Chapter 1

Introduction

In Taiwan'financial market, in order to adpat to "Globalization" and satisfy the demand of investors owning the knowledge and the training of modern investment, there are many financial innovations which are created and issued by securities companies, especially warrants, which are more popular than other derivatives. In our thesis, we are forcus on the warrants whose strike prices are related to the arithmetic moving average of the underlying stock price. The most prominent examples are moving-average-reset and moving-average-lookback warrants. A moving-average-reset warrant is struck at a series of decreasing contract-specified prices over a monitoring window based on the moving average. With the moving-specified-lookback condition the warrant becomes more complicated, which is struck at the minmum moving average of the underlying stock price over a monitoring window. There issues a great portion of these combounded warrants in Taiwan.

Moving average is often considered as a technical measure for short-term trends in stock prices. Hence, it is straightforward to associate the moving average with the strike price. The advantage is, first, it is too violently changing as only considering the stock price as the reset date approaches and, second, to provide an better way to determine the strike price of the options. Recently, there has been a little research on the pricing of moving-average-reset and moving-average-lookback options. In this thesis we will forcus on the moving-average-lookback option (henceforth MAL), as the slightly simpler moving-average-reset option can be handled similarly. Pricing moving-average-style options is difficult. Firs, let M_t^5 be the 5-day movingg average at the tth trading day, where $t \geq 4$ and S_t denotes the stock price at day t.

Then

$$M_t^5 = \frac{\sum_{i=t-4}^{t} S_i}{5}$$

The moving average M_{t+1}^5 is related to not M_t^5 but also M_{t-1}^5; M_{t-2}^5; ...; M_{t-4}^5. In pricingg arithmetic average MAL, we should solve the problem of the nonnormality of the sum of lognormal distribution, and the non-Morkovian property mentioned above. These two issues combine to increase the difficulty of pricing.

There are three major way to value derivatives. The first is to derive closed-form solutions of derivatives by partial-differential-equation (PDE) or martingale method. We can value the option from PDE by finite dfference techniques by transforming the problem from a path-dependent one to a Markovian problem. However, the PDE of arithmetic MAL has never been derived and the strike based on all past moving-average term is very difficult to value by finite difference approach.The second, we can price American-style derivatives on the tree algorithm, especially the CRR model (Cox, Ross, and Rubinstein (1979)). The last is Least-Squares Monte-Carlo (henceforth LSM) simulation approach we will forcus on, especailly used most effectively and powerful to not only price strongly path-denpendent and multifactor derivatives, but also to solve the problem of early exercise of American-style deratives on Monte-Carlo simulation before.

It is easy to solve the problem of path-dependence about the moving-average term by Monte-Carlo simulation. There exists, nevertheless, a bottleneck of pricing American-style derivatives on Monte-Carlo simulation approach all the time. The famous technology is introduced by Boyle, Broadie & Glasserman (1997). Their approach is more closely related to the tree method, and should produces an upward bias and a downward bias estimates and average both to obtain the unbias value. Besides the method needs to simulate several paths from each point to obtain an unbias estimator of the American option price, which resulting in the curse of dimesionality that the lattice methods also suffers from.

A new and simpler simulation based method to price American options has recently been proposed by Longstaff & Schwartz (2001). The idea is to estimate the conditional expectation of the payoff from continuing to keep the option alive at each possible exercise point from a cross-sectional regression using the simulated paths. Based on LSM methods, we can solve the problem of determining the optimal early exercise strategy of American-style options. We also consider the developed CRR model (Kao (2002)) as a benchmark on pricing arithmetic MALs. It will be found that the LSM approach is very close to the price calculated by the CRR model. With the LSM algorithm, a detail analysis of American-style AMALs will be presented to understand the properties of this derivatives. Besides we will find even if any changeable and complicated derivatives, such as MALs discussed in this thesis, it is not hard to price with the LSM method.

The remainder of this thesis is organized as follows. Chapter 2 reviews basic concepts and pricing technologies on simulation and tree model. Chapter 3 describes the underlying theoretical framework. Chapter 4 covers the pricing of arithmetic MALs and numerical analysis of different arithmetic MALs. Chapter 5 describes how to choose the number of regressors and alternative family of basis funtions. Chapter 6 summarizes results and concludes.

Chapter 2

Preliminaries on Options Pricing

In this chapter, we review fundamental concepts and pricing techniques used in later chapters. The …rst, the Monte-Carlo simulation technique must be introduced. The second, we will review the tree model.

2.1 Simulation and option pricing

There exists a major problem with numerical methods is that they are not easily extended to more than one of stochastic factors. In the Tree Model, the number of nodes grows exponentially as the number of stochastic factors increases. In the Finite Di¤erence, it can only calculate less than three stochastic factors generally. So it is possible method to solve the problem of multidimensional by using simulation. Furthermore, time should be divided into a number of segments in the simulation method. We get the next period price by a random walk, and the number of nodes remains constant through time. Besides there needs a large number M of simulated paths by Law of Large Number for convergence. At last the estimator is gotten by the average of the prices over the paths.

If we want to get the stochastic variables, such as stock prices, interest rate, volatility, or dependence on multiple stock prices, which can be included in the simulation. How many paths to use and how many steps to partition time to expiration into should be decided on. In general the more simulated paths , the more precise the estimator of stochastic factor is. In the same way, increasing the number of steps would confirm that the estimator converges to the exact true price.

We will show how to price options using simulation. The American option is assumed to be exercisable at a finite nubmer of equally spaced points in time. We can specify the risk discrete exercise feature by using of Geometric Brownian Motion (GBM). Finally, we describe how to price options using the simulation.

2.1.1 Simulating from a Geometric Brownian Motion

To simulate a GBM by the stochastic Differential Equation (SDE)

$$dS(t) = rS(t)dt + \sigma S(t)dW(t) \qquad (2.1)$$

where W is a standard Wiener process and r and σ are assumed constant, we use the well known solution to (1). Given a starting level of $S(0)$ this is

$$S(t) = S(0)\exp\{(r - \frac{1}{2}\sigma^2)t + \sigma W(t)\} \qquad (2.2)$$

From the propeties of the Wiener process simulated value of $S(t)$ at a a single point in time can be obtained from the formula

$$S(t) = S(0)\exp\{(r - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}Z\} \qquad (2.3)$$

where $Z \sim N(0, 1)$. A sequence of values at discrete date $0 \le t_1 \le t_2 \le \ldots \le t_N = T$ is obtained by setting

$$S(t_{i+1}) = S(t_i)\exp\{(r - \frac{1}{2}\sigma^2)(t_{i+1} - t_i) + \sigma\sqrt{(t_{i+1} - t_i)}Z(t_{i+1})\} \qquad (2.4)$$

where $Z(t_{i+1}) \sim IIN(0, 1)$.

2.1.2 Pricing European options using simulation

The price of European put option is the expectation under the risk neutral measure of the present value of its payoff given as

$$p \equiv p(S(0), T) = E[e^{-rT}\max(X - S(T), 0)]$$

And we can get an estimate of the price by the formula

$$\bar{P}_N = \frac{1}{N}\sum_{j=1}^{N} e^{-rT}\max(X - S_j(T), 0)$$

where N is the number of simulated paths and $S_j(T)$ is the value of the underlying stock at expiration of the option for path numbe j.

2.1.3 Pricing American options using simulation

The key of pricing American options with simulation is determining the optimal exercise strategy. We write the price of American put options as

$$P \equiv P(S(0), T) = \max_{0<\tau\leq T} E[e^{-rT}\max(X - S(\tau), 0)]$$

where the maximization is over stopping times $\tau \leq T$ adapted to filtration generated by the relevant stock price process $S(t)$. The problem is that at any possible exercise time, the holder of an American option should compare the payoff from immediate exercise to the expected payoff from continuation. The optimal decision is to exercise if the exercise value is positive and larger than the expected payoff from continuation.Using next period values of the underlying asset to determine the expected value along each path of continuing to keep the option alive would lead to biased price estimates. The main reason of making the estimator biased is to consider the expected payoff from continuation as perfect forsight (see Broadie & Glasserman(1997)). Hence, we can not simply estimate the price P by

$$\bar{P}_N = \frac{1}{N}\sum_{j=1}^{N} \max_{\tau}\,[e^{-r\tau}\max(X - S_j(\tau), 0)]$$

Note that we want to prevent this bias, but the best way is to simulate several paths from each possible exercise point thus resulting in multidimensionality. However, Longstaff & Schwartz (2001) provide a very powful idea to estimate the conditional expectation of the payoff from continuing to keep the option alive, using the cross-sectional information in the simulation.

The main motivation of the LSM approach can be given in terms of Hilbert Spaces, the space of square-integrable functions with the norm

$$\langle f(x), g(x)\rangle = \int f(x)g(x)dx$$

The theory of Hilber spaces tells us that any function $G(x_n)$ belonging to this space can be represented as a countable linear combination of bases for this vector space. We can write

$$G(x_n) = \sum_{k=0}^{\infty} a_k \phi_k(x_n) \qquad (2.5)$$

where $\{\phi_k(x)\}_{k=1}^{\infty}$ form a basis (See Royden(1988)). In pratice we use a finite linear combination to approximate $G(x_n)$ which we denote $G_K(x_n)$, where K is the number of basis functions used. The simplest approximation way is using least squares regression. when the coefficients $\{a_k\}_{k=0}^{K}$ in (15) are estimated, we have to simulate N paths s.t. $N \geq K+1$,i.e. there will exist data points $(y_j; x_j)$, $j = 1, \ldots, N$; by solving the minimization problem

$$\min_{\{a_k\}_{k=0}^{K}} \sum_{j=1}^{N} (a_0\phi_0(x_j) + a_1\phi_1(x_j) + \ldots + a_K\phi_K(x_j) - y_j)^2$$

With the parameter estimates $\{a_k\}_{k=0}^{K}$ we estimate $G_K(x)$ with

$$\hat{G}_K(x) = \sum_{k=0}^{K} \hat{a}_k \phi_k(x) \qquad (2.6)$$

In general, $\hat{G}_K(x) \to G_K(x)$ as $N \to \infty$. Letting $G(x) = E[y|x]$, where y is the payoff from continuing to keep the option alive, x represents the current state, and $\{\phi_k(x)\}_{k=0}^{K}$ is a set of independent variables, the conditional expectation function $G(x)$ can be arbitrarily approximated as N and K both tend to infinitely. And the approximated $G(x)$ is used to determine the optimal exercise strategy.

2.2 Tree Models and Auxiliary State Variable

In this Section, we review two usefule pricing techniques. The first, the CRR model, is mainly used to solve American-style options. The second, auxiliary state varialbes approach, is a general method to price path-dependent derivatives on the tree.

2.2.1 The CRR model

The CRR model is one simplest but very powerful of tree models introduced in Cox, Ross, and Rubinstein (1979).

Time to expiration is also divided into a number of segments, denote each unit of time as $\Delta t = \frac{T}{n}$, where T is time to expiration and n is the number of partitions. From Eq. (2.2), we can obtain the expected value of the stock price change after a small time Δt is $S_0 e^{r\Delta t}$ and the variance of the stock price change after Δt time is $\sigma^2 \Delta t$. Now consider the discrete-time version of Eq. (2.1) and change the normal diffusion to a discrete random variable, ΔW. It follows that

$$\Delta S_t = rS_{t-\Delta t}\Delta t + \sigma S_{t-\Delta t}\Delta W.$$

Assume ΔW follows the Bernoulli distribution such that

$$S_{t+\Delta t} = \begin{cases} S_t u, & \text{with probability } p, \\ S_t d, & \text{with probability } 1-p, \end{cases}$$

where u and d are the proportional change of S_t in the up and the down state. We let ΔW satisfy the mean and variance function mentioned above. This yields the following conditions,

$$\begin{aligned} e^{r\Delta t} &\approx pu + (1-p)d \\ \sigma^2 \Delta t &\approx pu^2 + (1-p)d^2 - [pu + (1-p)d]^2 \\ ud &= 1 \end{aligned}$$

We obtain a possible solution :

$$\begin{aligned} p &= \frac{e^{r\Delta t} - d}{u - d} \\ u &= e^{\sigma\Delta t} \\ d &= e^{-\sigma\Delta t} \end{aligned}$$

The stock price on node $N(i,j)$ reachable from the root with j up and $i-j$ down moves is

$$S(i,j) = S_0 u^j d^{i-j}$$

and the value of derivatives C on node $N(i;j)$ can be obtained by the backward induction formula :

$$C(i;j) = e^{-r\Delta t}[pC(i+1;j+1) + (1-p)C(i+1;j)] \qquad (2.5)$$

for $i = 0;1;\ldots;n$ and $j = 0;1;\ldots;i$; where $e^{-r\Delta t}[pC(i+1;j+1) + (1-p)C(i+1;j)]$ is called as the expected payoff from continuation on the CRR model. When pricing American-style options, we change Eq. (2.5) into

$$C(i;j) = \max(e^{-r\Delta t}[pC(i+1;j+1) + (1-p)C(i+1;j)];\ S(i;j) - X) \qquad (2.6)$$

where X is the strike price of call option, and $S(i;j) - X$ is the value of immediate exercise at node $N(i;j)$. The option value emerges in $C(0;0)$:

2.2.2 Auxiliary State Variable

This section draws on Dai (1999), which provides a general method for pricing path-dependent derivatives on tree Model. Auxiliary state varialbes are memory space to record the past information needed in dealing with the path dependency. Let $C(i;j;k)$ denote the option value on node $N(i;j)$. In addition to i and j, which provide the information of time and the current stock price, we need an additional k to record the information arising from path dependency.

To apply backward induction, we have to allocate enough auxiliary state variables for all the possible situations at each node. The size of auxiliary state variables depends on the number of possible situations determined by path dependency. This technique is not suitable for cases which need very large sizes of auxiliary state variables such as Asian options. However, the auxiliary state varialbes approach is useful in pricing longer time path-dependent derivatives, such as "weekly". When we allow approximation, the alleged shortcoming of this approach no longer exists.

Chapter 3

The LSM Valuation Algorithm

We will describe the general LSM algorithm in theory later. The valuation algorithm of LSM can be applied on the general derivative pricing paradigms, such as Black and Scholes (1973), Merton(1973), Cox, Ingersoll, and Ross (1985), Heath, and so on.We also present several convergence results for the algorithm.

3.1 The LSM valuation framework

Assuming an underlying complete probability space (¤; F,P) and …nite time [0; T], where the state space ¤ is the set of all possible realizations of the stochastic economy between time 0 and T and has typical element w representing a sample path, F is the sigma …eld of distinguishable events at tine T, and P is a probability measure de…ned on the elements of F. We de…ne F = fF_t; t 2 [0; T]g to be the augmented …ltration generated by the relevant price processes for the securities in the economy, and assume that F_T = F. Consistent with the no-arbitrage paradigm, we assume the existence of an equivalent martingale measure Q for this economy.

We restrict our attention to payo¤s that are elements of the space of square-integrable (or …nite-variance) functions L^2(¤; F; Q). The value of an American option equals the maximum is taken over all stopping times with respect to the …ltration F. We present the path of cash ‡ows generated by the option , denoted as C(w; s; t; T) , conditional on the option not being exercised at or before time t and on the optionholder following the optimal stopping strategy for all s, t < s · T.

The objective of the LSM algorithm is to provide a pathwise approximation to the optimal stopping rule that maximizes the value of the American option. In practice, many American options are continuously exercisable; the LSM algorithm can be used to approximate the value of these options by taking the exercising times to be sufficiently large.

At the final expiration date (T) of the option, the option is exercised if it is in the money, or expire if out of money. At exercise time $t_M < T$, however, the holder of an American option must determine whether to exercise immediately or to keep alive.

At time t_i, the payoff from immediate exercise is known to the investor, but the cash flow from continuation are unknown. No-arbitrage valuation theory, however, implies the expected payoff from continuation assuming that it cannot be exercised until after t_M, is given by taking the expectation of the remaining discounted cash flows $C(w; s; t; T)$ with respect to the risk-neutral pricing measure Q. Specifically, at time t_m, the value of continuation $G(w; t_m)$ can be represented as

$$G(w; t_m) = E_Q[\exp(-\int_{t_m}^{t_i} r(w; s)ds)C(w; t_i; t_m; T)|F_{t_m}]$$

where $r(w; t)$ is the riskfree rate, and the expectation is conditional on the information set F_{t_m} at time t_m. With this representation, the problem of optimal exercise reduces to comparing the immediate exercise value with this conditional expectation, and then exercise as soon as the immediate exercise value is positive and greater than and equal to the conditional expectation.

3.2 The LSM algorithm

The LSM approach uses least squares to approximate the conditional expectation function at $t_{M-1}; t_{M-2}; \ldots; t_1$. We work backwards to generate the cash flows $C(w; s; t; T)$ recursively. At a special time t_{M-1} we can represent the unknown $G(w; t_{M-1})$ as a linear combination of a countable set of $F_{t_{m-1}}$-measurable basis functions.

When the conditional expectation is an element of the L^2 space of square-integerable functions. Since L^2 is a Hilbert space, it has a countable orthonormal basis and the conditional expectation can be represented as a linear function of the elements of the basis.

As an example, assume that $x(t_i)$ is the value of the asset underlying the option and that X follows a Markov process[1]. We choose the set of laguerre polynomials as the basis functions (as Longstaff and Schwartz (2001)).

$$
\begin{aligned}
L_0(X) &= w(X) \\
L_1(X) &= w(X)(1 - X) \\
L_2(X) &= w(X))(1 - 2X + X^2/2) \\
L_n(X)) &= w(X)\frac{e^X}{n!}\frac{d^n}{dX^n}(X^n e^{-X})
\end{aligned}
$$

where $w(X) = \exp(-\frac{X}{2})$: With this specification, $G(w; t_{M-1})$ can be represented as

$$G(w; t_{M-1}) = \sum_{k=0}^{\infty} a_k L_j(X)$$

where the a_k coefficients are constants.

To implement the LSM approach, we approximate $G(w; t_{M-1})$ using $M < \infty$ basis functions mentioned above, and denote this approximation $G_M(w; t_{M-1})$. $G_M(w; t_{M-1})$ is estimated by regressing the discounted values of $C(w; s; t_{M-1}; T)$ on the basis functions across paths where the option is in the money. We use only in-the-money paths in the estimation since the exercise decision is only related with the in-the-money option. And we need a finite number of basis to obtain an accurate approximation to the conditional expectation function. Since the basis functions are independently and identically distributed across paths, the existence of moments of Theorem 3.5 of White (1984) shows that the fitted value of this regression $\hat{G}_K(w; t_{M-1})$ converges in mean square and in probability to $G(w; t_{M-1})$ as the number N of paths goes to infinity. Furthermore, Theorem 1.2.1 of Amemiya (1985) implies that $\hat{G}_K(w; t_{M-1})$ is the best linear unbiased estimator of $G_K(w; t_{M-1})$ based on a mean-squared metric.

Once the conditional expectation function at time t_{M-1} is estimated, we can determine whether early exercise at time t_{M-1} is optimal for in-the-money

[1] For Markovian problems, only current values of the state varialbes are necessary. For non-Markovain problems, both current ans past realizations of the state varialbes can be included in the basis functions and the regressions.

path w by comparing the immediate exercise value with $\hat{G}(w; t_{M-1})$, and repeating for each in-the-money path. Once exercise decision is indentified, the option payoff $C(w; s; t_{M-1}; T)$ can be approximated based on cash flows along path w after the determination of optimal exercise strategy at time t_{M-1}. The recursion process is rolling back and repeating until the exercise decisions at each exercise time along each path have been determined. The American option is then valued by strating at time 0, moving forward along each path until the first stopping time occurs, discounting the payoff from exercise back to time 0, and then averaging the payoff over all paths w.

When there are two state variables X and Y, the set of basis functions should include terms in X and in Y, as well as cross-products term, XY. Contrary to other methods with higher-dimensional problems, the number of basis functions does not grows exponentially but grow a slower rate with convergence result.

3.3 The LSM algorithm to pricing Amercian option in mathematics

3.3.1 The presentation of pricing American call options

The following is the detail of implementation of LSM algorithm.

1. Simulation of stock paths:

Simulate a large number of paths (N) of asset prices using an exact formula like (4), and choose the number of steps (M) sufficiently large to approximate continuous exercise. Following (a) let $S_j(t_i)$ denote the asset price along path j at time t_i corresponding to step i, where $j = 1; \ldots; N$ and $i = 1; \ldots; M$.

2. Calculation of the payoff matrix:

Let P (for payoff) be a $N \times M$ matrix, with typical element $f_{j;i}$. At time $t_M = T$ (the expiration date of the option) the payoff along each path is the maximum between zero and the value of exercising the option. Hence, we can define the elements of the last column as

$$f_{M;j} = \max(S_j(T) - X; 0); \quad 1 \le j \le N$$

OLS is used to estimate the conditional expectation of the payoff that the option is kept alive (see (14)) by working backwards at each time $t_i; 0 < i < M$. First, the rule for choosing the paths is where the option is in-the-money denoted as $\tilde{N}$. For a put option we define $\tilde{N} = \{j : S_j(t_i) - X(t) > 0; 1 \le j \le N\}$. For any $j \in \tilde{N}$, the payoff from continuation is the payoff along the path until expiration of the option discounted back using the risk-free interest rate

$$y_j(t_i) = \sum_{k=i+1}^{M} e^{-r(t_k - t_i)} f_{k;j}$$

This is the dependent variables. We need to transform these dependent variables to independent ones, such as $X_j(t_i) = h(x_j(t_i))$; where $h(x_j(t_j))$ is a transformation of the state variables. If the underlying asset is only one stochastic factor, it is suffice to explain variations in the dependent variable as $x_j(t_i) = S_j(t_j)$: Following (c) we approximate the conditional expectation $G(x_j(t_i)) \equiv E[y_j(t_i)|x_j(t_i)]$ as

$$\hat{G}(x) = X_j(t_i)\beta(t_i)$$

where $\beta(t_i)$ is a vector of coefficients. This is the linear regression model $y_i(t_i) = X_j(t_i)\beta(t_i) + u_i(t_i)$, the parameters can be estimated by

$$\hat{\beta}(t_i) = (X(t_i)'X(t_i))^{-1}X(t_i)'y(t_i)$$

The fitted values $\hat{y}(t_i) = X(t_i)\hat{\beta}(t_i)$, which corresponds to the estimated conditional expectation of the payoff when the option is kept alive, are used to determine if it is optimal to exercise the option at time t_i.If the fitted value is larger than the value of immediate exercise $X - S_j(t_i)$, $f_{i;j}$ are set equal to the value of immediate exercise $X - S_j(t_i)$ and in all other values $f_{n;j}$ $i < n \le N$, are set to equal to zero. That is,

$$f_{i;j} = \begin{cases} K - S_j(t_i) \text{ and } f_{j;n} = 0; i < n \le N, & X - S_j(t_i) > \hat{y}_j(t_i) \\ 0, & \text{otherwise} \end{cases} ; j \in \tilde{N}.$$

3. Calculating the value of the option:

When $t_i = 0$ the value of the option is calculated from the payoff matrix by discounting the payoffs to period zero using the risk-free rate and averaging across the simulated paths. Since there is at most one nonzero element along each path in P this can be written as

$$\bar{P}_N = \frac{1}{N}\sum_{j=1}^{N}\sum_{i=0}^{M}[e^{-rt_i}\max(f_{i;j};0)]$$

The neutral price process follows a GBM and the option has discrete exercise features.

3.4 Convergence results

How well the LSM algorithm performs is using a realistic number of paths and basis functions, it is useful to examine the theoretical convergence of the algorithm to the true value $G(X)$ of the American option.

First, we present the bias of the LSM algorithm when the American option is continuously exercisable.

Proposition 1 For any finite K, M, and vector $\theta \in R^{K\times(M-1)}$ representing the coefficients for the K basis functions at each of $M-1$ early exercise dates, let N denote the number of simulated paths, $G(X)$ denote the true value of the American-style option and $LSM(w;M;K)$ denote the discounted cash flow resulting from following the LSM rule of exercising when the immediate exercise value is positive and greater than or equal to $\hat{G}_K(w_i;t_m)$ as defined by θ. Then the following inequality holds almost surely,

$$G(X) \geq \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} LSM(w_i;K;M)$$

The LSM algorithm is considered as a stopping rule for an American-style option. The value of an American-style option is based on the stopping rule that maximizes the value of the option.

The result is particularly useful since it provides an objective criterion for convergence, that is any result simulated by the LSM algorithm has a upper bound. As a criterion example, we can increase K until the value implied by the LSM algorithm no longer increases. It is very useful and important property in the LSM algorithm.

The following convergence result for the LSM algorithm is difficult since we need to consider limits as the number of exercisable dates M; the number of basis functions K, the number of paths N go to infinitely. Consider the following proposition.

Proposition 2 Assume that the value of an American option depends on a single state variable X with support on $(0;\infty)$ which follows a Markov processes. Assume further that the option can only be exercised at times t_1 and t_2, and that the conditional expectation function $G(w;t_1)$ is absolutely continuous and

$$\int_0^\infty e^{-X} F^2(w;t_1)dX < \infty$$

$$\int_0^\infty e^{-X} F_K^2(w;t_1)dX < \infty$$

Then for any $\epsilon > 0$, there exists an $K < \infty$ such that

$$\lim_{N \to \infty} \Pr\left[\left|G(X) - \frac{1}{N}\sum_{i=1}^{N} LSM(w_i;K;M)\right| > \epsilon\right] = 0$$

Intuitively this result means that when K is large enough and $N \to \infty$, the LSM algorithm results in a value for American option within ϵ of the true value, where ϵ is selected arbitrarily.An important implication of this result is that the number of basis functions result in a desired of accuracy need not go to infinity.

Chapter 4

Pricing Moving-Average-Lookback options

There exists more than two variables, such as stock price , moving-average term, and strike price, etc... in valuing Moving-Average-Lookback options (MVALs), but it is easy to value with the LSM approach.In order to confirm that the value is almost approximate to the true value, the value of the CRR model is taken as a benchmark compared with the LSM approach. (see Kao (2002)). In Taiwan, the issued warrants are almost Arithmetic Moving-Average options, so we will forcus on and price Arithmatic Moving-Average-Lookback options (AMVALs). To the end, the empirical results about the different contracts of American-style AMALs will be presented for you.

4.1 Defining the AMVALs

Let $0 = t_0 < t_1 < t_2 < \cdots < t_{n_s} \leq T$; where n_s is the number of trading days before reset dates T_s, and t_i be the time points when the moving average is calculated. The t_i are expected to correspond to trading dates as closing prices. Assuming that the time interval between monitoring times are equal and $\Delta t = T_s/n_s$; i.e., $t_i = i\Delta t$. and $n = T/\Delta t$. Define $S_i \equiv S_{t_i}$, the stock price at time $i\Delta t$. The Arithmetic moving average at time t_i equals

$$m_a(i) \equiv \frac{\sum_{j=i-a+1}^{i} S_i}{a}, \quad a - 1 \leq i \leq n_s.$$

The minimum a-day Arithmetic moving average as of the reset date $t_{n_s} = T_s$ is defined as

$$m_a(k) \equiv \min_{a-1 \le t \le k} \frac{\sum_{i=t-a+1}^{t} S_i}{a}; a-1 \le t \le k; k = \frac{t_k}{\Delta t}$$

Note that time to expiration is divided into a number of periods, i.e., it is evaluated at discrete times. The payoff function of the MAL at expiration date t_i is

$$f_i = \begin{cases} \max(S_i - X; 0); n_s < i \le n \\ \max(S_i - X_i; 0); a-1 < i \le n_s \end{cases} \quad (4.1)$$

$$X_i = \begin{cases} \max(\min(m_a; UB); LB); n_s < i \le n \\ \max(\min(m_a(i); UB); LB); a-1 < i \le n_s \end{cases} \quad (4.2)$$

where X_s is the strike price of the option determined at reset date T_s: UB is the upper bound of the strike price is set to S_0; the initial stock price. The change of UB may happen at times t between t_{a-1} and T_s. LB, the lower bound of the strike price, is determined by the contract and fixed. Eq. (4.2) means that the strike price of the option, X, is struck at the minimum a-day moving average but range between LB and UB.

4.2 Pricing American-Style AMALs

We will described the details how to prcing the American-Style AMALs (AAMALs) by using the LSM approach. There are two scenarios defined for distinguishing the option is early exercised after reset date and before. We denote the former as scenario 1 and the latter as scenario 2. Note that we assume the strike price can be reset every day before T_s on scenario 2. The improved CRR model for AAMALs will be introduced simply.

4.2.1 The LSM methods

Now, we describe how to price AAMALs with LSM. As in section 3.3, the first step is to generate the stock price matix from t_1 to T: Simulate a large number of paths (M) of stock prices using the formula like (2.4). T_s is denoted as the reset date. The time must be classified into two parts, i.e., before the reset date and after the reset date, we set the former n_s steps and

the latter n' steps. Note that n is equal to n_s plus n'.
And we save the stock price matix $S(i;j)$, where i refers to stop at time t_i and j represent the j-th paths. The strike vector $M(i;j)$ is determined by the rule of $\max(\min(m_a(i;j);UB);LB)$ before T_s.

After reset date, the strike vector is determined at the reset date, denoted as M_{n_s}. Giving the expiration conditions, $S_{n;j} - M_{n_s;j}$; we use a constant, the first two Laguerre polynomials evaluated at the stock price, the first two Laguerre polynomials evaluated at the strike price, and the cross products of these Laguerre polynomials up to third-order terms. Thus we use a total of eight basis functions in the regressions. Thus, least squares regression is done on the following model after reset date T_s:

$$
\begin{aligned}
y_{i;j} &= \beta_0 + \beta_1 LS + \beta_2 LM_{n_s} + \beta_3 LSM_{n_s;j} \\
&\quad + \beta_4 LS(1 - S_{i;j}) + \beta_5 LM_{n'}(1 - M_{n_s;j}) \\
&\quad + \beta_6 LSM_{n_s}(S_{i;j} - \frac{1}{2}S_{i;j}^2 M_{n_s;j}) \\
&\quad + \beta_7 LSM_{n_s}(M_{n_s;j} - \frac{1}{2}M_{n_s;j}^2 S_{i;j}) \qquad (4.2) \\
LS &= \exp(-\frac{S_{i;j}}{2}) \\
LM_{n_s} &= \exp(-\frac{M_{n_s;j}}{2}) \\
LSM_{n_s} &= \exp(-\frac{S_{i;;j}M_{n_s;j}}{2})
\end{aligned}
$$

where $y_{i;j}$ is that the stock price vector of the j-th path after the time t_i is never early exercised at or before the time t_i based on the optimal exercise strategy of the LSM rule. To avoid any form of numerical overflow, and to get as precise results as possible, both payoff $y_{i;j}$ and the stock price $S_{i;j}$ and the strike $M_{n_s;j}$ are normalized by dividing the initial stock price S_0. Regressing with Eq. (4.2), we obtain this conditional expected payoff function at time $i\Delta t$.

$$
\begin{aligned}
\hat{E}_i(C_{i;j}|S_{i;j}; M_{n_s;j}) &= \hat{y}_{i;j} = \hat{\beta}_0 + \hat{\beta}_1 LS + \hat{\beta}_2 LM_{n_s} + \hat{\beta}_3 LSM_{n'} \\
&\quad + \hat{\beta}_4 LS(1 - S_{i;j}) + \hat{\beta}_5 LM_{n_s}(1 - M_{n_s;j}) \\
&\quad + \hat{\beta}_6 LSM_{n'}(S_{i;j} - \frac{1}{2}S_{i;j}^2 M_{n';j})
\end{aligned}
$$

$$+\hat{\beta}_7 LSM_{n_s}(M_{n_s;j} - \frac{1}{2}M^2_{n_s;j}S_{i;j}) \quad (4.3)$$

where $\hat{\beta}_i$ denotes the OLS estimator of β_i. Compare the exercise value, $S_{i;j} - M_{n_s;j}$, and the expected value of continuation, $E(C_{i;j}|S_{i;j};M_{n_s;j})$; to determine the option value with respect to each path at time $t_i \geq T_s$ as follows :

$$f_{i;j} = \begin{cases} S_{i;j} - M_{n_s;j}; \text{ if } S_{i;j} - M_{n_s;j} > E_i(C_{i;j}|S_{i;j};M_{n_s;j}) \\ 0 \text{ , otherwise} \end{cases} \quad (4.4)$$

After repeating the procedure in a backward fashion for $i = n - 1$ to n_s; we can get the value of scenario 1 by discounting the value in $C(n_s; j)$ for all j, averaging over all paths, and then discounting the value at time 0 with e^{-rT_s}.

The value of scenario 2 is stated two steps different from scenario 1. The first step is to replace $M_{i;j}$, $0 < i < n_s$ into $M_{n_s;j}$ in Equation (4.2), (4.3) and (4.4). The second step is repeating the first step until the time 0, discounting the value of all cash flows to time 0 and averaging all discounted payoff over all paths, which is the value of scenario 2.

4.2.2 The CRR Model

We proceed to price the American-style AMAL on the CRR model in this section. Recall that n_s is the number of trading days before the reset date. Let L denote the number of periods between two adjacent monitoring time points (which will coincide with daily closing times). By making $L\Delta t$ a day, we make L the number of trading points per day. The number of trading points before the reset date, N, is equal to n_sL. We will build the binomial tree up to the reset date.

In order to speed up the algorithm and becasue moving averages involes only daily closing prices, we simplify the N-period tree based on ideas from Ritchken and Trevor (1999). Although there are N periods before the reset date, we only care about nodes on monitoring days, i.e., at times 0, $L\Delta t$, $2L\Delta t$, ..., $nL\Delta t$. We therefore merge every L levels of the binomial tree into one, creating an $(L+1)$-ary tree with n periods in the process. There are more details introduced in Kao's Master thesis (2002).

4.3 Numerical Results

In this section, we do some empirical works and studies on the AAMALs with the LSM method. There are …ve cases for di¤erent contracts of AAMALs analysis, and we will take the value of scenario 1 on the CRR model on Kao (2002) as a benchmark.

4.3.1 Case 1 : Stock Price v.s. Volatility

We use the LSM method to price 5-day the scenario 1 and scenario 2 AAMALs and European-Style AMALs, and use the CRR method to price 5-day the scenario 1 AAMALs. Assume UB = 45, r = 3%; q = 0; T = 1; and T_s = 1=12. We will vary S_t and ¾ in the experiment and …x L = 3 on the CRR model. The results are tabulated in Table 4.1.

We obtain the following observation. First, the prices of senario 1 calculated by the LSM method is not di¤erenet from those by the CRR model within 0.08. Therefore, we can have the con…dence in the LSM algorithm. Second, the option value increases with ¾. Third, if the stock price is less than the UB at time 0; the special appearance is that the value of AAMALs calculated by the LSM method is undervalued to one by the CRR model; on the contraty, it is overestimated, which shows that there exists "slight" negative and positive bias in the LSM algorithm with the benchmark of the CRR model. If the CRR model is very close to the true value, the value calculated by LSM must be undervalued to one by the CRR model based on proposition 1. We think that maybe the more simulated paths in more complexed contract are needed to see the consistent result as proposition 1.

Next we check the relation between scenario 1 and scenario 2. Because the reset date T_s is one month, it is too short to see their di¤erence. The di¤erence between scenario 2 and scenario 1 only moves the highest up to 0.006. It means that when T_s is less than two months and the dividend yield is very low, the possibility of early exercise before T_s for the AAMAL is low. It is so interesting that the di¤erence between scenario 1 and European-style is very close to each other, which means an American-style AMALs call options will also not be exercised early with no dividend payment or low dividend rate.

4.3.2 Case 2 : Stock Price v.s. LB

Assume $UB = 50$, $r = 2\%$; $q = 4\%$; ¾ $= 50\%$; $T = 1$; and $T_s = 1{=}12$. Suppose there are 22 trading days in a month, so $n_s = 22$, and we set $n^0 = 50$: We will vary S_t and ¾ in the experiment and …x $L = 3$ on the CRR model. We will vary S_t and LB in the experiment. The results are tabulated in Table 4.2.

We make the following observations. Fisrt, not surprisingly, the option value decreases with LB, and increases with ¾. Second, compared with the tree algorithm, there also exists biases in the LSM method, but all of their di¤erences are less than the highest 0.16, although a large proporsition of the di¤erences are positive. Next, the di¤erences between scenario 1 and scenario 2 are also close to 0, due to the reset date is too short, and the option-holder would not exercise early before T_s. Besides, the European MALs value are similar to the value of scenario 1, too.

S_0	¾	Scenario1		Scenario2	European
		CRR	LSM		
40	0.3	5.0496	4.9931	4.9944	4.9511
			(0.0002)	(0.0002)	(0.0005)
	0.4	6.5839	6.5626	6.5651	6.5038
			(0.0022)	(0.0022)	(0.0017)
	0.5	8.1042	8.0247	8.0262	7.9423
			(0.0002)	(0.0002)	(0.0004)
45	0.3	6.7462	6.7828	6.7844	6.72409
			(0.0004)	(0.0004)	(0.0008)
	0.4	8.5769	8.6568	8.6612	8.6044
			(0.0030)	(0.0030)	(0.0028)
	0.5	10.3431	10.4185	10.4219	10.3414
			(0.0059)	(0.0062)	(0.0044)
50	0.3	9.3899	9.4242	9.4292	9.3691
			(0.0048)	(0.0045)	(0.0013)
	0.4	11.2034	11.2612	11.2642	11.1978
			(0.0009)	(0.0010)	(0.0007)
	0.5	13.0503	13.1141	13.1182	13.0427
			(0.0044)	(0.0043)	(0.0027)

Table 4.1 : The parameters are $UB = 45$, $LB = 40.5$, $r = 3\%$, ¾ $= 0$, $T = 1$, $T = 1$ ($n^0{=}50$), $T_s = 1/12$ ($n_s = 22$), $a = 5$ and $L = 3$ for the CRR

model. The numbers in parentheses are standard error of the price.In each simulation a total of 50,000 antithetic paths.

S_0	LB	Scenario1 CRR	Scenario1 LSM	Scenario2	European
45	42.5	8.7468	8.6955	8.6975	8.4149
			(0.0005)	(0.0006)	(0.0017)
55		12.7241	12.8498	12.9212	12.6029
			(0.0050)	(0.0039)	(0.0047)
65		19.4002	19.5083	19.5400	18.9262
			(0.0065)	(0.0070)	(0.0068)
45	45	8.1833	8.1394	8.1465	7.9350
			(0.0013)	(0.0014)	(0.0019)
55		12.6691	12.8097	12.8606	12.5923
			(0.0035)	(0.0034)	(0.0041)
65		19.3984	19.5575	19.6455	19.0297
			(0.0071)	(0.0076)	(0.0089)

Table 4.2 : he parameters are UB = 50, r = 2%, q = 4%, ¾ = 50%, a = 5, T = 1 (n^0=50), T_S = 1/12 (n_S = 22), and L = 3 for the CRR model. The numbers in parentheses are standard error of the price.In each simulation a total of 50,000 antithetic paths.

4.3.3 Case 3 : Dividend Rate v.s. Reset Date

In order to examine the option values of the AGMAL between scenario 1 and scenario 2 by the LSM method and the CRR method, the most important factors q and T_S are varied. Assume UB = 50, r = 2%; ¾ = 30%; LB = 45; T = 1; and a = 3. Suppose there are 22 trading days in a month, so n_S = 22 for T_S = 1=12, n_S = 44 for T_S = 2=12, and n_S = 66 for T_S = 3=12 and we set n^0 = 50 for all, and …x L = 3 on the CRR model. The pricing results by Monte Carlo simulation are based on 50,000 paths : 25,000 plus 25,000 antithetic, and the scenario 1 , scenario 2 and European-style by LSM are based on the same sample paths.

Table 4.3 shows that the prices are senstive to T_S and q, respectivelly. First, the option value incearses with T_S but decreases with q; and there is little di¤erence when T_S is at most two months whatever the value of q is: And even if T_S is three months long, the di¤erence is still insigni…cant.

Second, compared with the tree algorithm, as q increases at the same reset date T_s, the variations between the LSM and the CRR on sceanrio 1 raise more. The second result shows that after the company of the underlying stock pays the dividends, the original strike price UB must also be reset to the same proportinal change as the proportional change of the stock price; otherwise, as dividend rate increases, the stock price will drop o¤ much to reach the LB quickly before the reset date T_s, especially relatively long reset date, the probability of early exercise before T_s is large. Besides these results also show that the probability of early exercise for AAMAL before T_s with a relatively long reset period is larger than relatively short one.

T_s	1/12			2/12			3/12		
q	CRR	LSM	LSM2	CRR	LSM	LSM2	CRR	LSM	LSM2
2%	6.778	6.843 (0.001)	6.860 (0.001)	7.069	7.124 (0.001)	7.247 (0.001)	7.216	7.278 (0.001)	7.541 (0.000)
4%	6.322	6.380 (0.002)	6.400 (0.001)	6.607	6.656 (0.001)	6.838 (0.001)	6.750	6.808 (0.000)	7.157 (0.000)
6%	5.923	5.984 (0.000)	6.073 (0.000)	6.203	6.256 (0.002)	6.480 (0.002)	6.343	6.406 (0.000)	6.849 (0.000)

Table 4.3 : CRR and LSM are calculated on Scenario 1, and LSM2 is calculated on Scenario 2. The parameters are S_0 = UB = 50, LB = 45, r = 2%, ¾ = 30%, T = 1 (n^0=50), T_s = 1/12 (n_s = 22), a = 3 and L = 3 for the CRR model. The numbers in parentheses are standard error of the price.In each simulation a total of 50,000 antithetic paths.

4.3.4 Case 4 : Reset Date v.s. Di¤erent Reset Condition

In Taiwan, there are many various Moving-Average Options contract issued. We want to know the ralation of these di¤erent contracts, so vary two important factors di¤erenct reset conditions and the length of reset date. Two Moving-Average Reset Options are added. The …rst is that the AAMALs, denoted as RS9. The second is that it would be reset to 98%, 96%, 94%, 92%, and 90% of the initial strike price if the 3-day average price of the stock price would fall to 98%, 96%, 94%, 92%, and 90%, denoted as RS5. The last is that it would be reset to 95% and 90% of the initial strike price if the

3-day average price of the stock price would fall to 95% and 90%, denoted as RS2.

Assume S_t = 50; UB = 50, r = 2%; q = 0; ¾ = 30%; T = 1; and a = 3. Suppose there are 22 trading days in a month, so n_s = 22 for T_s = 1=12, n_s = 44 for T_s = 2=12, and n_s = 66 for T_s = 3=12 and we set n^0 = 50 for all, and …x L = 3 on the CRR model. The pricing results by Monte Carlo simulation are based on 50,000 paths : 25,000 plus 25,000 antithetic, and the scenario 1 , scenario 2 and European-style by LSM are based on the same sample paths. The pricing results appear in Table 4.4.

We make the following observations. First, the higher reset frequency, the more valuable the option value is, and we can …nd that the RS9 values are the highest in the three di¤erent reset conditions. Second, as reset frequency increases whatever the reset date T_s is, the premiums between scenario 2 and scenario 1 are not drawn out a conclusion. These results show that as the reset frequency become higher, the probability of Moving-Average options being in-the-money is larger, i.e., the AMAL call option would become more valuable.

4.3.5 Case 5 : Moving-Average Number v.s. Volatility

In Taiwan, the Securities often issue di¤erent moving-average number contracts. We want to know how the di¤erent moving-average number would a¤ect the option value. So the important factors a and ¾ are varied. Assume S_t = 45; UB = 45, r = 3%; q = 0; ¾ = 30%; T = 1; and T_s = 1=12. The pricing results by Monte Carlo simulation are based on 50,000 paths : 25,000 plus 25,000 antithetic, and the scenario 1 , scenario 2 and European-style by LSM are based on the same sample paths. The pricing results are on Table 4.5.

We make the following observations. The option value decreases with a, due to the strike price is reset smooth as a moves up. This result shows that as the monitoring interval becomes longer, the AMAL call option would become less valuable. Due to use a total of 100,000 antithetic paths more than 50,000, the estimates on the value are almost undervalued to those on the CRR model.

T_s	Contract	Scenario 1		Scenario 2	European
		CRR	LSM		
1	RS9	7.4514	7.4410	7.4384	7.3802
			(0.0021)	(0.0022)	(0.0037)
	RS5		6.9787	6.9726	6.8802
			(0.0014)	(0.0012)	(0.0019)
	RS2		6.9198	6.9172	6.8307
			(0.0029)	(0.0029)	(0.0026)
3	RS9	8.0087	7.9994	7.9237	7.8282
			(0.0032)	(0.0020)	(0.0031)
	RS5		7.1073	7.0064	6.9040
			(0.0011)	(0.0007)	(0.0012)
	RS2		7.0525	6.9518	6.8512
			(0.0027)	(0.0021)	(0.0032)

Table 4.4: The parameters are S_0 = UB = 50, LB = 45, r = 2%, q = 0, ¾ = 30%, T = 1, a = 3, T = 1 (n^0=50), n_S= 22 for T_S = 1/12 case and n_S = 66 for T_S = 3/12 case , and L = 3 for the CRR model. The numbers in parentheses are standard error of the price.In each simulation a total of 50,000 antithetic paths.

a	¾	Scenario 1		Scenario 2	European
		CRR	LSM		
3	0:3	6.8329	6.9044	6.9071	6.8625
			(0.0011)	(0.0010)	(0.0037)
	0:4	8.6679	8.7378	8.7428	8.6802
			(0.0022)	(0.0012)	(0.0019)
	0:5	10.4341	10.5083	10.5129	10.4462
			(0.0056)	(0.0054)	(0.0026)
6	0:3	6.7014	6.8197	6.8227	6.7212
			(0.0006)	(0.0006)	(0.0031)
	0:4	8.5285	8.6328	8.6356	8.5218
			(0.0035)	(0.0036)	(0.0012)
	0:5	10.2955	10.4100	10.4142	10.3062
			(0.0039)	(0.0028)	(0.0032)

Table 4.5: The parameters are S_0 = UB = 45, LB = 40.5, r = 3%, q = 0, T = 1, a = 3, T = 1 (n^0=50), T_S = 1/12 (n_S= 22), and L = 3 for the CRR

model. The numbers in parentheses are standard error of the price. In each simulation a total of 150,000 antithetic paths.

Chapter 5

What to choose the robustness of LSM?

In the previous sections we we showed that it is possible to value AAMAL using LSM method and that the price estimates are not different from the CRR method. We now examine alternative specifications of the cross-sectional regressions models. What we are looking for is the best way to approximate $g_K(x)$ in Eq. (2.6). To this end it is natural to work with members of different families of polynomials, $\{\phi_k\}_{k=0}^{\infty}$.

5.1 Altering the number of regressors

In LSM method, it is argued that increasing the number of regressors should be able to obtain an accurate approximation, and it is suggested that the number be increased until the option value implied by the LSM algorithm no longer increases. In order to examine the practical use of this suggestion we formulate the cross-sectional regressions of AAMALs as

$$\begin{aligned} y(t_i) &= \beta_0 + \sum_{k=1}^{K} [\beta_k w(S_{t_i}) LS_{k-1} + \alpha_k w(M(t_i)) LM_{k-1}] \\ &+ \sum_{k=1}^{K} \gamma_k w(S(t_i)M(t_i)) LSM_{k-1} + u(t_i) \qquad (5.1) \end{aligned}$$

where $w(S(t_i)) = \exp(-\frac{S(t_i)}{2})$; $w(M(t_i)) = \exp(-\frac{M(t_i)}{2})$; and $w(S(t_i)M(t_i))$

$= \exp(-\frac{S(t_i)M(t_i)}{2})$, $LS_k = \frac{e^{S_{t_i}}}{k!}\frac{d^k(S_{t_i}^k e^{S_{t_i}})}{dS_{t_i}^k}$; $LM_k = \frac{e^{M_{t_i}}}{k!}\frac{d^k(M_{t_i}^k e^{M_{t_i}})}{dM_{t_i}^k}$; $LSM_k = \sum_{i=0}^{k} \frac{e^{S_{t_i}M_{t_i}}}{k!}\frac{\partial^k(S_{t_i}^k e^{S_{t_i}M_{t_i}})}{\partial S_{t_i}^i \partial M_{t_i}^{k-i}}$ and consider increasing K from one to three.

Penal A of Table 5.6 shows the result for four specifications in the scenario 1 contitional on both of $S_t = 45$ and 50 associated with $\sigma = 0.3$ and 0.4 from Talbe 5.4. Changing the number K from one to two, the price estimate increases by significant amounts for all of the four specifications. And the value of scenario 1 is, obviously, significantly less than one of European-style when $K = 1$, which means that losing the correlation between the stock price and the strike price on the least square regreesion model would result in a very great amount of bias. Increasin K to three does not have the same large effect, although all the estimates increase. Thus, depending on how the suggestion in LS is interpreted we should choose $K = 2$ or 3.

5.2 Using alternative polynomial families

Even though the different elements of the family $\{L_k\}_{k=0}^{\infty}$ have the property of being mutually orthogonal with respect to the weighting function $\exp(-\frac{S}{2})$, it is not clear why using them. If there exists more than two stochastic factors, the number of regressors would increases with individual terms and the cross product terms. In this section, we try to work the simplest family of ordinary monomials Although they are not orthogonal, they produce very close approximations. Furthermore, they are much simple compared to the Laguerre polynomial. Thus, we formulate the cross-sectional regressions as

$$\begin{aligned} C(t_i) &= \beta_0 + \sum_{k=1}^{K} \beta_k S^k(t_i) + \sum_{k=1}^{K} \alpha_k M^k(t_i) \\ &\quad + \sum_{k=2}^{K-1} \sum_{i=1}^{k-1} \gamma_k S^i(t_i) M^{k-i}(t_i) + u(t_i) \end{aligned} \tag{5.2}$$

and again we increase K from one to three.

Panel B of Table 5.6 shows the effect of increasing K on the family of monomials. There exsits an interesting result that compared with the family of Laguerre polynomials, the option values with $K = 1$ are insignificantly different from one with $K = 2$: When $K = 1$, we never add the interset term into the regression model, i.e., $S_{t_i}M_{t_i}$ but the values appear more similar to those with $K = 2$ than the result of the family of Laguerre polynomials, but the values are also less than the European-style value. The reason, we think, is that there exists some correlation between the stock price and the strike price and the orthogonal work of both stochastic factors are not done on the family of monomials, so although we regress the stock price and the strike price without interset term, the model could still contribute their correlation e¤ect from individual stock price and strike price. Therefore, comparing the changes in the price when increasing K from two to three, the penal B also shows that monomials may converge faster, as the price estimates with K = 2 are signi…cant di¤erent from the CRR model.Thus, the rule of suggestion of increasing one more K is less important when replacing the Laguerre polynomials by ordinay monomials.

S_0	¾	CRR	K = 1	K = 2	K = 3	European
45	0.3	6.7462	6.0481	6.7844	6.7435	6.7409
			(0.0013)	(0.0020)	(0.0006)	(0.0004)
	0.5	10.3431	8.5768	10.4219	10.5622	10.3414
			(0.0028)	(0.0042)	(0.0029)	(0.0044)
50	0.3	9.3899	9.0532	9.4292	9.4839	9.3691
			(0.0011)	(0.0025)	(0.0007)	(0.0013)
	0.5	13.0503	11.4582	13.1182	13.2399	13.0427
			(0.0034)	(0.0043)	(0.0042)	(0.0047)

Panel A : Di¤erent numbers of weighted Laguerre polynomials

S_0	¾	CRR	K = 1	K = 2	K = 3	European
45	0.3	6.7462	6.7181	6.7629	6.7456	6.7409
			(0.0009)	(0.0012)	(0.0012)	(0.0004)
	0.5	10.3431	10.3378	10.4447	10.3433	10.3414
			(0.0056)	(0.0051)	(0.0054)	(0.0044)
50	0.3	9.3899	9.2560	9.3814	9.3890	9.3691
			(0.0007)	(0.0013)	(0.0011)	(0.0013)
	0.5	13.0503	12.8223	13.2527	13.0490	13.0427
			(0.0044)	(0.0043)	(0.0042)	(0.0047)

Penal B : Di¤erent numbers of Monomials

Table 5.6 : The parameters of scenario 1are UB = 45, LB = 40.5, r = 3%, q = 0, T = 1 (n^0=50), T_s = 1/12 (n_s= 22), and a = 3 , and L = 3 on the CRR model, The numbers in parentheses are standard error of the price. In each simulation a total of 100,000 antithetic paths.

Chapter 6

Conclusion

This thesis presents a simple new technology for approximating the value of the American-style AMALs on the LSM approach. It is not hard to …nd that the LSM algorithm not only help solve the pricing of complex derivatives, especially path-dependnce problem, and also disencumber the main problem of early exercise of American-style derivatives on the simulation all the time. We …nd, oh, My God, even if one does not learn any knowledge of …nancial engenering and other technologies of …nancial computation, he can only price any derivatives which is too complicated and hard to value with the LSM algorithm. This approach is intuitive, accurate, easy to apply and computationally e¢ cient. We illustrate this technique using a number analyses of complicated derivatives AAMALs to let us know the properties of this …nancial commodity popularly issued in Taiwan.

The family of basis functions of the cross-sectional regressions, i.e. the Laguerre polynomials, is compared with the simple family of ordinary Mononials, each of which leads to a trade-o¤ betwenn the time used to calculate a price and the precision of that price. Comparing the method-speci…c trade-o¤ reveals that the preferred basis functions uses $K = 2$ or 3 simple ordinary polynomials instead of $K = 3$ Laguerre polynomials.

At last, we make the comment that with the ability to value American options, the applicability of simulation techniques becomes much broader and more promising, particularly in multiple factors, such as more than three factors, the LSM method is much easier to extend than and superior to the tree models.

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