Introduction to Mortgage-Backed Securities

Anyone stupid enough to promise to be
responsible for a stranger's debts
deserves to have his own property
held to guarantee payment.
— Proverbs 27:13

I'm not putting my money in real estate.
I prefer bonds.
— Margaret Mitchell (1900–1949),
Gone with the Wind (1936)

Mortgages

- A mortgage is a loan secured by the collateral of real estate property.
- Suppose the borrower (the mortgagor) defaults, that is, fails to make the contractual payments.
- The lender (the mortgagee) can foreclose the loan by seizing the property.

Mortgage-Backed Securities

- A mortgage-backed security (MBS) is a bond backed by an undivided interest in a pool of mortgages.[a]
- MBSs traditionally enjoy high returns, wide ranges of products, high credit quality, and liquidity.
- The mortgage market has witnessed tremendous innovations in product design.

[a]They can be traced to 1880s (Levy, 2012).

Mortgage-Backed Securities (concluded)

- The complexity of the products and the prepayment option require advanced models and software techniques.
  - In fact, the mortgage market probably could not have operated efficiently without them.[a]
- They also consume lots of computing power.
- Our focus will be on residential mortgages.
- But the underlying principles are applicable to other types of assets.

[a]Merton (1994).

Types of MBSs

- An MBS is issued with pools of mortgage loans as the collateral.
- The cash flows of the mortgages making up the pool naturally reflect upon those of the MBS.
- There are three basic types of MBSs:
  1. Mortgage pass-through security (MPTS).
  2. Collateralized mortgage obligation (CMO).
  3. Stripped mortgage-backed security (SMBS).

Problems Investing in Mortgages

- The MBS sector is one of the largest in the debt market.[a]
- Individual mortgages are unattractive for many investors.
- Often at hundreds of thousands of U.S. dollars or more, they demand too much investment.
- Most investors lack the resources and knowledge to assess the credit risk involved.

[a]See p. 3 of the textbook. The outstanding balance was US$9.3 trillion as of 2017 vs. the US Treasury's US$14.5 trillion and corporate debt's US$9.0 trillion (SIFMA, 2018).

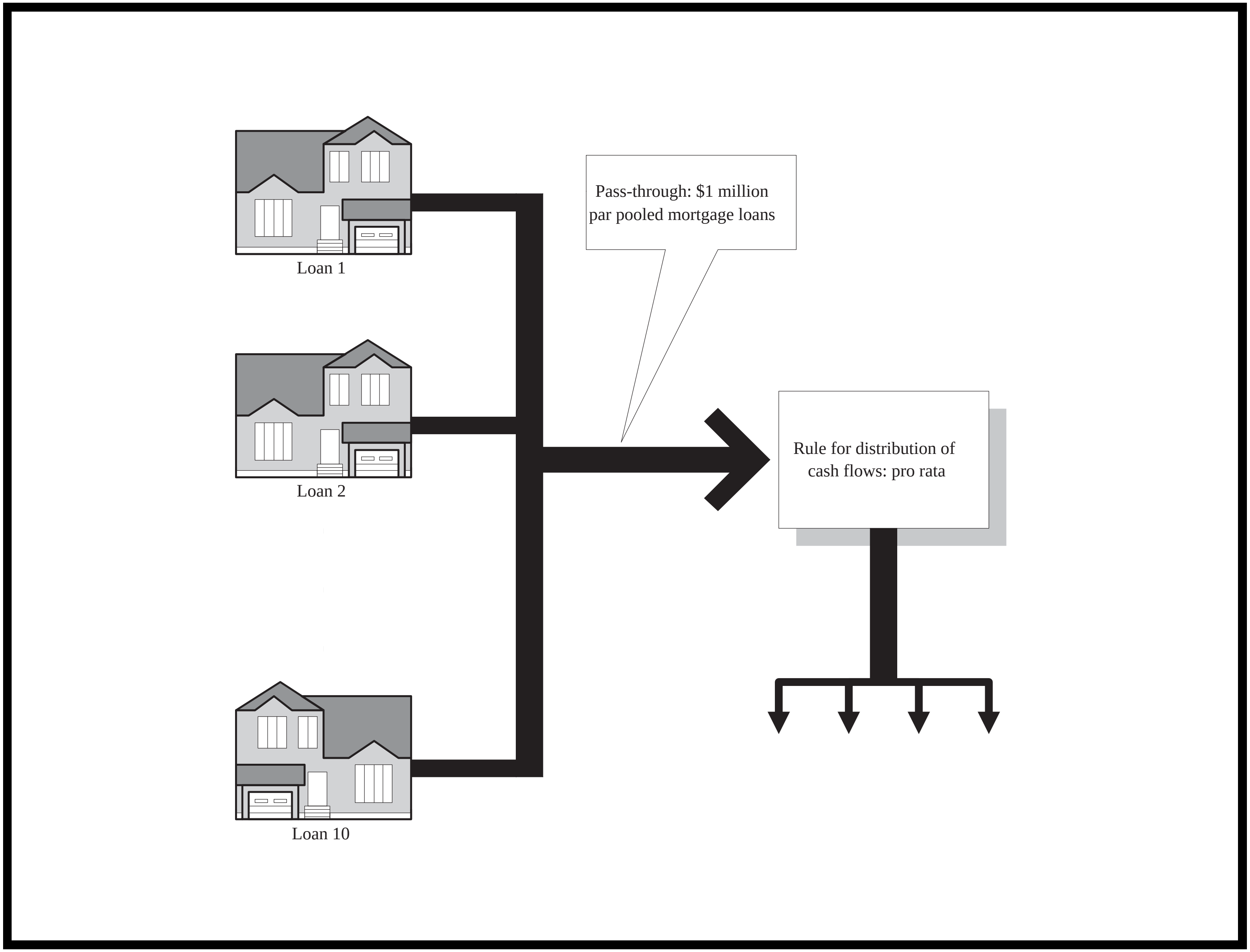
Problems Investing in Mortgages (concluded)

- Recall that a traditional mortgage is fixed rate, level payment, and fully amortized.
- So the percentage of principal and interest (P&I) varying from month to month, creating accounting headaches.
- Prepayment levels fluctuate with a host of factors.
- That makes the size and the timing of the cash flows unpredictable.

Mortgage Pass-Throughs[a]

- The simplest kind of MBS.
- Payments from the underlying mortgages are passed from the mortgage holders through the servicing agency, after a fee is subtracted.
- They are distributed to the security holder on a pro rata basis.
  - The holder of a \$25,000 certificate from a \$1 million pool is entitled to $2\frac{1}{2}\%$ (or 1/40th) of the cash flow.
- Because of higher marketability, a pass-through is easier to sell than its individual loans.

[a]First issued by Ginnie Mae (Government National Mortgage Association) in 1970.


Pass-through: $1 million
par pooled mortgage loans
Loan 1
Loan 2
Loan 10
Rule for distribution of
cash flows: pro rata

Collateralized Mortgage Obligations (CMOs)

- A pass-through exposes the investor to the total prepayment risk.
- Such risk is undesirable from an asset/liability perspective.
- To deal with prepayment uncertainty, CMOs were created.[a]
- Mortgage pass-throughs have a single maturity and are backed by individual mortgages.

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[a] In June 1983 by Freddie Mac (Federal Home Loan Mortgage Corporation), bailed out by the U.S. government in 2008, with the help of First Boston, which was acquired by Credit Suisse in 1990.

Collateralized Mortgage Obligations (CMOs) (continued)

- CMOs are *multiple*-maturity, *multi*class debt instruments collateralized by pass-throughs, stripped mortgage-backed securities, and whole loans.
- The total prepayment risk is now divided among classes of bonds called classes or tranches.[a]
- The principal, scheduled and prepaid, is allocated on a *prioritized* basis so as to redistribute the prepayment risk among the tranches in an unequal way.

[a] *Tranche* is a French word for “slice.”

Collateralized Mortgage Obligations (CMOs) (concluded)

- CMOs were the first successful attempt to alter mortgage cash flows in a security form that attracts a wide range of investors
  - The outstanding balance of agency CMOs was US$1.1 trillion as of the first quarter of 2015.[a]

[a] SIFMA (2015).

Sequential Tranche Paydown

- In the sequential tranche paydown structure, Class A receives principal paydown and prepayments before Class B, which in turn does it before Class C, and so on.
- Each tranche thus has a different effective maturity.
- Each tranche may even have a different coupon rate.

An Example

- Consider a two-tranche sequential-pay CMO backed by \$1,000,000 of mortgages with a 12% coupon and 6 months to maturity.
- The cash flow pattern for each tranche with zero prepayment and zero servicing fee is shown on p. 1240.
- The calculation can be carried out first for the Total columns, which make up the amortization schedule.
- Then the cash flow is allocated.
- Tranche A is retired after 4 months, and tranche B starts principal paydown at the end of month 4.

CMO Cash Flows without Prepayments

Month	Interest			Principal			Remaining principal		
	A	B	Total	A	B	Total	A	B	Tot
							500,000	500,000	1,000,0
1	5,000	5,000	10,000	162,548	0	162,548	337,452	500,000	837,4
2	3,375	5,000	8,375	164,173	0	164,173	173,279	500,000	673,2
3	1,733	5,000	6,733	165,815	0	165,815	7,464	500,000	507,4
4	75	5,000	5,075	7,464	160,009	167,473	0	339,991	339,9
5	0	3,400	3,400	0	169,148	169,148	0	170,843	170,8
6	0	1,708	1,708	0	170,843	170,843	0	0	
Total	10,183	25,108	35,291	500,000	500,000	1,000,000			

The total monthly payment is \$172,548. Month-i numbers reflect the ith monthly payment.

Another Example

- When prepayments are present, the calculation is only slightly more complex.
- Suppose the single monthly mortality (SMM) per month is 5%.
- This means the prepayment amount is 5% of the *remaining* principal.
- The remaining principal at month i *after* prepayment then equals the scheduled remaining principal as computed by Eq. (7) on p. 53 times $(0.95)^i$.
- This done for all the months, the interest payment at any month is the remaining principal of the previous month times 1%.

Another Example (continued)

- The prepayment amount equals the remaining principal times $0.05/0.95$.
  - The division by 0.95 yields the remaining principal *before* prepayment.
- Page 1243 tabulates the cash flows of the same two-tranche CMO under 5% SMM.

Another Example (continued)

Month	Interest			Principal			Remaining principal		
	A	B	Total	A	B	Total	A	B	Total
							500,000	500,000	1,000,00
1	5,000	5,000	10,000	204,421	0	204,421	295,579	500,000	795,5
2	2,956	5,000	7,956	187,946	0	187,946	107,633	500,000	607,6
3	1,076	5,000	6,076	107,633	64,915	172,548	0	435,085	435,0
4	0	4,351	4,351	0	158,163	158,163	0	276,922	276,9
5	0	2,769	2,769	0	144,730	144,730	0	132,192	132,1
6	0	1,322	1,322	0	132,192	132,192	0	0	
Total	9,032	23,442	32,474	500,000	500,000	1,000,000			

Month-i numbers reflect the ith monthly payment.

Another Example (continued)

- For instance, the total principal payment at month one, \$204,421, can be verified as follows.
- The *scheduled* remaining principal is \$837,452 from p. 1240.
- The remaining principal is hence

$$837452 \times 0.95 = 795579.$$

- That makes the total principal payment

$$1000000 - 795579 = 204421.$$

Another Example (concluded)

- As tranche A's remaining principal is \$500,000, all 204,421 dollars go to tranche A.
  - Incidentally, the prepayment is

$$837452 \times 5\% = 41873.$$

- Tranche A is retired after 3 months, and tranche B starts principal paydown at the end of month 3.

Stripped Mortgage-Backed Securities (SMBSs)[a]

- The principal and interest are divided between the PO strip and the IO strip.
- In the scenarios on p. 1239 and p. 1241:
  - The IO strip receives all the interest payments under the Interest/Total column.
  - The PO strip receives all the principal payments under the Principal/Total column.

[a]They were created in February 1987 when Fannie Mae issued its Trust 1 stripped MBS. Fannie Mae was bailed out by the U.S. government in 2008.

Stripped Mortgage-Backed Securities (SMBSs) (concluded)

- These new instruments allow investors to better exploit anticipated changes in interest rates.[a]
- The collateral for an SMBS is a pass-through.
- CMOs and SMBSs are usually called derivative MBSs.

[a]See p. 357 of the textbook.

Prepayments

- The prepayment option sets MBSs apart from other fixed-income securities.
- The exercise of options on most securities is expected to be "rational."
- This kind of "rationality" is weakened when it comes to the homeowner's decision to prepay.
- For example, even when the prevailing mortgage rate exceeds the mortgage's loan rate, some loans are prepaid.

Prepayment Risk

- Prepayment risk is the uncertainty in the amount and timing of the principal prepayments in the pool of mortgages that collateralize the security.
- This risk can be divided into contraction risk and extension risk.[a]
- Contraction risk is the risk of having to reinvest the prepayments at a rate lower than the coupon rate when interest rates decline.

[a]Similar to mortality risk and longevity risk in life insurance.

Prepayment Risk (continued)

- Extension risk is due to the slowdown of prepayments when interest rates climb, making the investor earn the security's lower coupon rate rather than the market's higher rate.
- Prepayments can be in whole or in part.
  - The former is called liquidation.
  - The latter is called curtailment.

Prepayment Risk (concluded)

- The holder of a pass-through security is exposed to the total prepayment risk associated with the underlying pool of mortgage loans.
- CMOs are designed to alter the distribution of that risk among investors.

Other Risks

- Investors in mortgages are exposed to at least three other risks.
  - Interest rate risk is inherent in any fixed-income security.
  - Credit risk is the risk of loss from default.
    - For privately insured mortgage, the risk is related to the credit rating of the company that insures the mortgage.
  - Liquidity risk is the risk of loss if the investment must be sold quickly.

Prepayment: Causes

Prepayments have at least five components.

Home sale ("housing turnover"). The sale of a home generally leads to the prepayment of mortgage because of the full payment of the remaining principal.

Refinancing. Mortgagors can refinance their home mortgage at a lower mortgage rate. This is the most volatile component of prepayment and constitutes the bulk of it when prepayments are extremely high.

Default. Caused by foreclosure and subsequent liquidation of a mortgage. Relatively minor in most cases.[a]

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[a]Except the subprime mortgage crisis that started the financial crisis of 2007–2008 (Zuckerman, 2010; Lewis, 2011).

Prepayment: Causes (concluded)

Curtailment. As the extra payment above the scheduled payment, curtailment applies to the principal and shortens the maturity of fixed-rate loans. Its contribution to prepayments is minor.

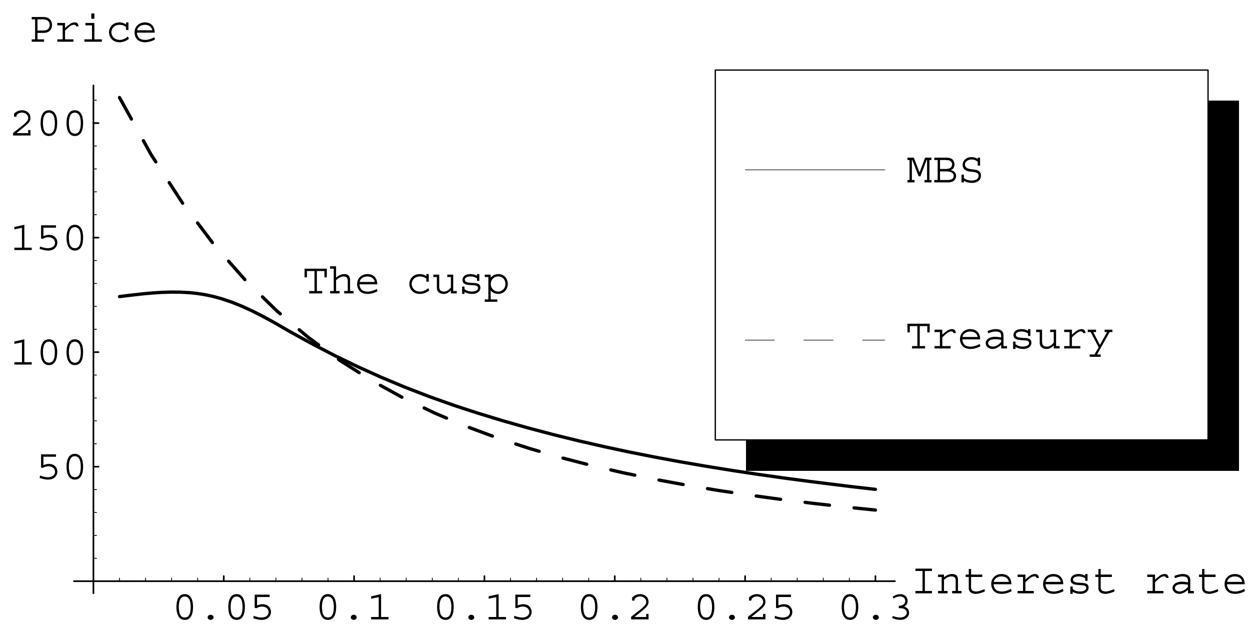
Full payoff (liquidation). There is evidence that many mortgagors pay off their mortgage completely when it is very seasoned and the remaining balance is small. Full payoff can also be due to natural disasters.

Prepayment: Characteristics

- Prepayments usually increase as the mortgage ages — first at an increasing rate and then at a decreasing rate.
- They are higher in the spring and summer and lower in the fall and winter.
- They vary by the geographic locations of the underlying properties.
- They increase when interest rates drop but with a time lag.

Prepayment: Characteristics (continued)

- If prepayments were higher for some time because of high refinancing rates, they tend to slow down.
  - Perhaps, homeowners who do not prepay when rates have been low for a prolonged time tend never to prepay.
- Plot on p. 1257 illustrates the typical price/yield curves of the Treasury and pass-through.



Price compression occurs as yields fall through a threshold. The cusp represents that point.

Prepayment: Characteristics (concluded)

- As yields fall and the pass-through's price moves above a certain price, it flattens and then follows a downward slope.
- This phenomenon is called the price compression of premium-priced MBSs.
- It demonstrates the negative convexity of such securities.

Analysis of Mortgage-Backed Securities

Oh, well, if you cannot measure,

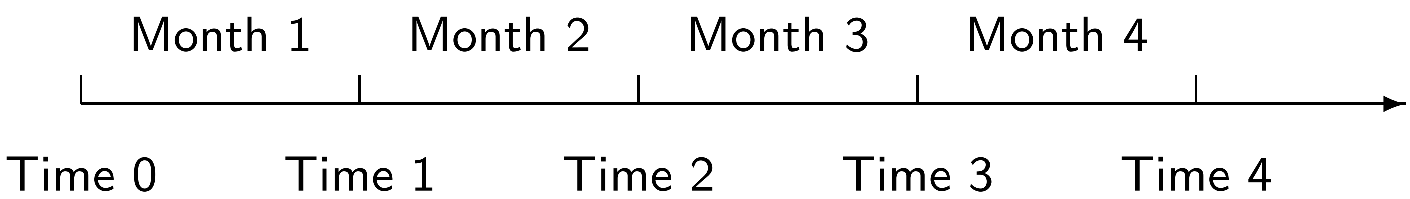
measure anyhow.

— Frank H. Knight (1885–1972)

Uniqueness of MBS

- Compared with other fixed-income securities, the MBS is unique in two respects.
- Its cash flow consists of principal and interest (P&I).
- The cash flow may vary because of prepayments in the underlying mortgages.

Time Line

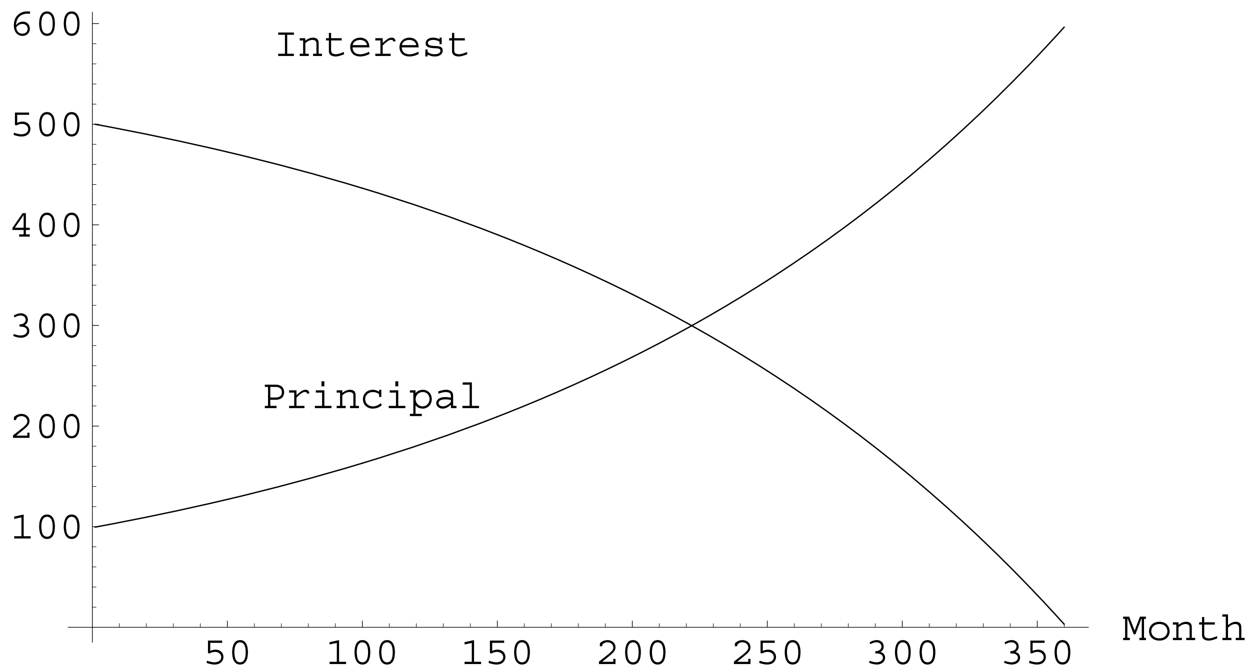


- Mortgage payments are paid in arrears.
- A payment for month i occurs at time i, that is, end of month i.
- The end of a month will be identified with the beginning of the coming month.

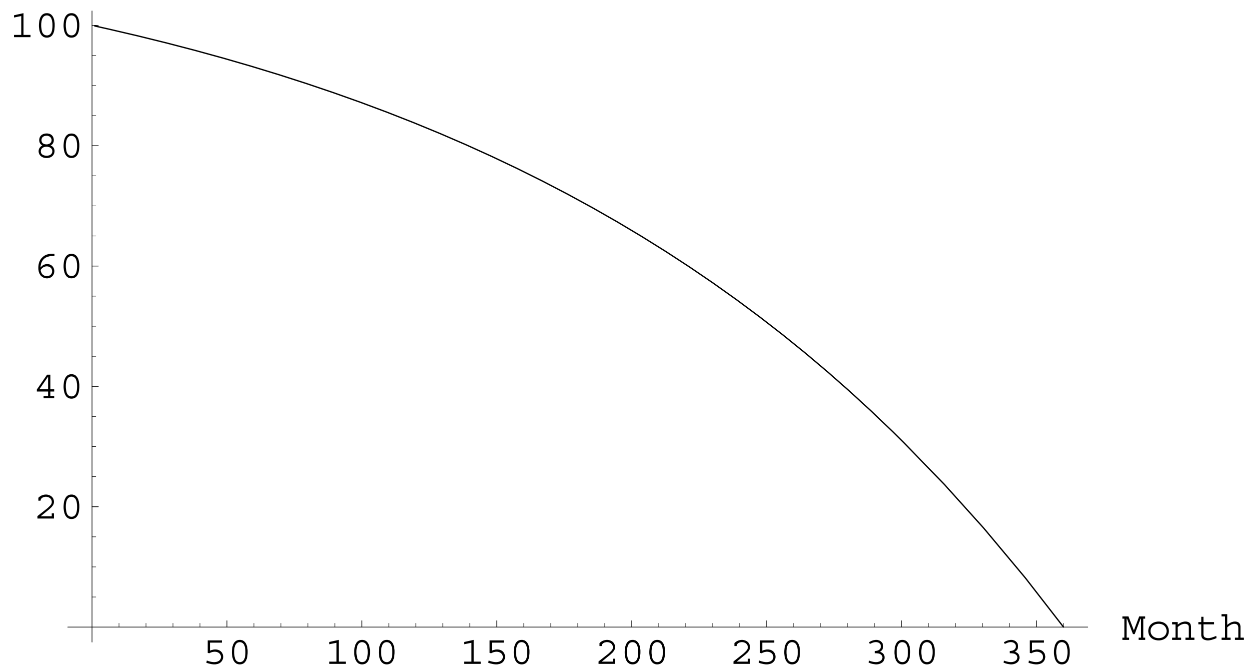
Cash Flow Analysis

- A traditional mortgage has a fixed term, a fixed interest rate, and a fixed monthly payment.
- Page 1264 illustrates the scheduled P&I for a 30-year, 6% mortgage with an initial balance of $100,000.
- Page 1265 depicts how the remaining principal balance decreases over time.

Scheduled Principal and Interest Payments



Scheduled Remaining Principal Balances (in 1000s)



Cash Flow Analysis (continued)

- In the early years, the P&I consists mostly of interest.
- Then it gradually shifts toward principal payment with the passage of time.
- However, the total P&I payment remains the same each month, hence the term *level* pay.
- In the absence of prepayments and servicing fees, identical characteristics hold for the pool's P&I payments.

Cash Flow Analysis (continued)

- From Eq. (7) on p. 53 the remaining principal balance *after* the kth payment is

$$C\,\frac{1-(1+r/m)^{-n+k}}{r/m}. \tag{173}$$

  - C is the scheduled P&I payment of an n-month mortgage making m payments per year.[a]
  - r is the annual mortgage rate.

- For mortgages, $m = 12$.

[a] See Eq. (6) on p. 45.

Cash Flow Analysis (continued)

- The scheduled remaining principal balance after k payments can be expressed as a portion of the original principal balance:

$$
\begin{aligned}
\text{Bal}_k &\stackrel{\Delta}{=} 1 - \frac{(1+r/m)^k - 1}{(1+r/m)^n - 1} \\
&= \frac{(1+r/m)^n - (1+r/m)^k}{(1+r/m)^n - 1}. \qquad (174)
\end{aligned}
$$

- This equation can be verified by dividing Eq. (173) (p. 1267) by the same equation with $k = 0$.

Cash Flow Analysis (continued)

- The remaining principal balance after k payments is

$$\mathrm{RB}_k \stackrel{\Delta}{=} \mathcal{O} \times \mathrm{Bal}_k,$$

  where $\mathcal{O}$ will denote the original principal balance.

- The term factor denotes the portion of the remaining principal balance to its original principal balance.
- So Bal_k is the monthly factor when there are no prepayments.
- It is also known as the amortization factor.

Cash Flow Analysis (concluded)

- The idea of factor can also be applied to a mortgage pool.
- Then it is called the paydown factor on the pool or simply the pool factor.

An Example

- The remaining balance of a 15-year mortgage with a 9% mortgage rate after 54 months is

$$
\begin{aligned}
& \mathcal{O} \times \frac{[\,1+(0.09/12)\,]^{180}-[\,1+(0.09/12)\,]^{54}}{[\,1+(0.09/12)\,]^{180}-1} \\
= \quad & \mathcal{O} \times 0.824866.
\end{aligned}
$$

- In other words, roughly 82.49% of the original loan amount remains after 54 months.

P&I Analysis

- By the amortization principle, the tth interest payment equals

$$I_t \stackrel{\Delta}{=} \mathrm{RB}_{t-1} \times \frac{r}{m} = \mathcal{O} \times \frac{r}{m} \times \frac{(1+r/m)^n - (1+r/m)^{t-1}}{(1+r/m)^n - 1}.$$

- The principal part of the tth monthly payment is

$$\begin{aligned} P_t &\stackrel{\Delta}{=} \mathrm{RB}_{t-1} - \mathrm{RB}_t \\ &= \mathcal{O} \times \frac{(r/m)(1+r/m)^{t-1}}{(1+r/m)^n - 1}. \end{aligned} \tag{175}$$

P&I Analysis (concluded)

- The scheduled P&I payment at month t, or $P_t + I_t$, is

$$
\begin{aligned}
& (\mathrm{RB}_{t-1} - \mathrm{RB}_t) + \mathrm{RB}_{t-1} \times \frac{r}{m} \\
= \quad & \mathcal{O} \times \left[\frac{(r/m)(1 + r/m)^n}{(1 + r/m)^n - 1} \right], \qquad (176)
\end{aligned}
$$

  indeed a level pay independent of t.[a]

- The term within the brackets is called the payment factor or annuity factor.

- It is the monthly payment for each dollar of mortgage.

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[a]This formula is identical to Eq. (6) on p. 45.

An Example

- The mortgage on pp. 47ff has a monthly payment of

$$250000 \times \frac{(0.08/12) \times (1 + (0.08/12))^{180}}{(1 + (0.08/12))^{180} - 1} = 2389.13$$

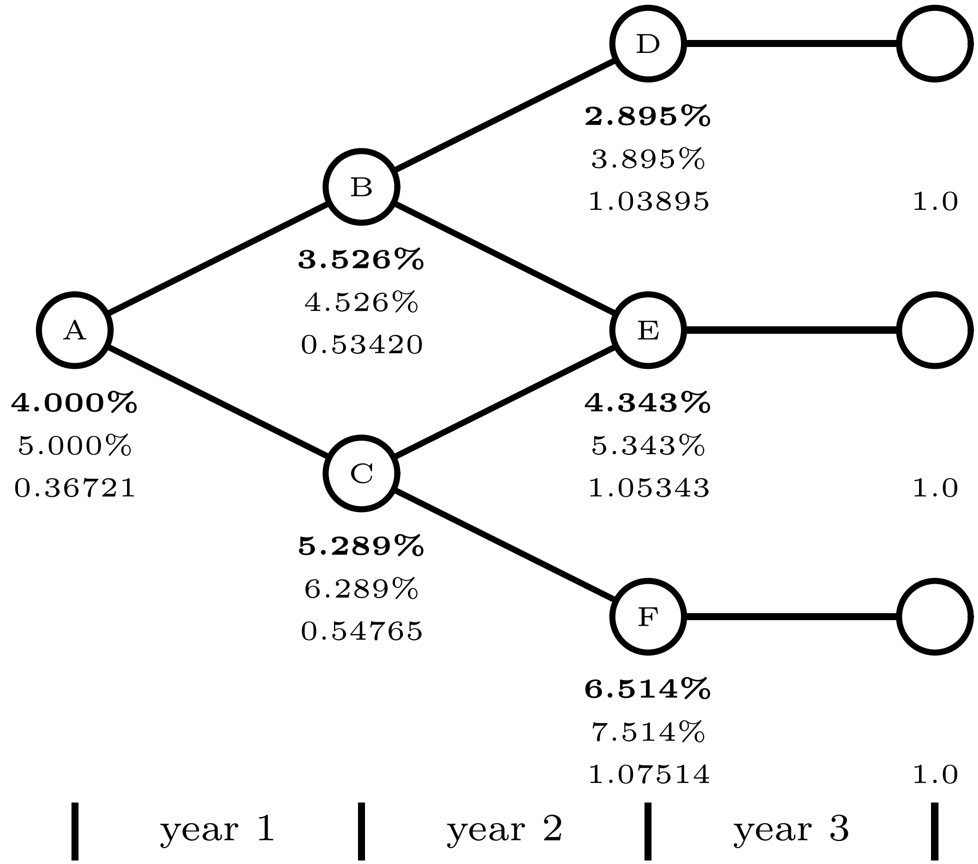
  by Eq. (176) on p. 1273.

- This number agrees with the number derived earlier on p. 47.

Pricing Adjustable-Rate Mortgages

- We turn to ARM pricing as an interesting application of derivatives pricing and the analysis above.
- Consider a 3-year ARM with an interest rate that is 1% above the 1-year T-bill rate at the beginning of the year.
- This 1% is called the margin.
- Assume this ARM carries annual, not monthly, payments.
- The T-bill rates follow the binomial process, in boldface, on p. 1276,[a] and the risk-neutral probability is 0.5.

[a]They are from p. 1015.



Stacked at each node are the T-bill rate, the mortgage rate, and the payment factor for a mortgage initiated at that node and ending at year 3 (based on the mortgage rate at the same node).

Pricing Adjustable-Rate Mortgages (continued)

- How much is the ARM worth to the issuer?
- Each new coupon rate at the reset date determines the level mortgage payment for the months until the next reset date as if the ARM were a fixed-rate loan with the new coupon rate and a maturity equal to that of the ARM.
- For example, for the interest rate tree on p. 1276, the scenario $A \rightarrow B \rightarrow E$ will leave our three-year ARM with a remaining principal at the end of the second year different from that under the scenario $A \rightarrow C \rightarrow E$.

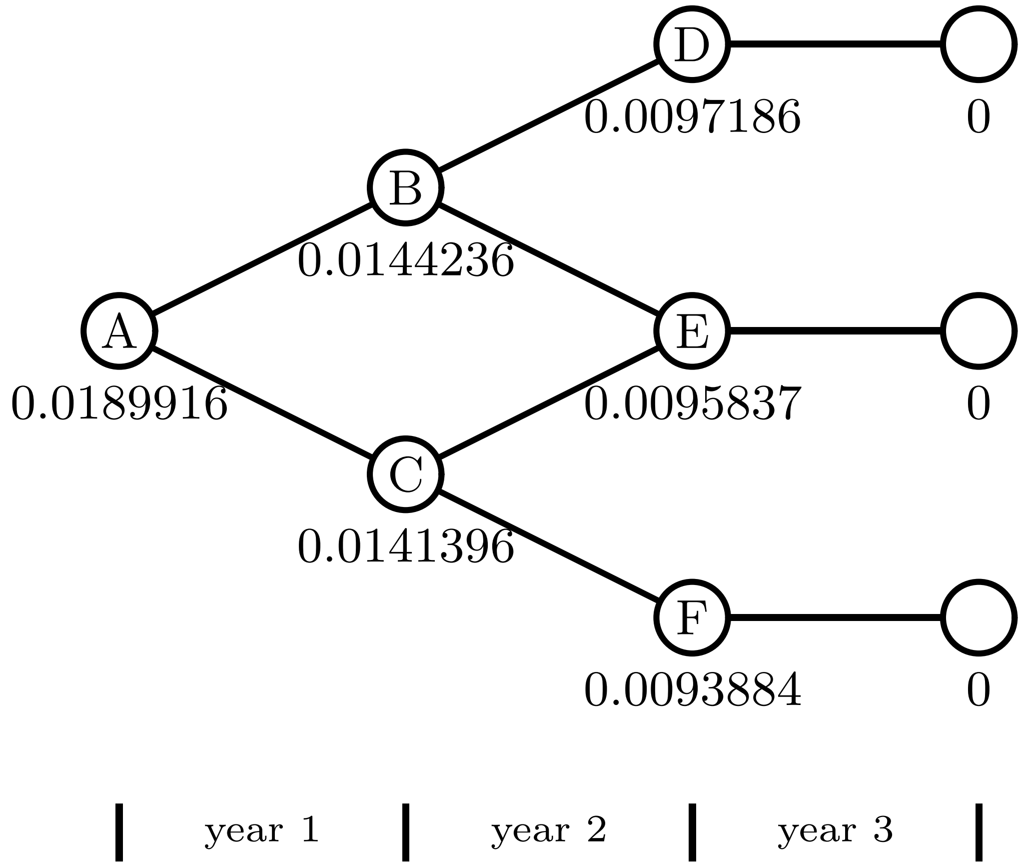
Pricing Adjustable-Rate Mortgages (continued)

- This path dependency calls for care in algorithmic design to avoid exponential complexity.
- Attach to each node on the binomial tree the annual payment per \$1 of principal for a mortgage initiated at that node and ending at year 3.
  - In other words, the payment factor.
- At node B, for example, the annual payment factor can be calculated by Eq. (176) on p. 1273 with $r = 0.04526$, $m = 1$, and $n = 2$ as

$$\frac{0.04526 \times (1.04526)^2}{(1.04526)^2 - 1} = 0.53420.$$

Pricing Adjustable-Rate Mortgages (continued)

- The payment factors for other nodes on p. 1276 are calculated in the same manner.
- We now apply backward induction to price the ARM (see p. 1280).
- At each node on the tree, the *net* value of an ARM of value \$1 initiated at that node and ending at the end of the third year is calculated.
- For example, the value is zero at terminal nodes since the ARM is immediately repaid.


A
0.0189916
B
0.0144236
C
0.0141396
D
0.0097186
E
0.0095837
F
0.0093884
0
0
0
year 1
year 2
year 3

Pricing Adjustable-Rate Mortgages (continued)

- At node D, the value is

$$\frac{1.03895}{1.02895} - 1 = 0.0097186,$$

which is simply the net present value of the payment 1.03895 next year.

  – Recall that the issuer makes a loan of \$1 at D.

- The values at nodes E and F can be computed similarly.

Pricing Adjustable-Rate Mortgages (continued)

- At node B, we first figure out the remaining principal balance after the payment one year hence as

$$1 - (0.53420 - 0.04526) = 0.51106,$$

because \$0.04526 of the payment of \$0.53426 constitutes the interest.

- The issuer will receive \$0.01 above the T-bill rate next year, and the value of the ARM is either \$0.0097186 or \$0.0095837 per \$1, each with probability 0.5.

Pricing Adjustable-Rate Mortgages (continued)

- The ARM's value at node B thus equals

$$\frac{0.51106 \times (0.0097186 + 0.0095837)/2 + 0.01}{1.03526} = 0.0144236.$$

- The values at nodes C and A can be calculated similarly as

$$\begin{aligned}
& \frac{(1 - (0.54765 - 0.06289)) \times (0.0095837 + 0.0093884)/2 + 0.01}{1.05289} \\
= \quad & 0.0141396 \\
& \frac{(1 - (0.36721 - 0.05)) \times (0.0144236 + 0.0141396)/2 + 0.01}{1.04} \\
= \quad & 0.0189916,
\end{aligned}$$

respectively.

Pricing Adjustable-Rate Mortgages (concluded)

- The value of the ARM to the issuer is hence \$0.0189916 per \$1 of loan amount.
- The above idea of scaling has wide applicability in pricing certain classes of path-dependent securities.[a]

[a] For example, newly issued lookback options in Programming Assignment 11.7.12(2) of the textbook.

More on ARMs

- ARMs are indexed to publicly available indices such as:
  - LIBOR.
  - The constant maturity Treasury rate (CMT).
  - The Cost of Funds Index (COFI).
- COFI is based on an average cost of funds.
- So it moves relatively sluggishly compared with LIBOR.

More on ARMs (continued)

- Since 1990, the need for securitization gradually shift in LIBOR's favor.[a]
- That will change because of scandals.[b]
- As a result, LIBOR will be faced out by 2021.[c]
- Alternatives include the Secured Overnight Financing Rate (SOFR).

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[a]There are roughly $400 trillion LIBOR-indexed assets as of 2017 (Durden, 2017).

[b]Morgenson (2012). The LIBOR rate-fixing scandal broke in June 2012. A sample email on August 20, 2007: "ok, i will move the curve down 1bp maybe more if I can".

[c]Durden (2017).

More on ARMs (continued)

- If the ARM coupon reflects fully and instantaneously current market rates, then the ARM security will be priced close to par and refinancings rarely occur.
- In reality, adjustments are imperfect in many ways.
- At the reset date, a margin is added to the benchmark index to determine the new coupon.
- ARMs often have periodic rate caps that limit the amount by which the coupon rate may increase or decrease at the reset date.
- They also have lifetime caps and floors.

More on ARMs (concluded)

- To attract borrowers, mortgage lenders usually offer a below-market initial rate (the “teaser” rate).
- The reset interval, the time period between adjustments in the ARM coupon rate, is often annual, which is not frequent enough.
- These terms are easy to incorporate into the pricing algorithm.

Expressing Prepayment Speeds

- The cash flow of a mortgage derivative is determined from that of the mortgage pool.
- The single most important factor complicating this endeavor is the unpredictability of prepayments.
- Recall that prepayment represents the principal payment made in excess of the scheduled principal amortization.

Expressing Prepayment Speeds (concluded)

- Compare the amortization factor Bal_t of the pool with the reported factor to determine if prepayments have occurred.
- The amount by which the reported factor is exceeded by the amortization factor is the prepayment amount.

Single Monthly Mortality

- A SMM of ω means $\omega\%$ of the scheduled remaining balance at the end of the month will prepay.[a]
- In other words, the SMM is the percentage of the remaining balance that prepays for the month.
- Suppose the remaining principal balance of an MBS at the beginning of a month is \$50,000, the SMM is 0.5%, and the scheduled principal payment is \$70.
- Then the prepayment for the month is

$$0.005 \times (50{,}000 - 70) \approx 250$$

dollars.

[a] Recall p. 1241.

Single Monthly Mortality (concluded)

- If the same monthly prepayment speed s is maintained since the issuance of the pool, the remaining principal balance at month i will be

$$\mathrm{RB}_i \times \left(1 - \frac{s}{100}\right)^i. \tag{177}$$

- It goes without saying that prepayment speeds must lie between 0% and 100%.

An Example

- Take the mortgage on p. 1271.
- Its amortization factor at the 54th month is 0.824866.
- If the actual factor is 0.8, then the (implied) SMM for the initial period of 54 months is

$$100 \times \left[1 - \left(\frac{0.8}{0.824866} \right)^{1/54} \right] = 0.0566677\%.$$

- In other words, roughly 0.057% of the remaining principal is prepaid per month.

Conditional Prepayment Rate

- The conditional prepayment rate (CPR) is the annualized equivalent of a SMM,

$$\text{CPR} = 100 \times \left[1 - \left(1 - \frac{\text{SMM}}{100} \right)^{12} \right].$$

- Conversely,

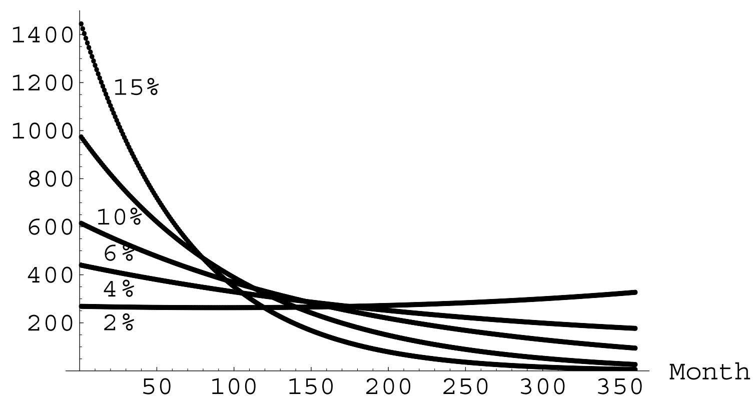
$$\text{SMM} = 100 \times \left[1 - \left(1 - \frac{\text{CPR}}{100} \right)^{1/12} \right].$$

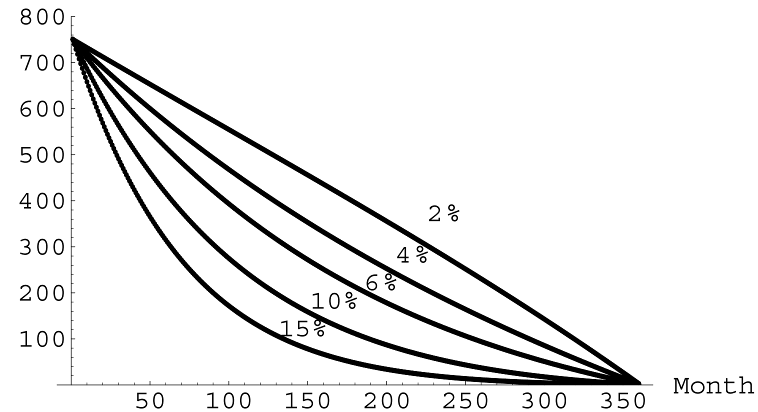
Conditional Prepayment Rate (concluded)

- For example, the SMM of 0.0566677 on p. 1293 is equivalent to a CPR of

$$100 \times \left[1 - \left(1 - \left(\frac{0.0566677}{100}\right)^{12}\right)\right] = 0.677897\%.$$

- Roughly 0.68% of the remaining principal is prepaid annually.
- The figures on 1296 plot the principal and interest cash flows under various prepayment speeds.
- With accelerated prepayments, the principal cash flow is shifted forward in time.





Principal (left) and interest (right) cash flows at various CPRs. The 6% mortgage has 30 years to maturity and an original loan amount of \$100,000.

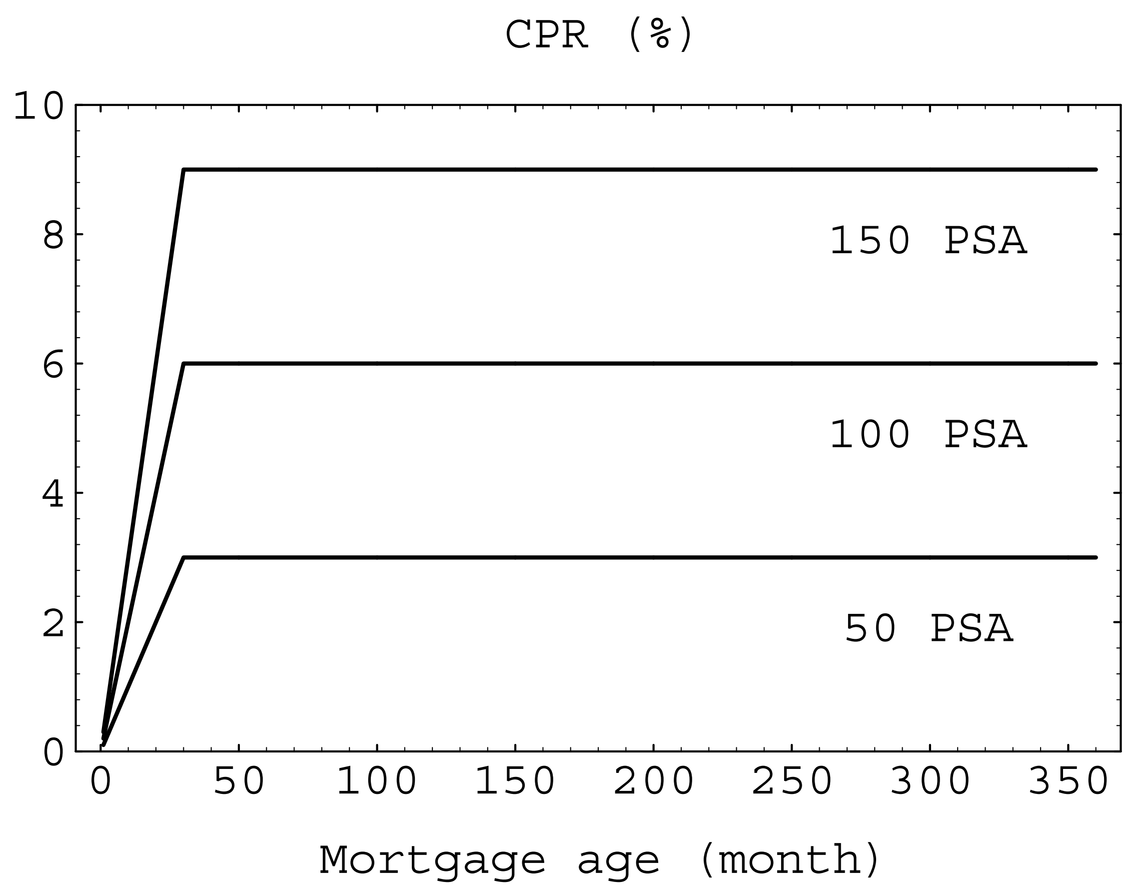
PSA

- In 1985 the Public Securities Association[a] (PSA) standardized a prepayment model.
- The PSA standard is expressed as a monthly series of CPRs.
  - It reflects the increase in CPR that occurs as the pool seasons.
- At the time the PSA proposed its standard, a seasoned 30-year GNMA's typical prepayment speed was $\sim 6\%$ CPR.

[a]It became the Bond Market Association in 1997, which merged with the Securities Industry Association (SIA) to form the Securities Industry and Financial Markets Association (SIFMA) in 2006.

PSA (continued)

- The PSA standard postulates the following prepayment speeds:
  - The CPR is 0.2% for the first month.
  - It increases thereafter by 0.2% per month until it reaches 6% per year for the 30th month.
  - It then stays at 6% for the remaining years.
- The PSA benchmark is also referred to as 100 PSA.
- Other speeds are expressed as some percentage of PSA.
  - 50 PSA means one-half the PSA CPRs.
  - 150 PSA means one-and-a-half the PSA CPRs.


CPR (%)
10
8
6
4
2
0
0
50
100
150
200
250
300
350
150 PSA
100 PSA
50 PSA
Mortgage age (month)

PSA (concluded)

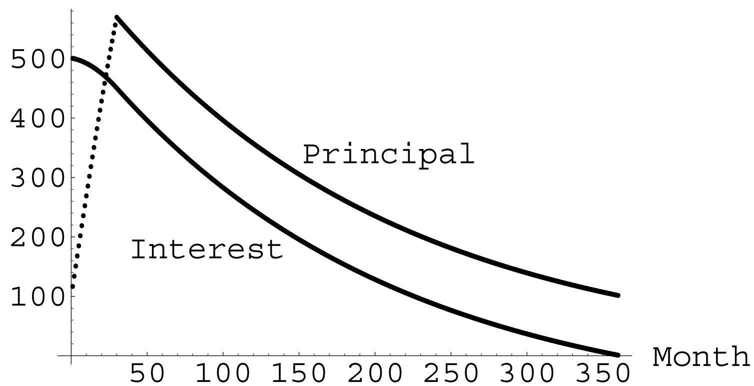
- Mathematically,

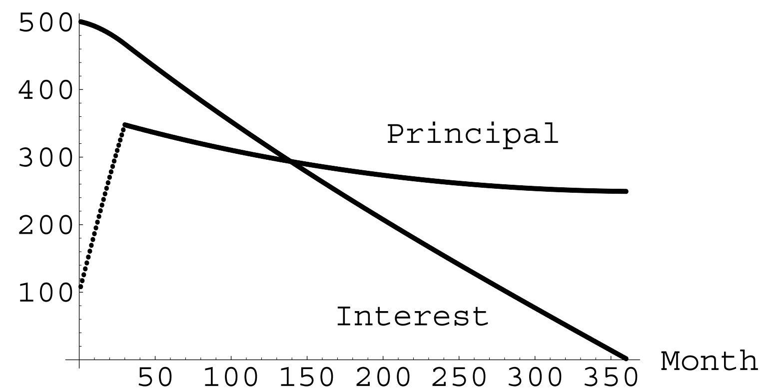
$$\text{CPR} = \begin{cases} 6\% \times \frac{\text{PSA}}{100} & \text{if the pool age exceeds 30 months} \\ \\ 0.2\% \times m \times \frac{\text{PSA}}{100} & \text{if the pool age } m \leq 30 \text{ months} \end{cases}$$

- Conversely,

$$\text{PSA} = \begin{cases} 100 \times \frac{\text{CPR}}{6} & \text{if the pool age exceeds 30 months} \\ \\ 100 \times \frac{\text{CPR}}{0.2 \times m} & \text{if the pool age } m \leq 30 \text{ months} \end{cases}$$

Cash Flows at 50 and 100 PSAs





The 6% mortgage has 30 years to maturity and an original loan amount of \$100,000. The 100 PSA scenario is on the left, and the 50 PSA is on the right.

Prepayment Vector

- The PSA tries to capture how prepayments vary with age.
- But it should be viewed as a market convention rather than a model.
- A vector of PSAs generated by a prepayment model should be used to describe the monthly prepayment speed through time.
- The monthly cash flows can be derived thereof.

Prepayment Vector (continued)

- Similarly, the CPR should be seen purely as a measure of speed rather than a model.
- If one treats a single CPR number as the true prepayment speed, that number will be called the constant prepayment rate.
- This simple model is inconsistent with the empirical fact that pools with new production loans typically prepay at a slower rate than seasoned pools.
- A vector of CPRs should be preferred.

Prepayment Vector (concluded)

- A CPR/SMM vector is easier to work with than a PSA vector because of the lack of dependence on the pool age.
- But they are all equivalent as a CPR vector can always be converted into an equivalent PSA vector and vice versa.

Cash Flow Generation

- Each cash flow is composed of the principal payment, the interest payment, and the principal prepayment.
- Let B_k denote the actual remaining principal balance at month k.
- Then the pool's actual remaining principal balance at time $i-1$ is B_{i-1}.

Cash Flow Generation (continued)

- The *scheduled* principal and interest payments at time i are

$$\overline{P_i} \triangleq B_{i-1}\left(\frac{\text{Bal}_{i-1} - \text{Bal}_i}{\text{Bal}_{i-1}}\right) \tag{178}$$

$$= B_{i-1}\,\frac{r/m}{(1+r/m)^{n-i+1} - 1} \tag{179}$$

$$\overline{I_i} \triangleq B_{i-1}\,\frac{r-\alpha}{m} \tag{180}$$

  - Bal_k is the scheduled remaining principal balance after k payments (p. 1268).
  - α is the servicing spread (or servicing fee rate).
  - It consists of the servicing fee for the servicer as well as the guarantee fee.

Cash Flow Generation (continued)

- The prepayment at time i is

$$\mathrm{PP}_i = B_{i-1}\,\frac{\mathrm{Bal}_i}{\mathrm{Bal}_{i-1}} \times \mathrm{SMM}_i.$$

  - SMM_i is the prepayment speed for month i.

- If the total principal payment from the pool is $\overline{P_i} + \mathrm{PP}_i$, the remaining principal balance is

$$\begin{aligned} B_i &= B_{i-1} - \overline{P_i} - \mathrm{PP}_i \\ &= B_{i-1}\left[1 - \left(\frac{\mathrm{Bal}_{i-1} - \mathrm{Bal}_i}{\mathrm{Bal}_{i-1}}\right) - \frac{\mathrm{Bal}_i}{\mathrm{Bal}_{i-1}} \times \mathrm{SMM}_i\right] \\ &= \frac{B_{i-1} \times \mathrm{Bal}_i \times (1 - \mathrm{SMM}_i)}{\mathrm{Bal}_{i-1}}. \end{aligned} \tag{181}$$

Cash Flow Generation (continued)

- Equation (181) generalizes Eq. (178) on p. 1306.
- Equation (181) can be applied iteratively to yield[a]

$$B_i = \mathrm{RB}_i \times \prod_{j=1}^{i} (1 - \mathrm{SMM}_j). \tag{182}$$

  - The above formula generalizes Eq. (177) on p. 1292.
- Define

$$b_i \triangleq \prod_{j=1}^{i} (1 - \mathrm{SMM}_j).$$

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[a] RB_i is defined on p. 1269.

Cash Flow Generation (continued)

- $I'_i \triangleq \text{RB}_{i-1} \times (r-\alpha)/m$ is the scheduled interest payment.
- The scheduled P&I is[a]

$$\overline{P_i} = b_{i-1} P_i \text{ and } \overline{I_i} = b_{i-1} I'_i. \tag{183}$$

- The scheduled cash flow and the b_i determined by the prepayment vector are all that are needed to calculate the projected actual cash flows.

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[a] P_i and I_i are defined on p. 1272.

Cash Flow Generation (concluded)

- If the servicing fees do not exist (that is, $\alpha = 0$), the projected monthly payment *before* prepayment at month i becomes

$$\overline{P_i} + \overline{I_i} = b_{i-1}(P_i + I_i) = b_{i-1} C. \qquad (184)$$

  - C is the scheduled monthly payment on the original principal.[a]

- See Figure 29.10 in the text (p. 437) for a linear-time algorithm for generating the mortgage pool's cash flow.

[a] See Eq. (176) on p. 1273.

