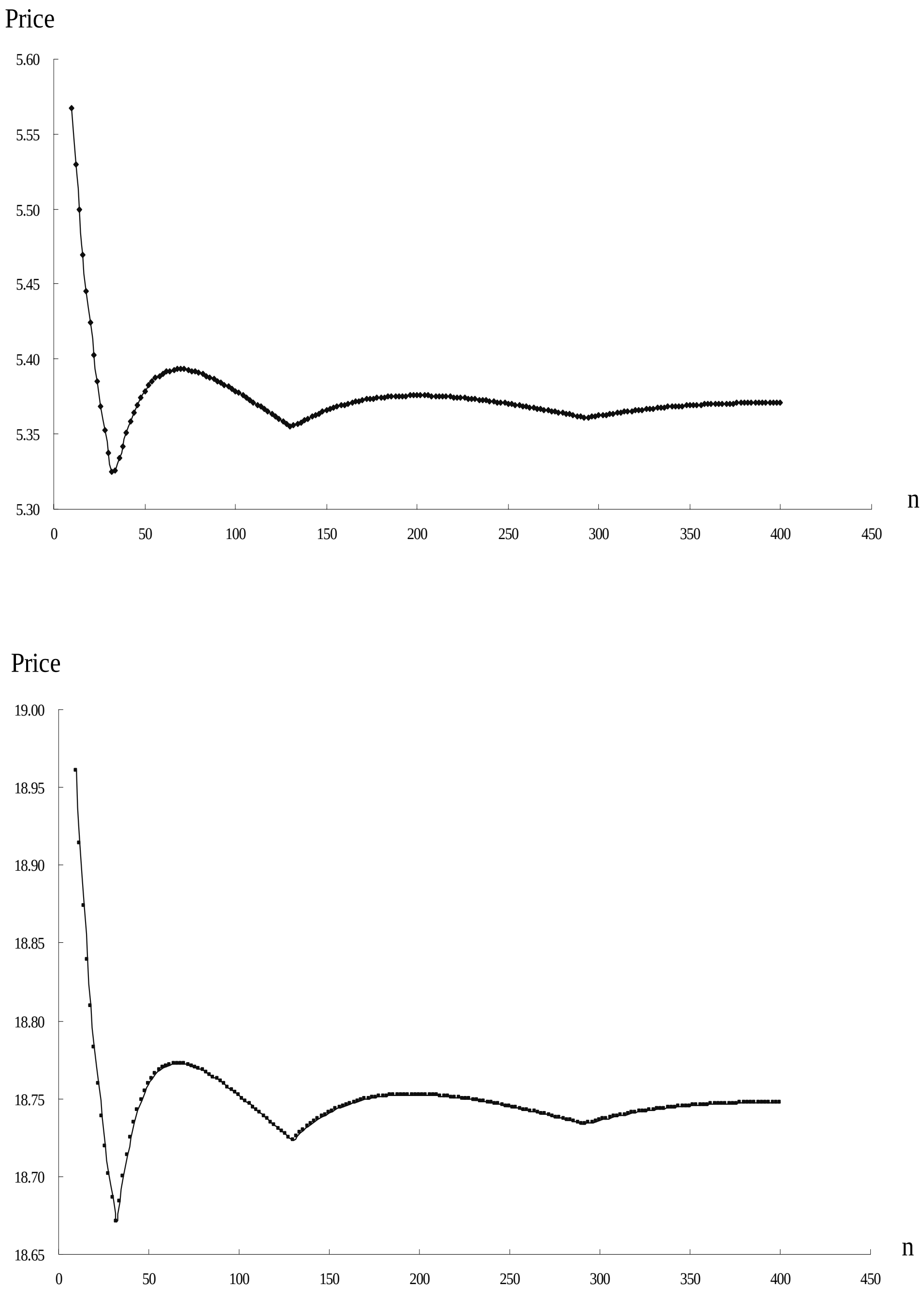
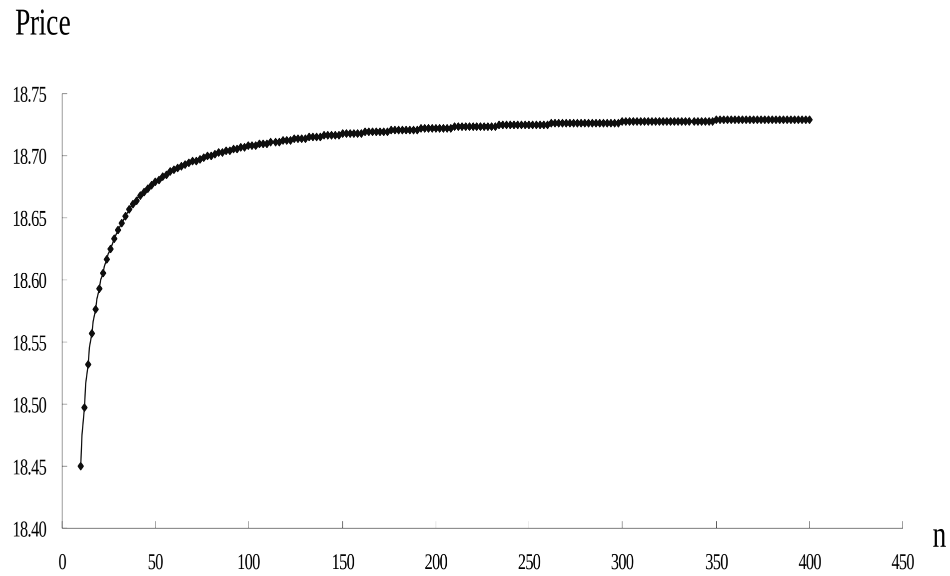


Figure 3.4: AMERICAN PUT AND CALL UNDER EVEN NUMBER OF PERIODS.
A graph of price against even number of periods, n, for an American put (up) and call (down), under the CRR method.



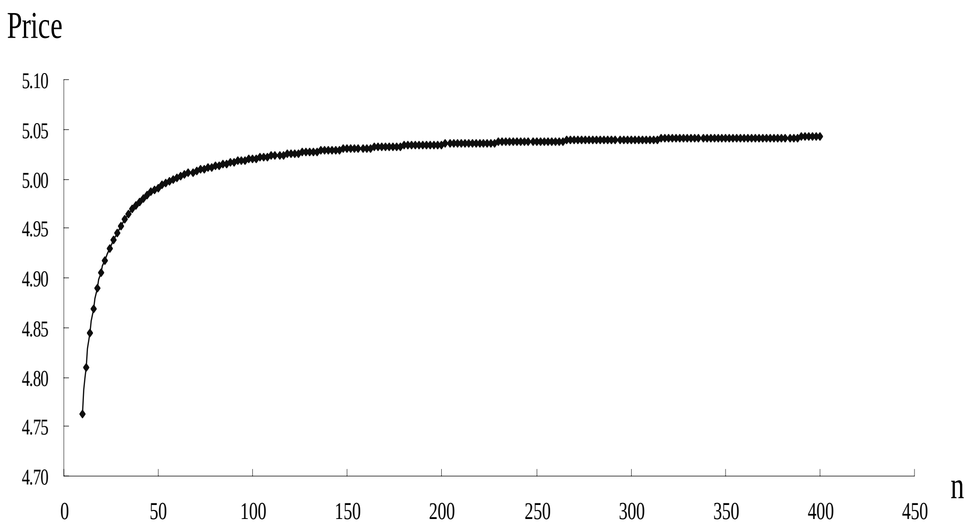


Figure 3.5: European call and put (pegging the strike price).
A graph of price against even number of periods, n, for a European call (up) and put (down), under the modified CRR method.

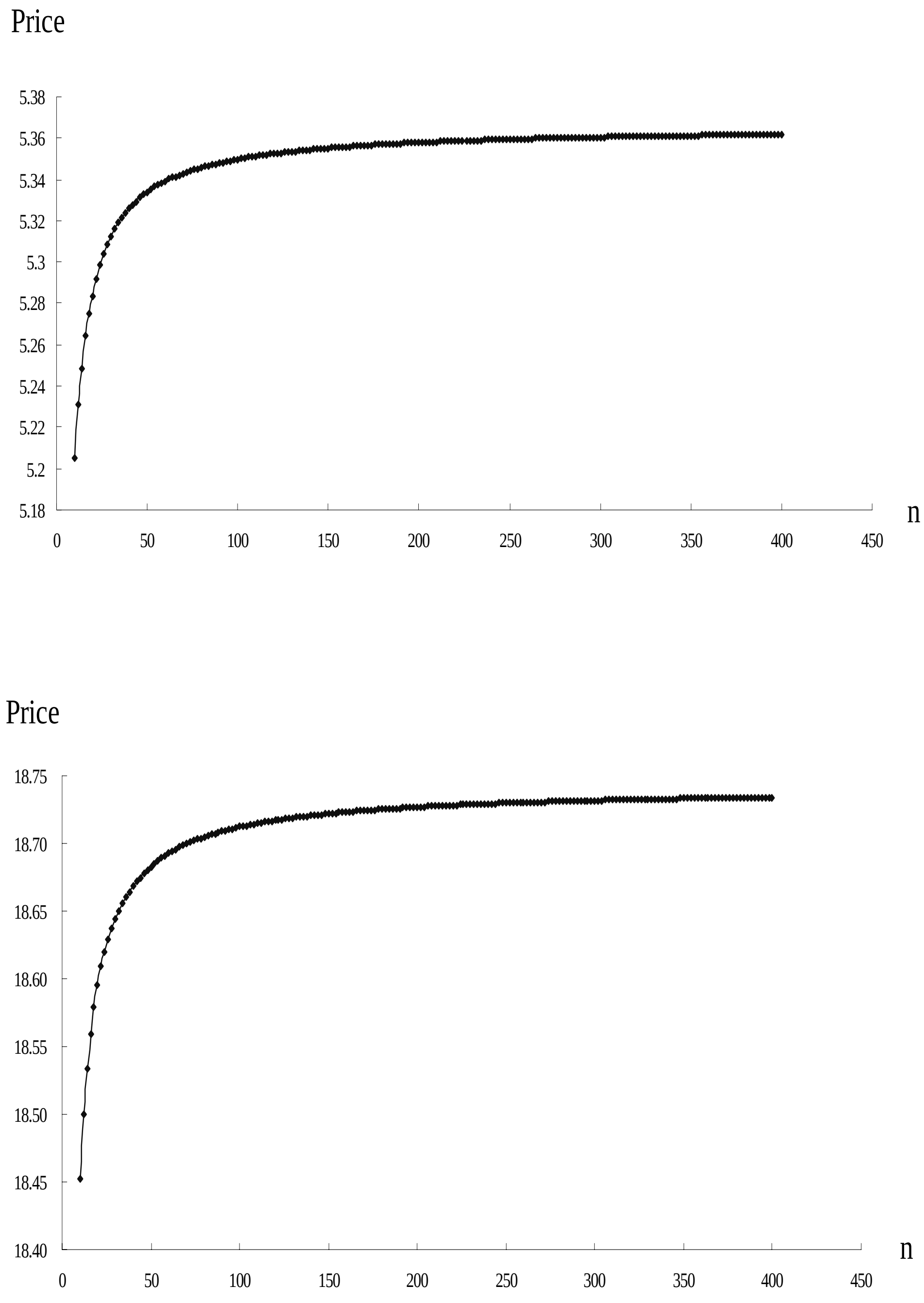


Figure 3.6: AMERICAN PUT AND CALL (PEGGING THE STRIKE PRICE).
A graph of price against even number of periods, n, for an American put (up) and call (down),under the modified CRR method.

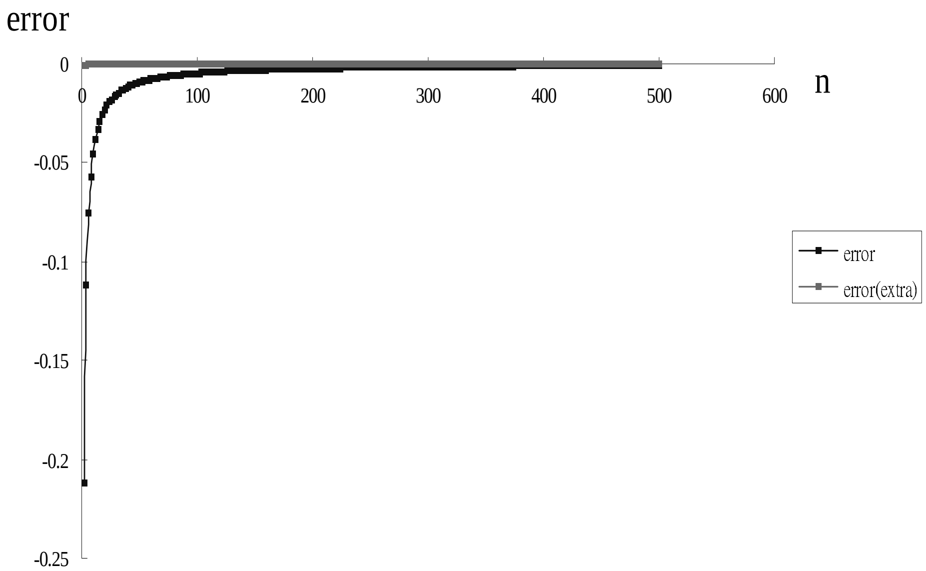


Figure 3.7: Relative error of European call (out of money).
A graph of error against even number of periods, n, for a European call (out of the money), under the modified CRR method. $S = 90$, $X = 100$, $\sigma = 0.3$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

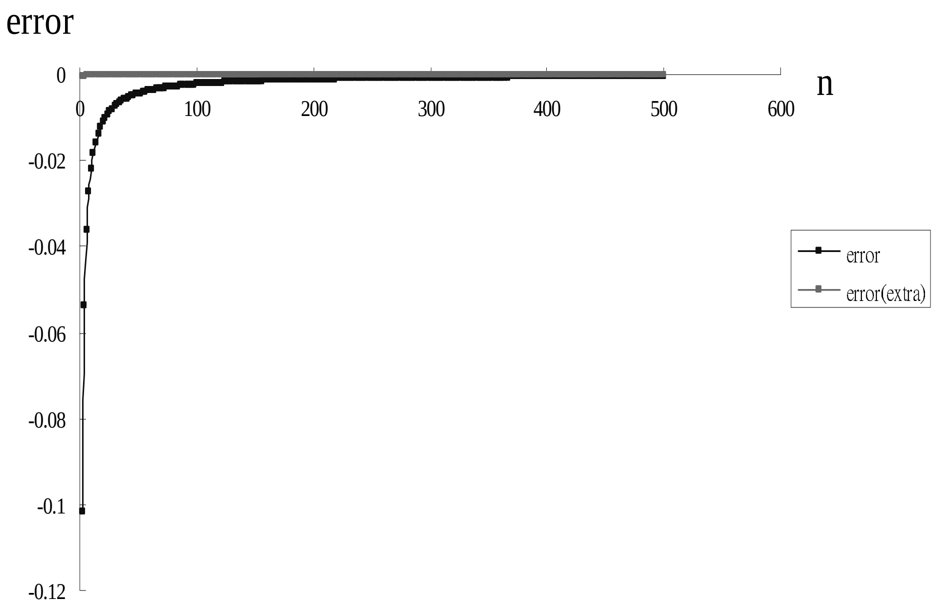


Figure 3.8: Relative error of European call (at the money).
A graph of error against even number of periods, n, for a European call (at the money), under the modified CRR method. $S = 100$, $X = 100$, $\sigma = 0.3$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

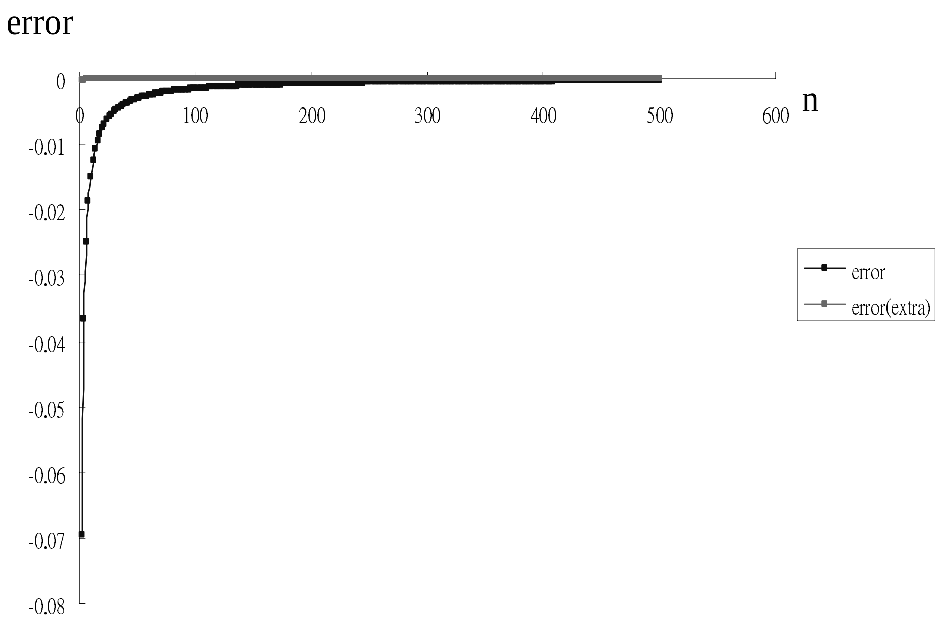


Figure 3.9: RELATIVE ERROR OF EUROPEAN CALL (IN THE MONEY).
A graph of error against even number of periods, n, for a European call (in the money), under the modified CRR method. $S = 110$, $X = 100$, $\sigma = 0.3$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

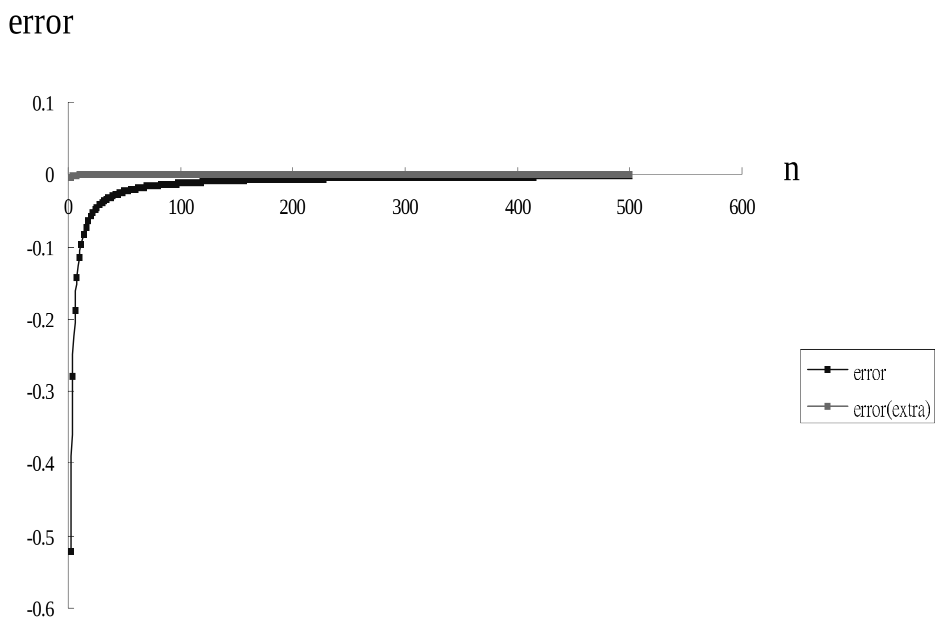


Figure 3.10: RELATIVE ERROR OF EUROPEAN PUT (OUT OF THE MONEY).
A graph of error against even number of periods, n, for a European put (out of the money), under the modified CRR method. $S = 100$, $X = 90$, $\sigma = 0.2$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

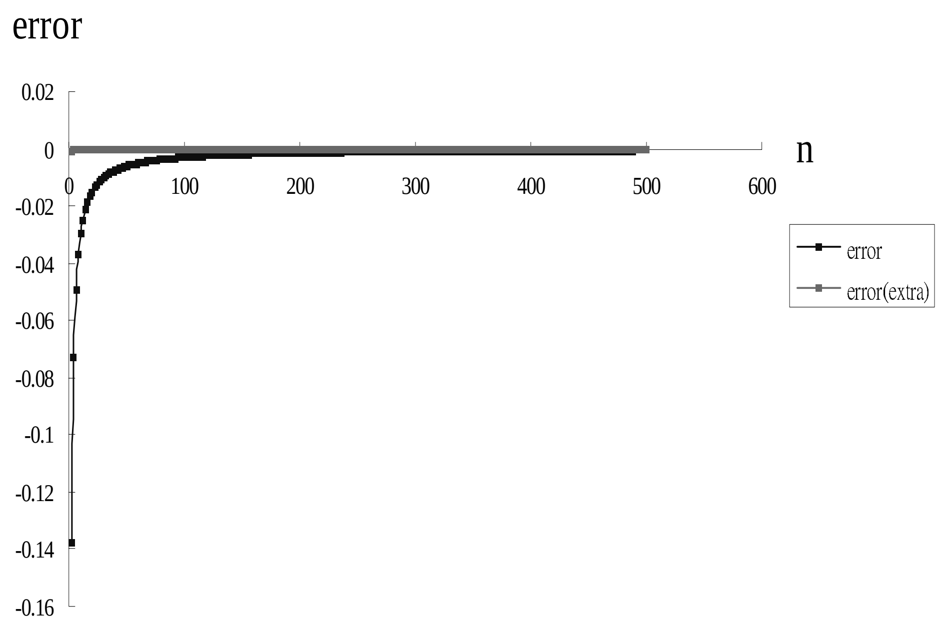


Figure 3.11: Relative error of European put (at the money).
A graph of error against even number of periods, n, for a European put (at the money), under the modified CRR method. $S = 100$, $X = 100$, $\sigma = 0.2$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

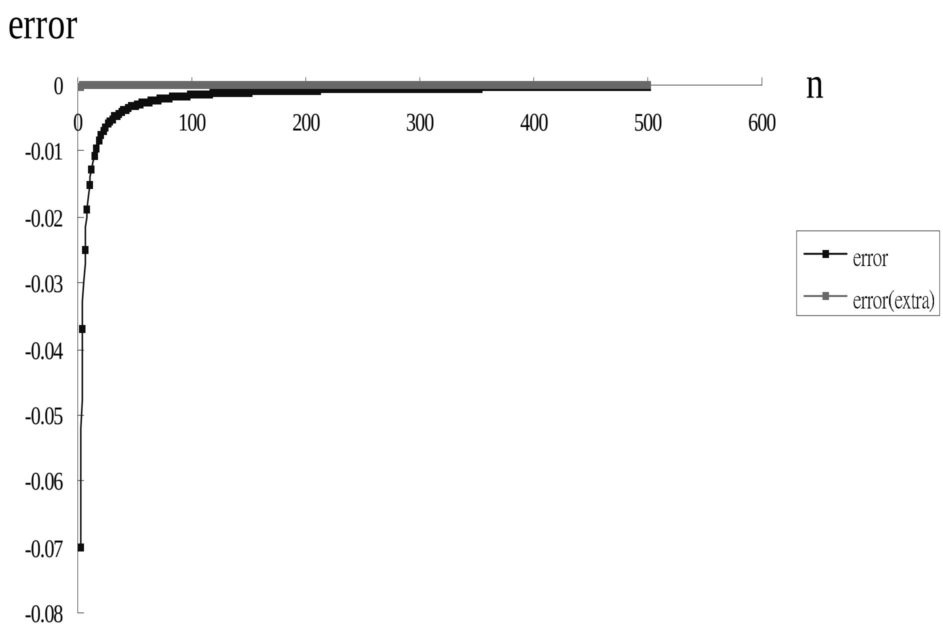


Figure 3.12: Relative error of European put (in the money).
A graph of error against the even number of periods, n, for a European put (in the money), under the modified CRR method. $S = 100$, $X = 110$, $\sigma = 0.2$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

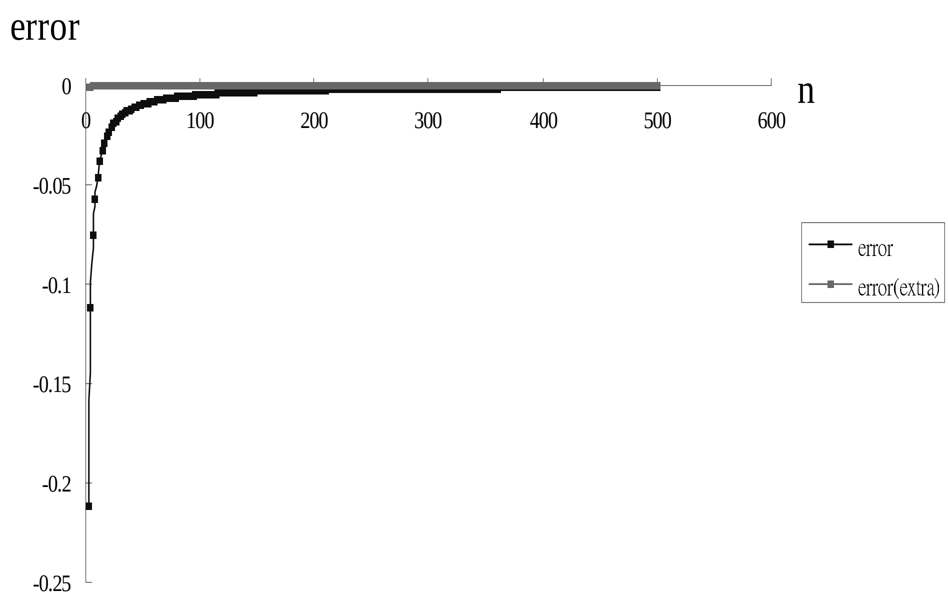


Figure 3.13: RELATIVE ERROR OF AMERICAN CALL (OUT OF THE MONEY).
A graph of error against the even number of periods, n, for an American call (out of the money), under the modified CRR method. $S = 90$, $X = 100$, $\sigma = 0.3$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

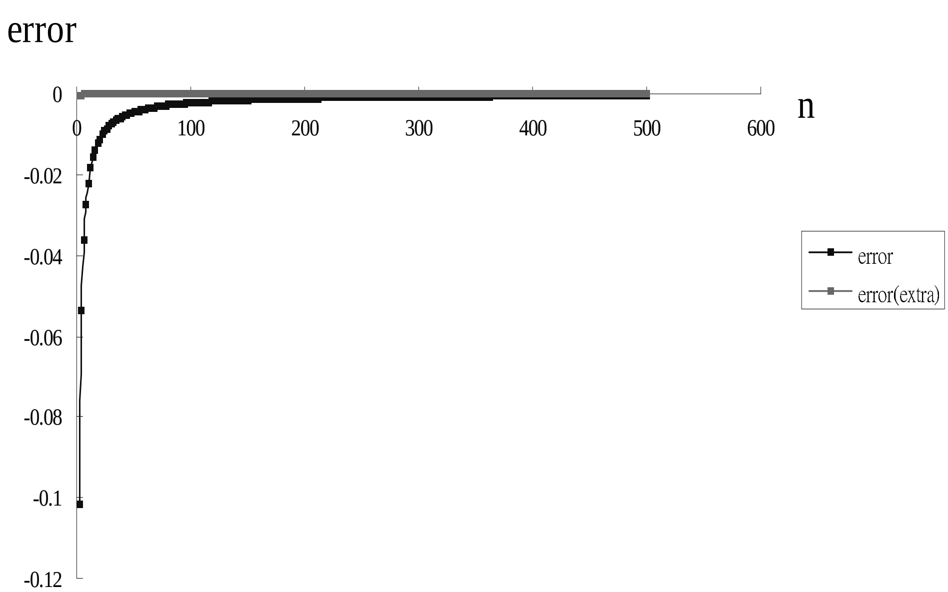


Figure 3.14: RELATIVE ERROR OF AMERICAN CALL (AT THE MONEY).
A graph of error against the even number of periods, n, for an American call (at the money), under the modified CRR method. $S = 100$, $X = 100$, $\sigma = 0.3$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

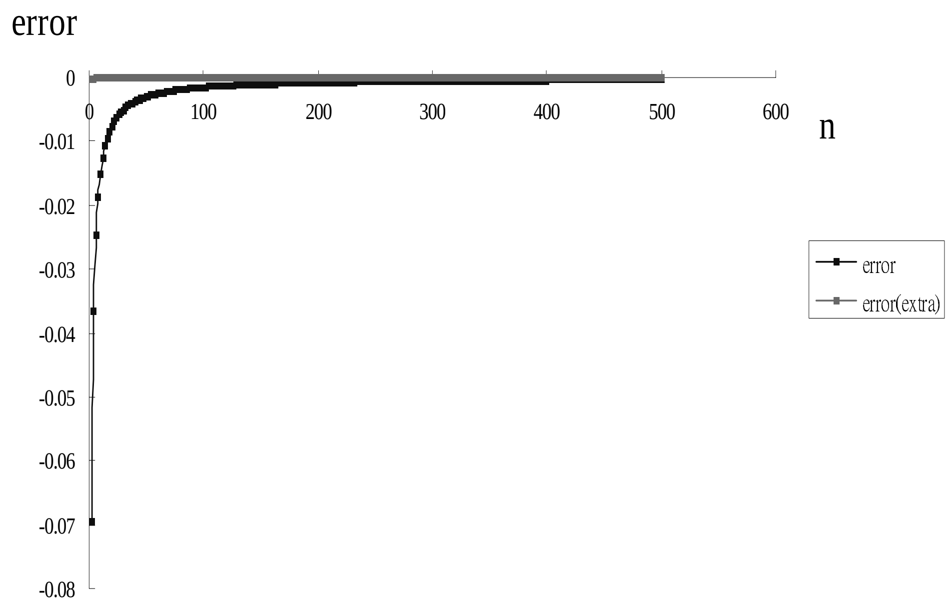


Figure 3.15: Relative error of American call (in the money).
A graph of error against the even number of periods, n, for a American call (in the money), under the modified CRR method. $S = 110$, $X = 100$, $\sigma = 0.3$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

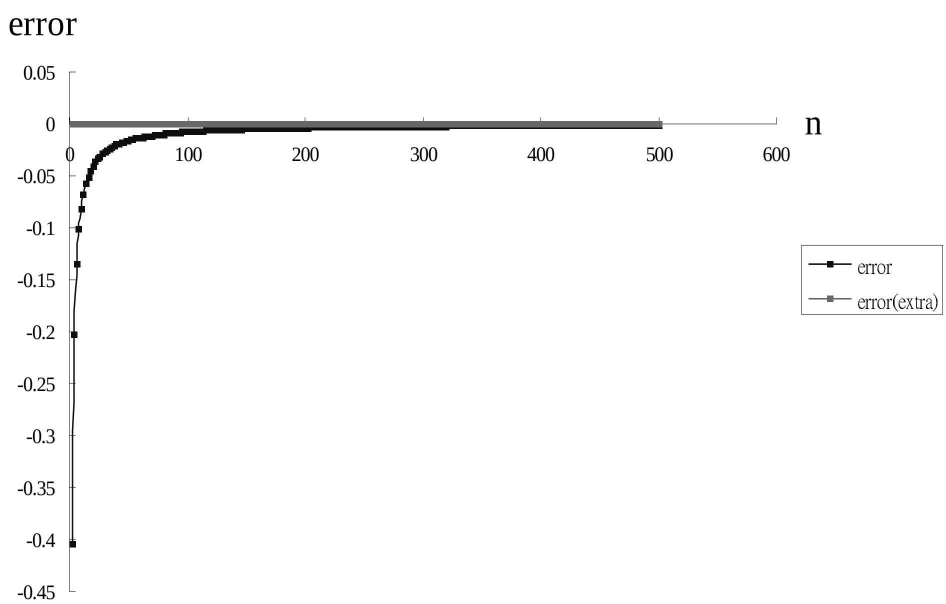


Figure 3.16: Relative error of American put (out of the money).
A graph of error against the even number of periods, n, for an American put (out of the money), under the modified CRR method. $S = 100$, $X = 90$, $\sigma = 0.2$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

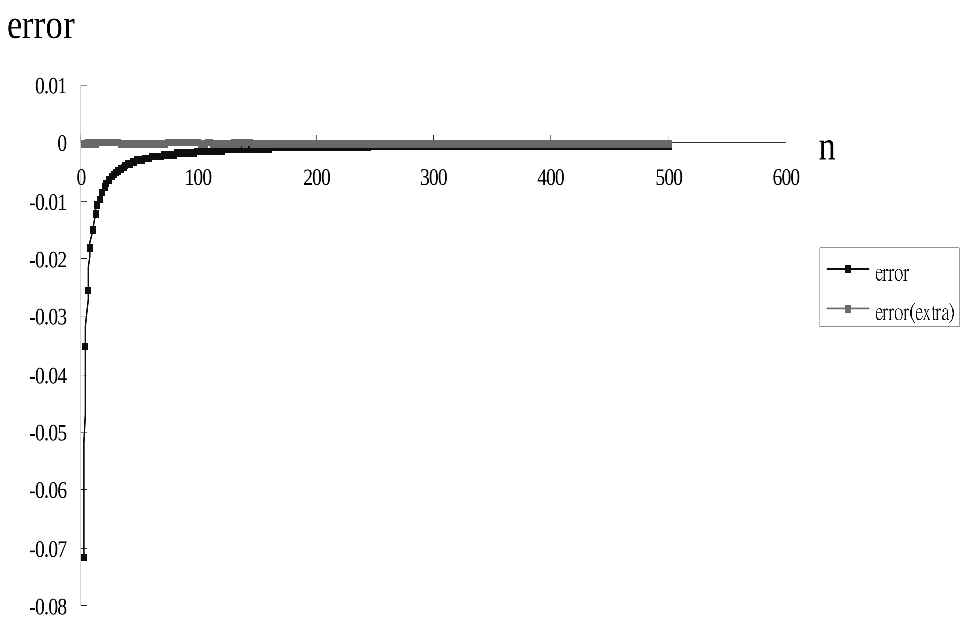


Figure 3.17: RELATIVE ERROR OF AMERICAN PUT (AT THE MONEY).
A graph of error against the even number of periods, n, for an American put (at the money), under the modified CRR method. $S = 100$, $X = 100$, $\sigma = 0.2$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

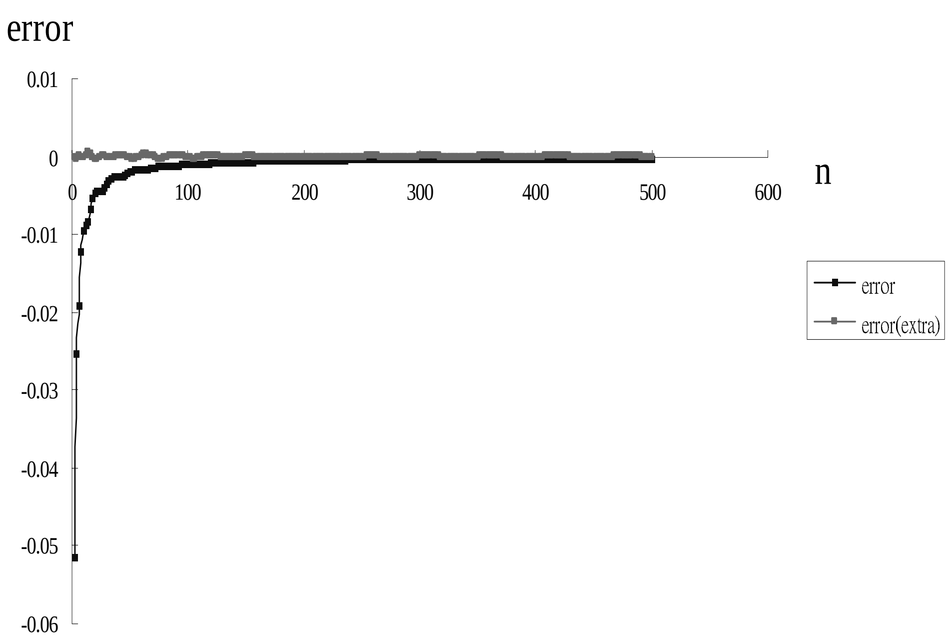


Figure 3.18: RELATIVE ERROR OF AMERICAN PUT (IN THE MONEY).
A graph of error against the even number of periods, n, for a American put (in the money), under the modified CRR method. $S = 100$, $X = 110$, $\sigma = 0.2$, $r = 0.07$, $q = 0.03$, $T = 0.5$.

Chapter 4

Conclusions and Future Work

4.1 Conclusions

This thesis deals with European and American options with tree methods via extrapolation and really provides an efficient methodology. By pegging the strike price, we modify the CRR method and then apply extrapolation. We find that the relative errors of extrapolated European and American options are extremely small even if n is very small. Therefore, we do not have to build the binomial tree for too many steps; instead, we simply apply extrapolation to get extremely accurate answers.

4.2 Future Work

We modified the CRR method to make extrapolation applicable under the condition of an even n. In order to peg the strike price and keep the strike price at the center of the final nodes, we have to choose the even number of periods so that there will be an odd number of final nodes. What will happen if we choose the trinomial tree to price European and American options instead? Does sawtooth effect exist under the trinomial tree? If the sawtooth effect exist, we may peg the strike price and see if it works in smoothing the sawtooth effect. We do not have to limit the number n because the final nodes will always be odd in the trinomial tree. If we peg the strike price to modify the trinomial tree, what will happen to the setting in the trinomial model? Is there a unique solution or maybe we may do something to fix the parameter suitable for extrapolation? These questions are interesting to explore in the future.

Bibliography

[1] Amin, K., and A. Khanna. "Convergence of American Option Values from Discrete- to Continuous-Time Financial Models." *Mathematical Finance*, 4 (1994), pp. 289–304.

[2] Barone-Adesi., and R.E. Whaley. "Efficient Analytic Approximation of American Options Values." *Journal of finance*, 42 (1987), pp. 301–320.

[3] Broadie, M., and J.B. Detemple. "American Capped Call Options on Dividend Paying Assets." *Review of Financial Studies*, 8 (1995), pp. 161–191.

[4] Broadie, M., and J.B. Detemple. "American Options Valuation: New Bounds. Approximations and a Comparison of Existing Methods." *Review of Financial Studies*, 9, No. 4 (1996), pp. 1211–1250

[5] Black, F., and Scholes, M. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* , 81, No. 3 (1973), pp. 637–659.

[6] Cox, J. C., Ross, S. A., and Rubinstein, M. "Option Pricing: a Simplified Approach." *Journal of Financial Economics* , 7 (1979), pp. 229–264.

[7] Figlewski, S., and Gao, B. "The Adaptive Mesh Model: a New Approach to Efficient Option Pricing." *Journal of Financial Economics* , 53 (1999), pp. 313–351.

[8] Geske, R. "A Note on an Analytic Valuation Formula for Unprotected American Call Options on Stocks with Known Dividends." *Journal of Financial Economics* , 7 (1979), pp. 375–380.

[9] Hull, John. *Options, Futures, and Other Derivatives.* 4th edition. Prentice-Hall, 2000.

[10] Leisen, D. P. J. "Pricing the American Put Option: a Detailed Convergence Analysis for Binomial Models." *Journal of Economic Dynamics and Control*, 22 (1998), pp. 1419–1444.

[11] Leisen, D. P. J. "The Random-time Binomial Model." *Journal of Economic Dynamics and Control*, 23 (1999), pp. 1355–1386.

[12] Leisen, D. P. J., and Reimer, M. "Binomial Models for Option Valuation — Examining and Improving Convergence." *Applied Mathematical Finance*, 3 (1996), pp. 319–346.

[13] Lyuu, Yuh-Dauh. *Financial Engineering and Computaion: Principles, Mathematics, Algorihms.* Cambridge, 2002.

[14] Whaley, R. "On the Valuation of American Call Options on Stocks with Known Dividends." *Journal of Financial Economics*, 9 (1981), pp. 207–211.

[15] Widdicks, Martin, Ari D. Andricopoulos, David P. Newton, and Peter W. Duck. "On the Enhanced Convergence of Standard Lattice Methods for Option Pricing." *Journal of Futures Markets*, 22, No. 4 (2002), pp. 315–338.

[16] Ritchen, P. "On Pricing Barrier Options." *Journal of Derivatives*, 4 (1995), pp. 19–28.

[17] Tain, Y. "A Flexible Binomial Option Pricing Model." *Journal of Futures Markets*, 19, (1999), pp. 817–843.