Cash Dividends

- Exchange-traded stock options are not cash dividend-protected (or simply protected).
  - The option contract is not adjusted for *cash* dividends.
- The stock price falls by an amount roughly equal to the amount of the cash dividend as it goes ex-dividend.
- Cash dividends are detrimental for calls.
- The opposite is true for puts.

Stock Splits and Stock Dividends

- Options are adjusted for stock splits.
- After an n-for-m stock split, m shares become n shares.
- Accordingly, the strike price is only m/n times its previous value, and the number of shares covered by one option becomes n/m times its previous value.
- Exchange-traded stock options are adjusted for stock dividends.
- We assume options are unprotected.

Example

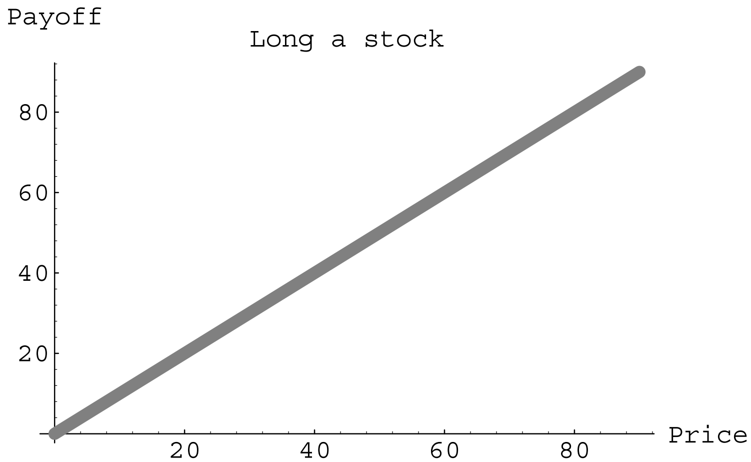
- Consider an option to buy 100 shares of a company for \$50 per share.
- A 2-for-1 split changes the term to a strike price of \$25 per share for 200 shares.

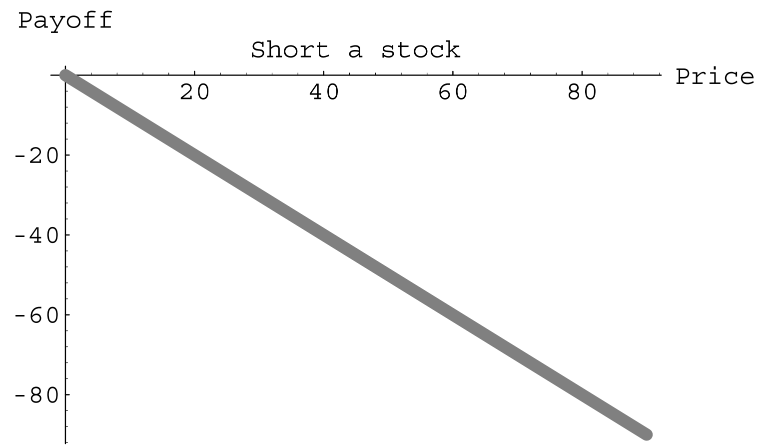
Short Selling

- Short selling[a] involves selling an asset that is *not* owned with the intention of buying it back later.
  - If you short 1,000 XYZ shares, the broker borrows them from another client to sell them in the market.
  - This action generates proceeds for the investor.
  - The investor can close out the short position by buying 1,000 XYZ shares.
- Clearly, the investor profits if the stock price falls.

[a]Or shorting. It was invented by Le Maire (1558–1624) in 1608.

Payoff of Stock





Short Selling (concluded)

- Not all assets can be shorted.
- In reality, short selling is not simply the opposite of going long.[a]

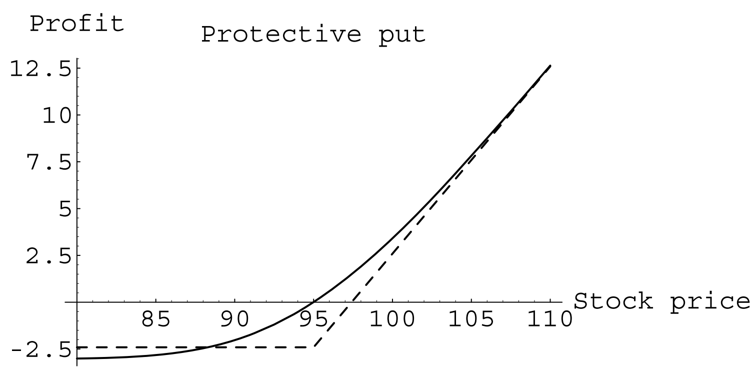
[a] Kosowski & Neftci (2015). See https://tw.news.appledaily.com/headline/daily/20180307/37950481/ for an example in Taiwan on February 6, 2018.

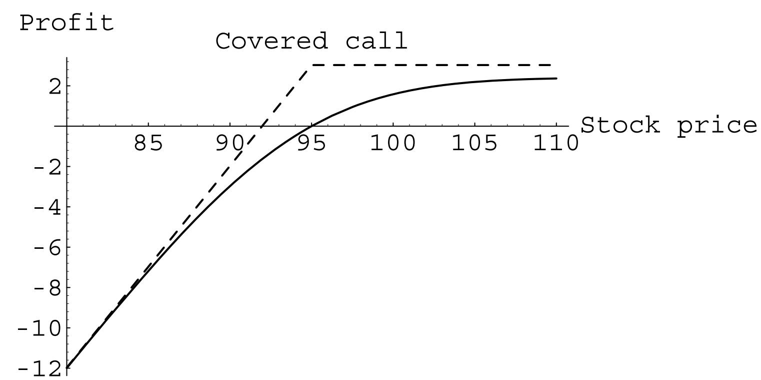
Covered Position: Hedge

- A hedge combines an option with its underlying stock in such a way that one protects the other against loss.
- Covered call: A long position in stock with a short call.[a]
  - It is "covered" because the stock can be delivered to the buyer of the call if the call is exercised.
- Protective put: A long position in stock with a long put.
- Both strategies break even only if the stock price rises above a certain level, so they are bullish.

---

[a] A short position has a payoff opposite in sign to that of a long position. Some ETFs offer this payoff, such as the Global X Nasdaq 100 Covered Call ETF (QYLD).





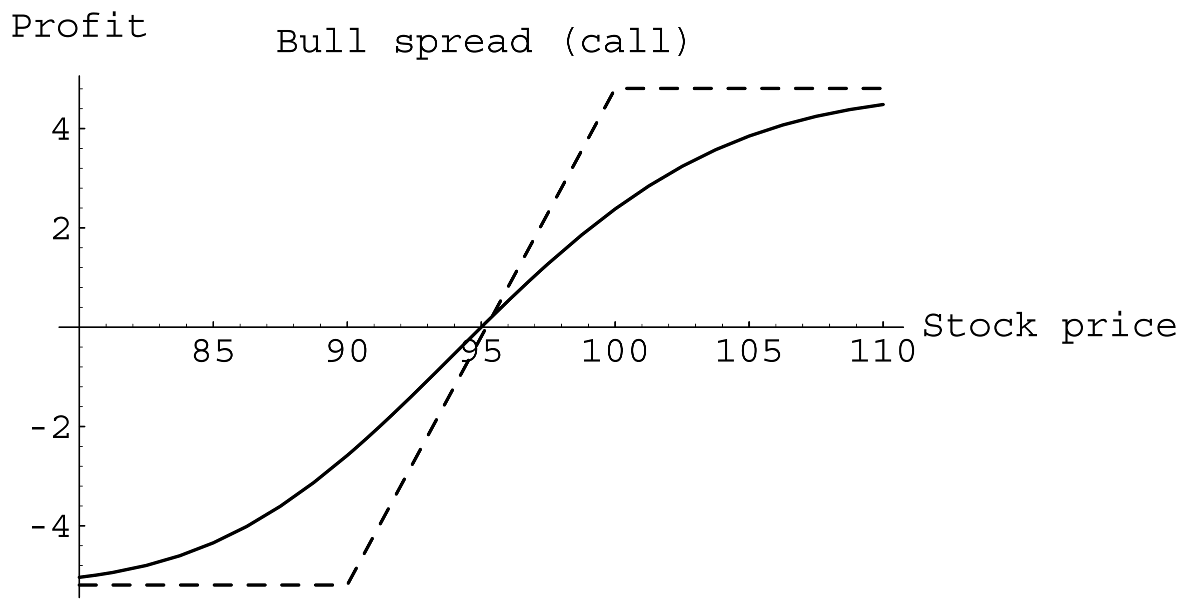
Solid lines are profits of the portfolio one month before maturity, assuming the portfolio is set up when $S = 95$ then.

Covered Position: Spread

- A spread consists of options of the same type and on the same underlying asset but with different strike prices or expiration dates.
- We use X_L, X_M, and X_H to denote the strike prices with
$$X_L < X_M < X_H.$$

Covered Position: Spread (continued)

- A bull call spread consists of a long X_L call and a short X_H call with the same expiration date.
  - The initial investment is $C_L - C_H$.
  - The payoff is nonnegative.
  - The maximum payoff is $X_H - X_L$.
    - When both are exercised at expiration.
  - The maximum profit is $(X_H - X_L) - (C_L - C_H)$.
  - The maximum loss is $C_L - C_H$.
    - When neither is exercised at expiration.



Profit
Bull spread (call)
4
2
-2
-4
85
90
95
100
105
110
Stock price

Covered Position: Spread (continued)

- If we buy $(X_H - X_L)^{-1}$ units of the bull call spread and $X_H - X_L \to 0$, a (Heaviside) step function emerges as the payoff.
- This payoff defines the binary (or digital) call.
- The binary call thus costs
$$-\frac{\partial C}{\partial X}$$
today.
  - Recall that C is the (standard) call's price.
- This formula is model independent!

Covered Position: Spread (continued)

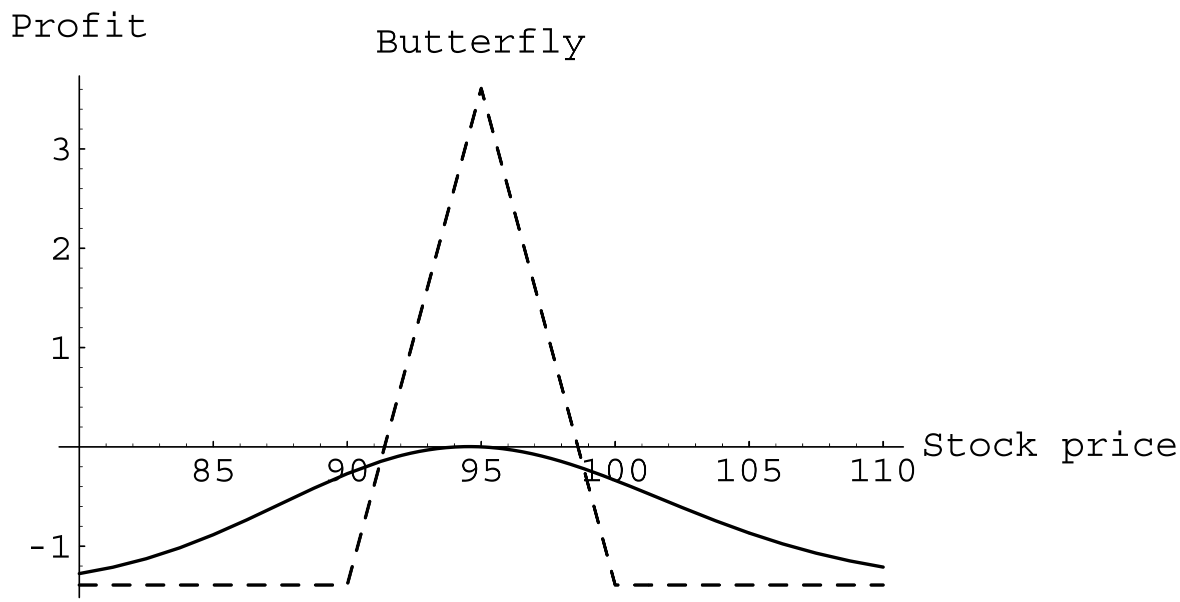
- Writing an X_H put and buying an X_L put with identical expiration date creates the bull put spread.[a]
- A bear spread amounts to selling a bull spread.
- It profits from declining stock prices.

---

[a] See `https://www.businesstoday.com.tw/article/category/80392/post/201803070` for a sad example in Taiwan on February 6, 2018.

Covered Position: Spread (continued)

- Three calls or three puts with different strike prices and the same expiration date create a butterfly spread.
  - The spread is long one X_L call, long one X_H call, and short *two* X_M calls.
- Same as long a bull call spread with strike prices X_L and X_M and short a bull call spread with strike prices X_M and X_H.
- A butterfly spread has a positive payoff at expiration only if the asset price falls between X_L and X_H.


Profit
Butterfly
3
2
1
-1
85
90
95
100
105
110
Stock price

Covered Position: Spread (continued)

- Assume $X_M = (X_H + X_L)/2$.
- Take a position in $(X_M - X_L)^{-1}$ units of the butterfly spread.
- When $X_H - X_L \to 0$, it approximates a state contingent claim,[a] which pays $1 only in the state $S = X_M$.[b]

---

[a] Alternatively, Arrow security in honor of Kenneth Arrow (1921–2017), co-winner of the 1972 Nobel Prize in Economic Sciences.

[b] See Exercise 7.4.5 of the textbook.

Covered Position: Spread (concluded)

- The price of a state contingent claim is called a state price.
- The state price equals[a]

$$\frac{\partial^2 C}{\partial X^2}.$$

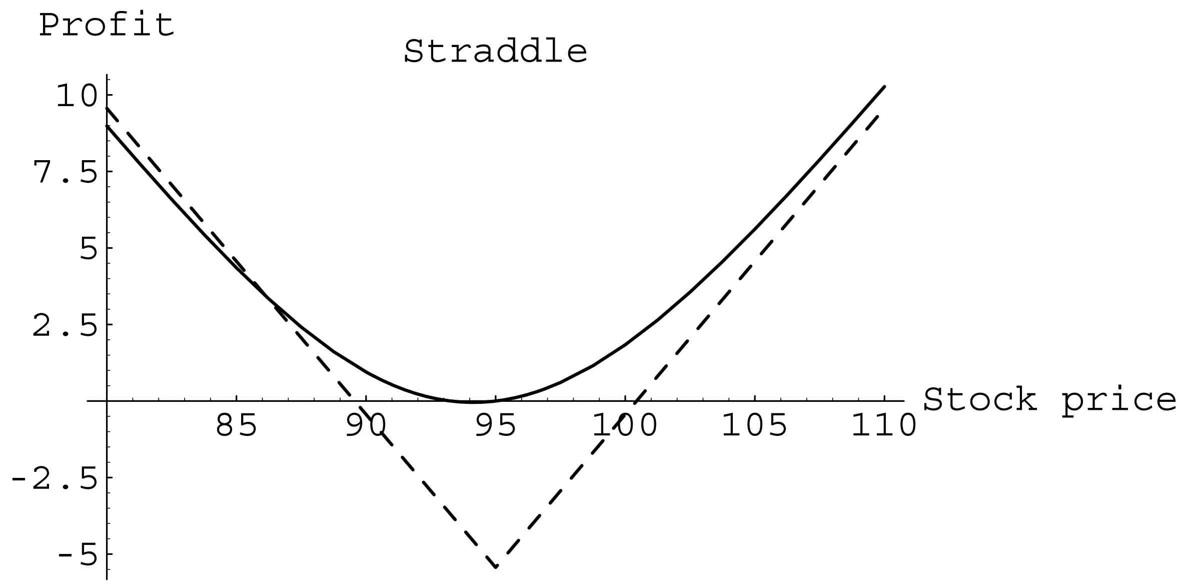
- In fact, the FV of $\partial^2 C/\partial X^2$ is the probability density of the stock price $S_T = X$ at option's maturity.[b]
- You can buy a butterfly spread if you believe the probability of $S_T \approx X$ is higher than this probability.

---

[a]One can also use the put (see Exercise 9.3.6 of the textbook).
[b]Breeden & Litzenberger (1978). This formula is model independent!

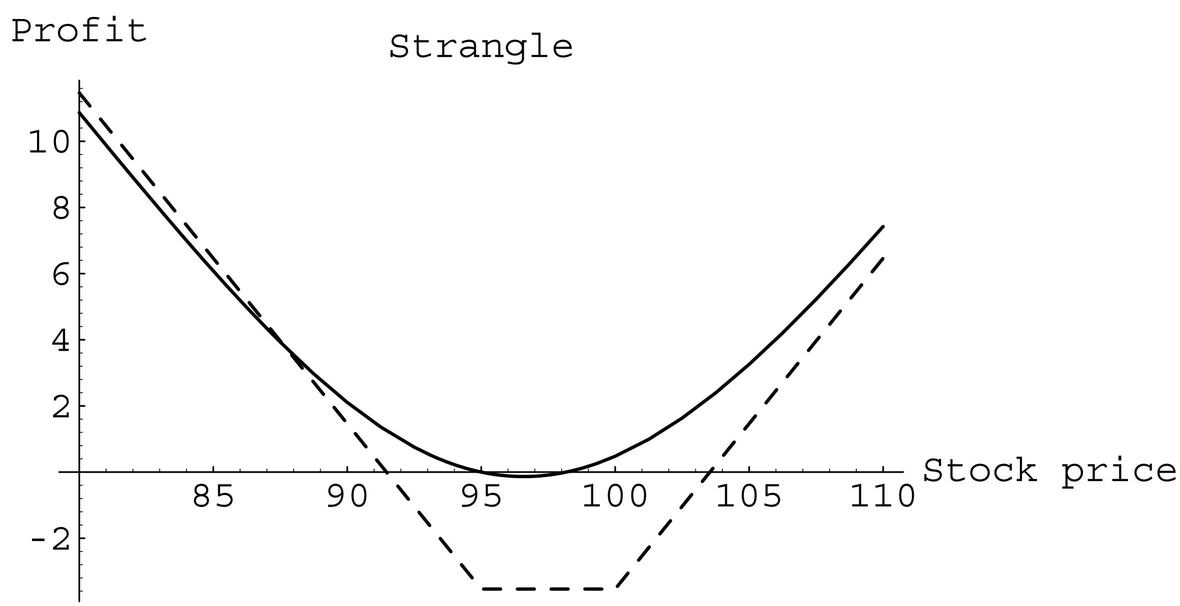
Covered Position: Combination

- A combination consists of options of different types on the same underlying asset.
  - These options must be either all bought or all written.
- Straddle: A long call and a long put with the same strike price and expiration date.
  - Since it profits from high volatility, a person who buys a straddle is "long volatility."
  - Selling a straddle benefits from low volatility.


Profit
Straddle
10
7.5
5
2.5
-2.5
-5
85
90
95
100
105
110
Stock price

Covered Position: Combination (concluded)

- Strangle: Identical to a straddle except that the call's strike price is higher than the put's.


Profit
Strangle
10
8
6
4
2
-2
85
90
95
100
105
110
Stock price

Arbitrage in Option Pricing

All general laws are
attended with inconveniences,
when applied to particular cases.
— David Hume (1711–1776)

The problem with QE is
it works in practice,
but it doesn't work in theory.
— Ben Bernanke[a] (2014)

[a]Co-winner of the 2022 Nobel Prize in Economic Sciences.

Arbitrage

- The no-arbitrage principle says there is no free lunch.
- It supplies the argument for option pricing.
- A riskless arbitrage opportunity is one that, without any initial investment, generates nonnegative returns under all circumstances and positive returns under some.
- In an efficient market, such opportunities do not exist (for long).[a]

---

[a] Heakal (2013), "In the real world of investments, however, there are obvious arguments against the EMH [efficient market hypothesis]. There are investors who have beaten the market—Warren Buffett."

Portfolio Dominance Principle

- Consider two portfolios A and B.
- Suppose A's payoff is at least as good as B's under *all* circumstances and better under *some*.
- Then A should be more valuable than B.

Two Simple Corollaries

- A portfolio yielding a zero return in every possible scenario must have a zero PV.[a]
  - Short the portfolio if its PV is positive.
  - Buy it if its PV is negative.
  - In both cases, a free lunch is created.
- Two portfolios that yield the same return at time T must have the same price before time T.[b]

---

[a] Lyft, Inc. (2019), "We have incurred net losses each year since our inception and we may not be able to achieve or maintain profitability in the future."

[b] Aristotle, "those who are equal should have everything alike."

The PV Formula (p. 41) Justified

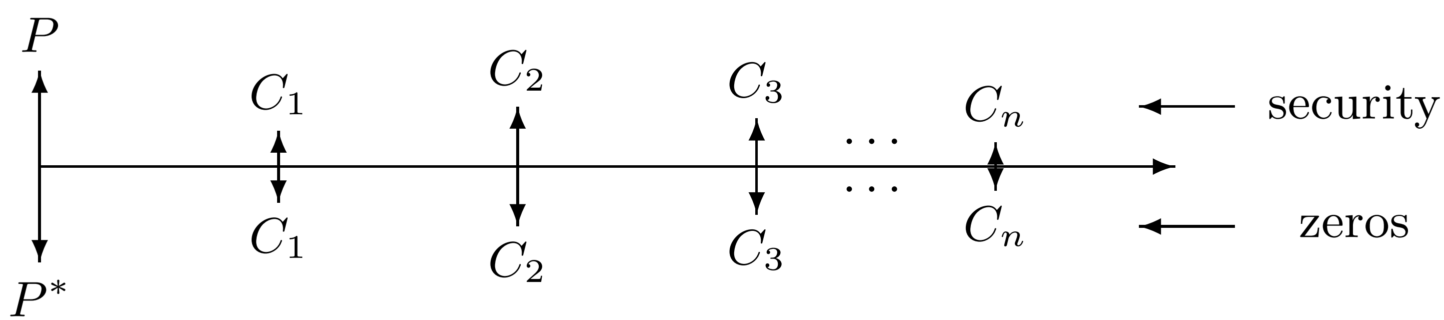
Theorem 1 *For a certain cash flow* $C_1, C_2, \ldots, C_n$,

$$P = \sum_{i=1}^{n} C_i d(i).$$

- Suppose the price $P^* < P$.
- Short[a] the n zeros that match the security's n cash flows.
- The proceeds are P dollars.

---

[a] A key assumption.


P
C_1
C_2
C_3
$\cdots$
C_n
security
$\cdots$
C_1
C_2
C_3
C_n
zeros
P^*

The Proof (concluded)

- Then use P^* of the proceeds to buy the security.
- The cash inflows of the security will offset exactly the obligations of the zeros.
- A riskless profit of $P - P^*$ dollars has been realized now.
- If $P^* > P$, just reverse the trades.

One More Example

Theorem 2 *A put or a call must have a nonnegative value.*

- Suppose otherwise and the option has a negative price.
- Buy the option for a positive cash flow now.
- It will end up with a nonnegative amount at expiration.
- So an arbitrage profit is realized now.

Relative Option Prices

- These relations hold regardless of the model for stock prices.
- Assume, among other things, that there are no transactions costs[a] or margin requirements, borrowing and lending are available at the riskless interest rate, interest rates are *nonnegative*, and there are no arbitrage opportunities.

[a]Schwab cut the fees of online trades of stocks and ETFs to zero on October 7, 2019.

Relative Option Prices (concluded)

- Let the current time be time zero.
- $\text{PV}(x)$ stands for the PV of x dollars at expiration.
- Hence

$$\text{PV}(x) = xd(\tau),$$

where τ is the time to expiration.

Put-Call Parity[a]

$$C = P + S - \mathrm{PV}(X). \tag{31}$$

- Consider the portfolio of:
  - One short European call;
  - One long European put;
  - One share of stock;
  - A loan of $\mathrm{PV}(X)$.
- All options are assumed to carry the same strike price X and time to expiration, τ.
- The initial cash flow is therefore

$$C - P - S + \mathrm{PV}(X).$$

[a]Castelli (1877).

The Proof (continued)

- At expiration, if the stock price $S_\tau \leq X$, the put will be worth $X - S_\tau$ and the call will expire worthless.
- The loan is now X.
- The net future cash flow is zero:

$$0 + (X - S_\tau) + S_\tau - X = 0.$$

- On the other hand, if $S_\tau > X$, the call will be worth $S_\tau - X$ and the put will expire worthless.
- The net future cash flow is again zero:

$$-(S_\tau - X) + 0 + S_\tau - X = 0.$$

The Proof (concluded)

- The net future cash flow is zero in either case.
- The no-arbitrage principle[a] implies that the initial investment to set up the portfolio must be nil as well.

---

[a] Recall p. 223.

Consequences of Put-Call Parity

- There is only one kind of European option.
  - The other can be replicated from it in combination with stock and riskless lending or borrowing.
  - Combinations such as this create synthetic securities.[a]
- $S = C - P + \text{PV}(X)$: A stock is equivalent to a portfolio containing a long call, a short put, and lending $\text{PV}(X)$.
- $C - P = S - \text{PV}(X)$: A long call and a short put amount to a long position in stock and borrowing the PV of the strike price (buying stock on margin).

---

[a]Like the synthetic bonds on p. 150.

Intrinsic Value

Lemma 3 *An American call or a European call on a non-dividend-paying stock is never worth less than its intrinsic value.*

- An American call cannot be worth less than its intrinsic value.[a]
- For European options, the put-call parity implies

$$C = (S - X) + (X - \mathrm{PV}(X)) + P \geq S - X.$$

- Recall $C \geq 0$ (p. 227).
- It follows that $C \geq \max(S - X, 0)$, the intrinsic value.

[a]See Lemma 8.3.1 of the textbook.

Intrinsic Value (concluded)

A European *put* on a non-dividend-paying stock may be worth less than its intrinsic value $X - S$.

Lemma 4 *For European puts,* $P \geq \max(\text{PV}(X) - S, 0)$.

- Prove it with the put-call parity.[a]
- Can explain the right figure on p. 197 why $P < X - S$ when S is small.

[a]See Lemma 8.3.2 of the textbook.

Early Exercise of American Calls

European calls and American calls are identical when the underlying stock pays no dividends!

Theorem 5 (Merton, 1973) *An American call on a non-dividend-paying stock should not be exercised before expiration.*

- By Exercise 8.3.2 of the text, $C \geq \max(S - \mathrm{PV}(X), 0)$.
- If the call is exercised, the value is $S - X$.
- But

$$\max(S - \mathrm{PV}(X), 0) \geq S - X.$$

Remarks

- The above theorem does *not* mean American calls should be kept until maturity.
- What it does imply is that when early exercise is being considered, a *better* alternative is to sell it.
- Early exercise may become optimal for American calls on a dividend-paying stock, however.
  - Options are assumed to be unprotected.
  - Stock price declines as the stock goes ex-dividend.

Early Exercise of American Calls: Dividend Case

Surprisingly, an American call should be exercised only at a few dates.[a]

Theorem 6 (Merton, 1973) *An American call will only be exercised at expiration or just before an ex-dividend date.*

In contrast, it might be optimal to exercise an American put even if the underlying stock does not pay dividends.

[a]See Theorem 8.4.2 of the textbook.

A General Result[a]

Theorem 7 (Cox & Rubinstein, 1985) *Any piecewise linear payoff function can be replicated using a portfolio of calls and puts.*

Corollary 8 *Any sufficiently well-behaved payoff function can be approximated by a portfolio of calls and puts.*

Theorem 9 (Bakshi & Madan, 2000) *Any payoff function with bounded expectation can be replicated by a continuum of out-of-the-money European calls and puts.*

---

[a]See Exercise 8.3.6 of the textbook.

Option Pricing Models

Black insisted that anything one could do
with a mouse could be done better
with macro redefinitions
of particular keys on the keyboard.
— Emanuel Derman (2004),
My Life as a Quant

So we would bring in smart folks.
They didn't know anything about finance.[a]
James Simons[b] (2015, May 13, 33:27)

---

[a] `https://www.youtube.com/watch?v=QNznD9hMEh0`

[b] James Harris Simons (1938–2024) was the founder of Renaissance Technologies. Its Medallion Fund had a 66.1% average gross annual return rate in 1988–2018!

The Setting

- The no-arbitrage principle is insufficient to pin down the exact option value.
- Need a model of probabilistic behavior of stock prices.
- An obstacle is that it seems a risk-adjusted interest rate is needed to discount the option's expected payoff.[a]
- Breakthrough came in 1973 when Black (1938–1995) and Scholes with help from Merton published their celebrated option pricing model.[b]
  - Known as the Black-Scholes option pricing model.

---

[a]Like Eq. (30) on p. 184.

[b]The results were obtained as early as June 1969. Merton and Scholes were winners of the 1997 Nobel Prize in Economic Sciences.

Fischer Black (1938–1995)

Myron Scholes (1941–)

Robert C. Merton (1944–)

Terms and Approach

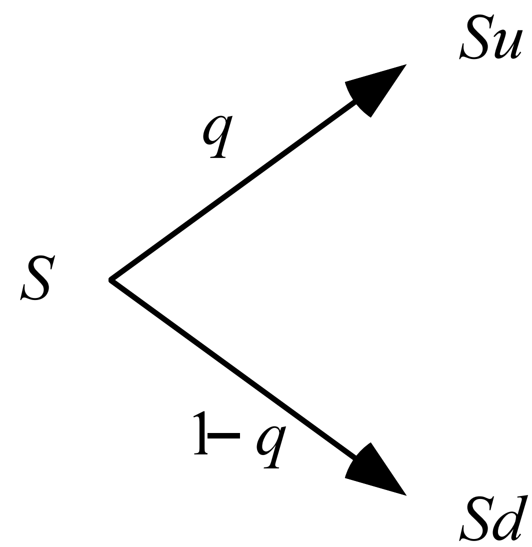
- C: call value.
- P: put value.
- X: strike price
- S: stock price
- $\hat{r} > 0$: the continuously compounded riskless rate per period.
- $R \stackrel{\Delta}{=} e^{\hat{r}}$: gross return.
- Start from the discrete-time binomial model.

Binomial Option Pricing Model (BOPM)

- Time is discrete and measured in periods.
- If the current stock price is S, it can go to Su with probability q and Sd with probability $1 - q$, where $0 < q < 1$ and $d < u$.
  - In fact, $d \leq R \leq u$ must hold to rule out arbitrage.[a]
- Six pieces of information will suffice to determine the option value based on arbitrage considerations:

  S, u, d, X, $\hat{r}$, and the number of periods to expiration.

[a]See Exercise 9.2.1 of the textbook. The sufficient condition is $d < R < u$ (Björk, 2009), which we shall assume.

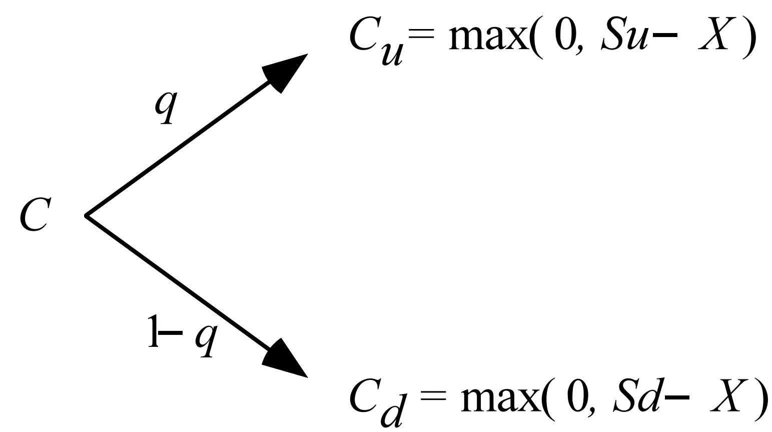


Su
q
S
1 − q
Sd

Call on a Non-Dividend-Paying Stock: Single Period

- The expiration date is only one period from now.
- C_u is the call price at time 1 if the stock price moves to Su.
- C_d is the call price at time 1 if the stock price moves to Sd.
- Clearly,

$$\begin{aligned} C_u &= \max(0, Su - X), \\ C_d &= \max(0, Sd - X). \end{aligned}$$


$C_u = \max(0, Su - X)$
q
C
$1 - q$
$C_d = \max(0, Sd - X)$

Call on a Non-Dividend-Paying Stock: Single Period (continued)

- Set up a portfolio of h shares of stock and B dollars in riskless bonds.
  - This costs $hS + B$.
  - We call h the hedge ratio or delta.
- The value of this portfolio at time one is

$$\begin{aligned} & hSu + RB, && \text{up move,} \\ & hSd + RB, && \text{down move.} \end{aligned}$$

Call on a Non-Dividend-Paying Stock: Single Period (continued)

- Choose h and B such that the portfolio *replicates* the payoff of the call,

$$\begin{aligned} hSu + RB &= C_u, \\ hSd + RB &= C_d. \end{aligned}$$

Call on a Non-Dividend-Paying Stock: Single Period (concluded)

- Solve the above equations to obtain

$$h = \frac{C_u - C_d}{Su - Sd} \geq 0, \tag{32}$$

$$B = \frac{uC_d - dC_u}{(u-d)\,R}. \tag{33}$$

- By the no-arbitrage principle, the European call should cost the same as the equivalent portfolio,[a]

$$C = hS + B.$$

- As $uC_d - dC_u < 0$, the equivalent portfolio is a *levered* long position in stocks.

[a] Or the replicating portfolio, as it replicates the option.

American Call Pricing in One Period

- Have to consider immediate exercise.
- $C = \max(hS + B, S - X)$.
  - When $hS + B \geq S - X$, the call should not be exercised immediately.
  - When $hS + B < S - X$, the option should be exercised immediately.
- For non-dividend-paying stocks, early exercise is not optimal by Theorem 5 (p. 236).
- So

$$C = hS + B.$$

Put Pricing in One Period

- Puts can be similarly priced.
- The delta for the put is $(P_u - P_d)/(Su - Sd) \leq 0$, where

$$\begin{aligned} P_u &= \max(0, X - Su), \\ P_d &= \max(0, X - Sd). \end{aligned}$$

- Let $B = \frac{uP_d - dP_u}{(u-d)\,R}$.
- The European put is worth $hS + B$.
- The American put is worth $\max(hS + B, X - S)$.
  - Early exercise can be optimal with American puts.

Risk

- Surprisingly, the option value is independent of q.[a]
- Hence it is independent of the expected value of the stock,
$$qSu + (1 - q)\, Sd.$$
- The option value depends on the sizes of price changes, u and d, which the investors must agree upon.
- Then the set of possible stock prices is the same whatever q is.

[a] More precisely, not directly dependent on q. Thanks to a lively class discussion on March 16, 2011.

Pseudo Probability

- After substitution and rearrangement,

$$hS + B = \frac{\left(\frac{R-d}{u-d}\right) C_u + \left(\frac{u-R}{u-d}\right) C_d}{R}.$$

- Rewrite it as

$$hS + B = \frac{pC_u + (1-p)\, C_d}{R},$$

where

$$p \triangleq \frac{R-d}{u-d}. \tag{34}$$

Pseudo Probability (concluded)

- As $0 < p < 1$, it may be interpreted as probability.
- Alternatively,

$$\left(\frac{R-d}{u-d}\right) C_u + \left(\frac{u-R}{u-d}\right) C_d$$

  interpolates the value at SR through points (Su, C_u) and (Sd, C_d).

Risk-Neutral Probability

- The expected rate of return for the stock is equal to the riskless rate $\hat{r}$ under p as

$$pSu + (1 - p)\, Sd = RS. \tag{35}$$

- The expected rates of return of all securities must be the riskless rate when investors are risk-neutral.
- For this reason, p is called the risk-neutral probability.
- The value of an option is the expectation of its discounted future payoff in a risk-neutral economy.
- So the rate used for discounting the FV is the riskless rate[a] *in a risk-neutral economy.*

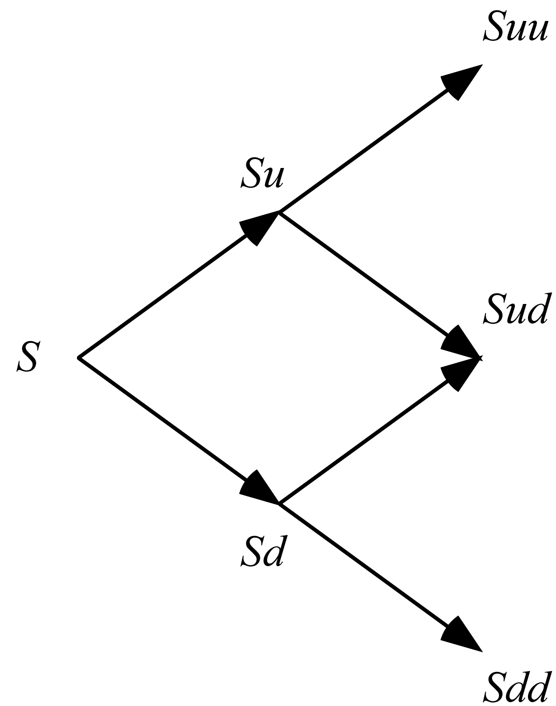
---

[a]Recall the question on p. 242.

Option on a Non-Dividend-Paying Stock: Multi-Period

- Consider a call with two periods remaining before expiration.
- Under the binomial model, the stock can take on 3 possible prices at time two: Suu, Sud, and Sdd.
  - There are 4 paths.
  - But the tree *combines* or *recombines*; hence there are only 3 terminal prices.
- At any node, the next two stock prices only depend on the current price, not the prices of earlier times.[a]

[a]It is Markovian.


Suu
Su
Sud
S
Sd
Sdd

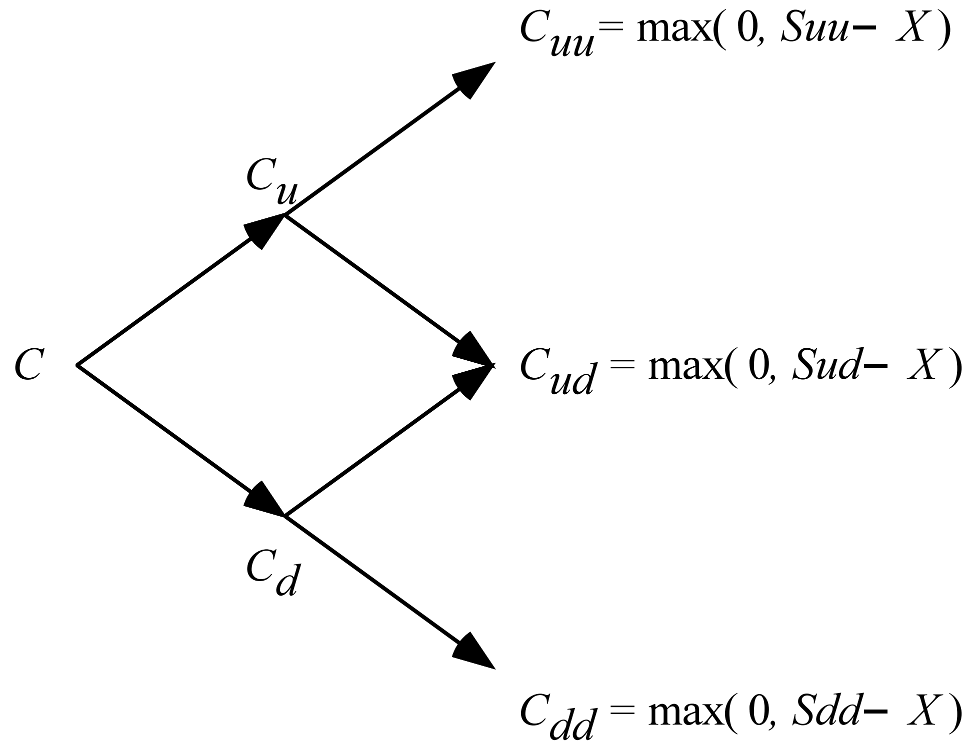
Option on a Non-Dividend-Paying Stock: Multi-Period (continued)

- Let C_{uu} be the call's value at time two if the stock price is Suu.
- Thus,

$$C_{uu} = \max(0, Suu - X).$$

- C_{ud} and C_{dd} can be calculated analogously,

$$\begin{aligned} C_{ud} &= \max(0, Sud - X), \\ C_{dd} &= \max(0, Sdd - X). \end{aligned}$$


$C_{uu} = \max(0, Suu - X)$
C_u
C
$C_{ud} = \max(0, Sud - X)$
C_d
$C_{dd} = \max(0, Sdd - X)$

Option on a Non-Dividend-Paying Stock: Multi-Period (continued)

- The call values at time 1 can be obtained by applying the same logic:

$$C_u = \frac{pC_{uu} + (1-p)\, C_{ud}}{R}, \tag{36}$$

$$C_d = \frac{pC_{ud} + (1-p)\, C_{dd}}{R}.$$

- Deltas can be derived from Eq. (32) on p. 253.
- For example, the delta at C_u is

$$\frac{C_{uu} - C_{ud}}{Suu - Sud}.$$

Option on a Non-Dividend-Paying Stock: Multi-Period (concluded)

- We now reach the current period.
- Compute
$$\frac{pC_u + (1-p)\,C_d}{R}$$
as the option price.
- Again, the values of delta h and B can bederived from Eqs. (32)–(33) on p. 253.

Early Exercise

- Since the call will not be exercised at time 1 even if it is American, $C_u \geq Su - X$ and $C_d \geq Sd - X$.
- Therefore,

$$\begin{aligned} hS + B &= \frac{pC_u + (1-p)\, C_d}{R} \geq \frac{[\, pu + (1-p)\, d\,]\, S - X}{R} \\ &= S - \frac{X}{R} > S - X. \end{aligned}$$

  - The call again will not be exercised at present.[a]
- So

$$C = hS + B = \frac{pC_u + (1-p)\, C_d}{R}.$$

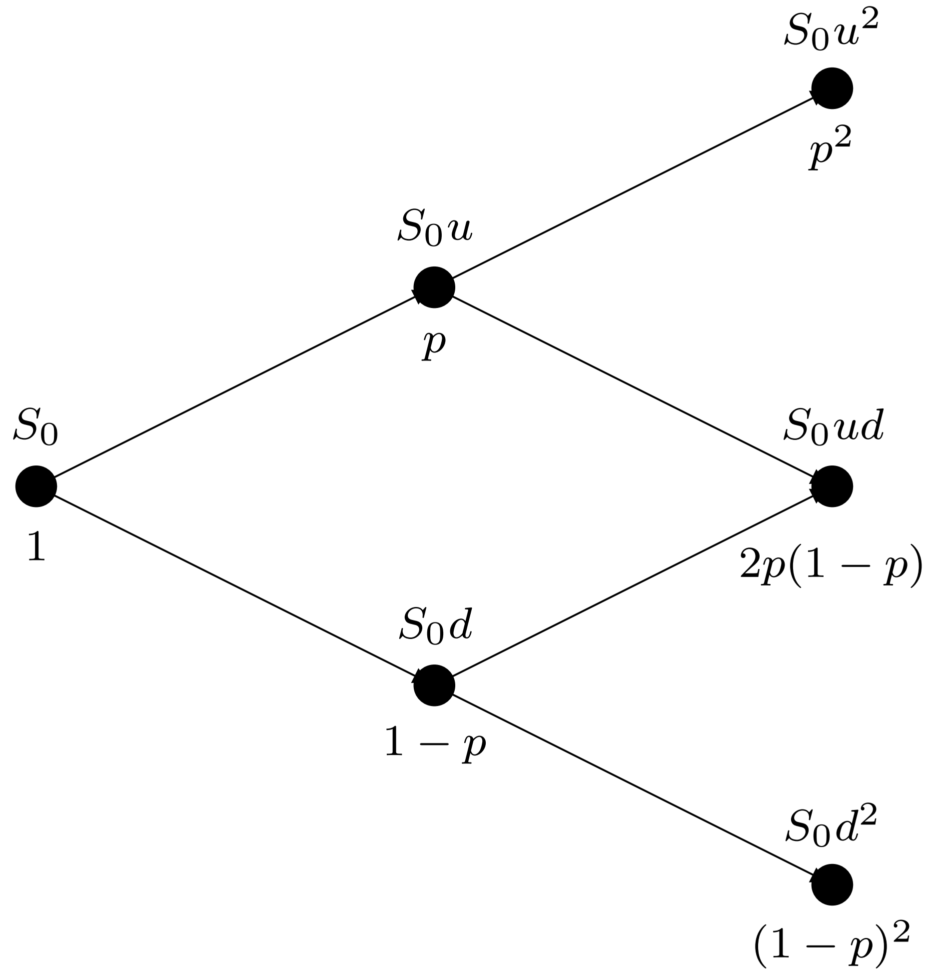
[a]Consistent with Theorem 5 (p. 236).

Backward Induction[a]

- The above expression calculates C from the two successor nodes C_u and C_d and none beyond.
- The same computation happened at C_u and C_d, too, as demonstrated in Eq. (36) on p. 264.
- This recursive procedure is called backward induction.
- C equals

$$
\begin{aligned}
& [\, p^2 C_{uu} + 2p(1-p)\, C_{ud} + (1-p)^2 C_{dd}](1/R^2) \\
= \;& [\, p^2 \max\big(0, Su^2 - X\big) + 2p(1-p) \max\,(0, Sud - X) \\
& + (1-p)^2 \max\big(0, Sd^2 - X\big)\,]/R^2.
\end{aligned}
$$

[a]Ernst Zermelo (1871–1953).


S_0u^2
p^2
S_0u
p
S_0
1
S_0ud
$2p(1-p)$
S_0d
$1-p$
S_0d^2
$(1-p)^2$

Backward Induction (continued)

- In the n-period case,

$$C = \frac{\sum_{j=0}^{n} \binom{n}{j} p^j (1-p)^{n-j} \times \max\left(0, Su^j d^{n-j} - X\right)}{R^n}.$$

  – The value of a call on a non-dividend-paying stock is the expected discounted payoff at expiration in a risk-neutral economy.

- Similarly,

$$P = \frac{\sum_{j=0}^{n} \binom{n}{j} p^j (1-p)^{n-j} \times \max\left(0, X - Su^j d^{n-j}\right)}{R^n}.$$

Backward Induction (concluded)

- Note that
$$p_j \triangleq \frac{\binom{n}{j} p^j (1-p)^{n-j}}{R^n}$$
is the state price[a] for the state $Su^j d^{n-j}$, $j = 0, 1, \ldots, n$.

- In general,
$$\text{option price} = \sum_j (p_j \times \text{payoff at state } j).$$

---

[a]Recall p. 214. One can obtain the *undiscounted* state price $\binom{n}{j} p^j (1-p)^{n-j}$—the risk-neutral probability—for the state $Su^j d^{n-j}$ with $(X_M - X_L)^{-1}$ units of the butterfly spread where $X_L = Su^{j-1} d^{n-j+1}$, $X_M = Su^j d^{n-j}$, and $X_H = Su^{j-1+1} d^{n-j-1}$ (Bahra, 1997).

Risk-Neutral Pricing Methodology

- Every derivative can be priced as if the economy were risk-neutral.
- For a European-style derivative with the terminal payoff function $\mathcal{D}$, its value is

$$e^{-\hat{r}n} E^{\pi}[\,\mathcal{D}\,]. \tag{37}$$

  - E^{π} means the expectation is taken under the risk-neutral probability.
- The "equivalence" between arbitrage freedom in a model and the existence of a risk-neutral probability is called the (first) fundamental theorem of asset pricing.[a]

[a]Dybvig & Ross (1987).

Philip H. Dybvig[a] (1955–)

[a]Co-winner of the 2022 Nobel Prize in Economic Sciences.

Stephen Ross (1944–2017)