

Problems; the Smile

- Options written on the same underlying asset usually do not produce the same implied volatility.
- A typical pattern is a “smile” in relation to the strike price.
 - The implied volatility is lowest for at-the-money options.
 - It becomes higher the further the option is in- or out-of-the-money.
- Other patterns have also been observed.

Problems; the Smile (concluded)

- To address this issue, volatilities are often combined to produce a composite implied volatility.
- This practice is not sound theoretically.
- The existence of different implied volatilities for options on the same underlying asset shows the Black-Scholes model cannot be literally true.

Trading Days and Calendar Days

- Interest accrues based on the calendar day.
- But σ is usually calculated based on trading days only.
 - Stock price seems to have lower volatilities when the exchange is closed.^a
- How to incorporate these two different ways of day count into the Black-Scholes formula and binomial tree algorithms?

^aFama (1965); French (1980); French and Roll (1986).

Trading Days and Calendar Days (concluded)

- Think of σ as measuring the volatility of stock price one year from now (regardless of what happens in between).
- Suppose a year has 260 trading days.
- So a heuristic is to replace σ in the Black-Scholes formula with^a

$$\sigma \sqrt{\frac{365}{260} \times \frac{\text{number of trading days to expiration}}{\text{number of calendar days to expiration}}}.$$

- How about binomial tree algorithms?

^aFrench (1984).

Binomial Tree Algorithms for American Puts

- Early exercise has to be considered.
- The binomial tree algorithm starts with the terminal payoffs

$$\max(0, X - Su^j d^{n-j})$$

and applies backward induction.

- At each intermediate node, it checks for early exercise by comparing the payoff if exercised with the continuation value.

Bermudan Options

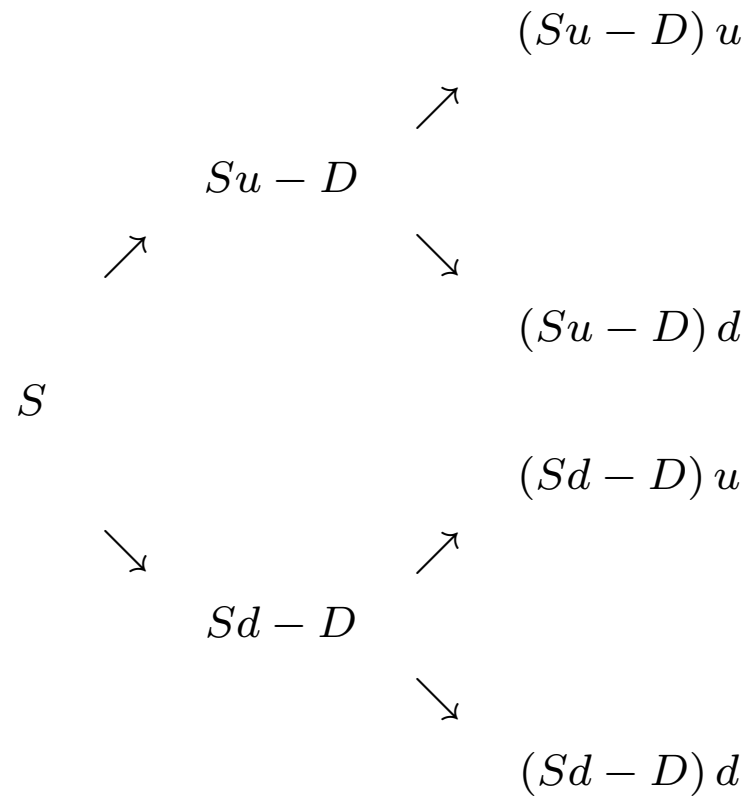
- Some American options can be exercised only at discrete time points instead of continuously.
- They are called Bermudan options.
- Their pricing algorithm is identical to that for American options.
- But early exercise is considered for only those nodes when early exercise is permitted.

Options on a Stock That Pays Dividends

- Early exercise must be considered.
- Proportional dividend payout model is tractable (see text).
 - The dividend amount is a constant proportion of the *prevailing* stock price.
- In general, the corporate dividend policy is a complex issue.

Known Dividends

- Constant dividends introduce complications.
- Use D to denote the amount of the dividend.
- Suppose an ex-dividend date falls in the first period.
- At the end of that period, the possible stock prices are $S_u - D$ and $S_d - D$.
- Follow the stock price one more period.
- The number of possible stock prices is not three but four: $(S_u - D)u$, $(S_u - D)d$, $(S_d - D)u$, $(S_d - D)d$.
 - The binomial tree no longer combines.



An Ad-Hoc Approximation

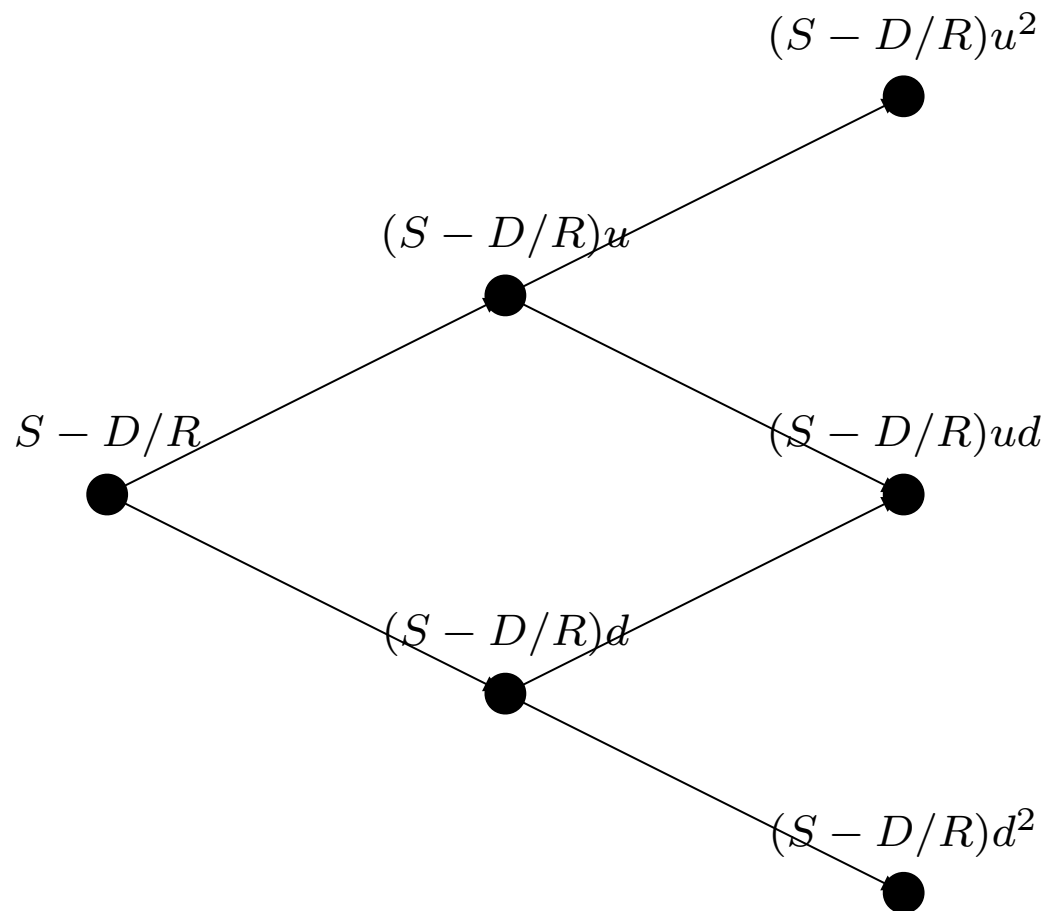
- Use the Black-Scholes formula with the stock price reduced by the PV of the dividends.^a
- This essentially decomposes the stock price into a riskless one paying known dividends and a risky one.
- The riskless component at any time is the PV of future dividends during the life of the option.
 - σ is the volatility of the process followed by the risky component.
- The stock price, between two adjacent ex-dividend dates, follows the same lognormal distribution.

^aRoll (1977).

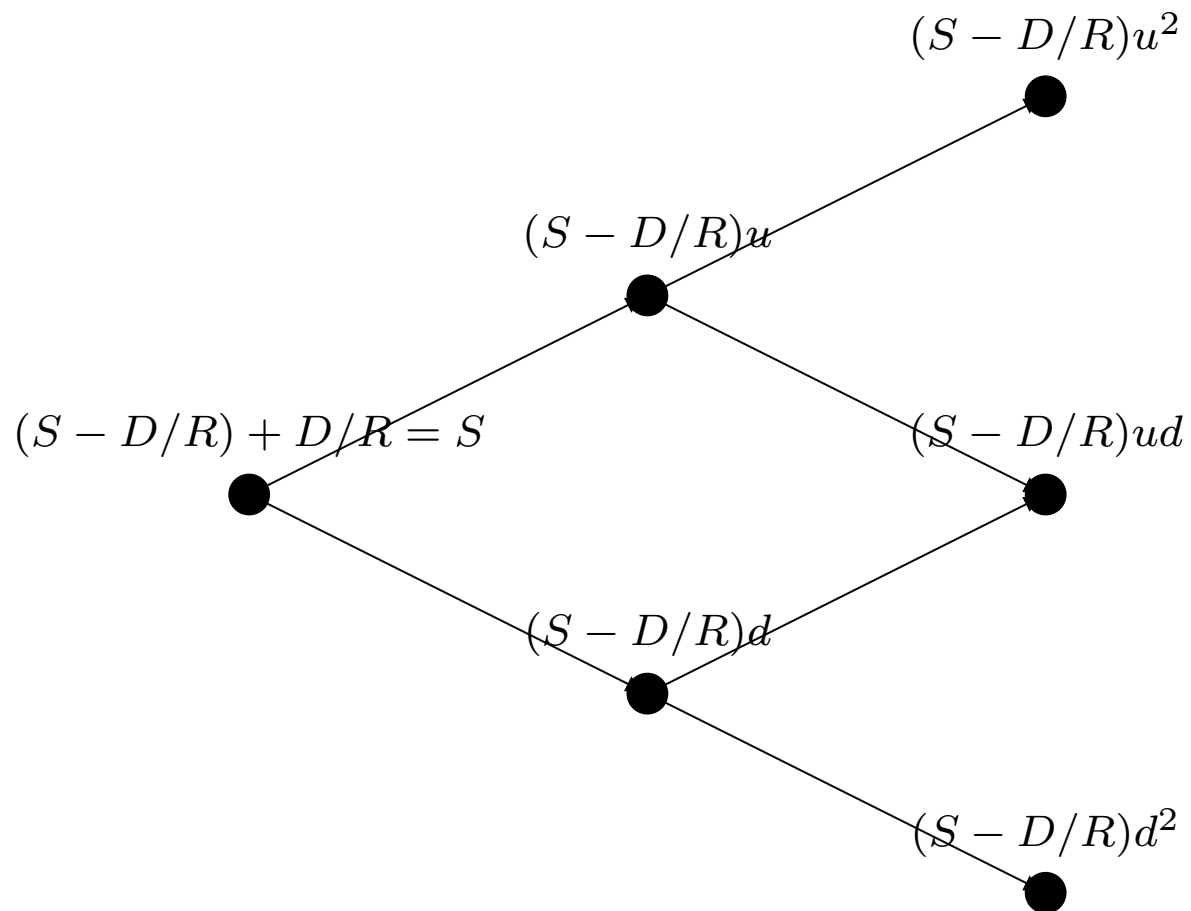
An Ad-Hoc Approximation (concluded)

- Start with the current stock price minus the PV of future dividends before expiration.
- Develop the binomial tree for the new stock price as if there were no dividends.
- Then add to each stock price on the tree the PV of all *future* dividends before expiration.
- American option prices can be computed as before on this tree of stock prices.

An Ad-Hoc Approximation vs. P. 273 (Step 1)



An Ad-Hoc Approximation vs. P. 273 (Step 2)



An Ad-Hoc Approximation vs. P. 273^a

- The trees are different.
- The stock prices at maturity are also different.
 - $(Su - D)u, (Su - D)d, (Sd - D)u, (Sd - D)d$ (p. 273).
 - $(S - D/R)u^2, (S - D/R)ud, (S - D/R)d^2$ (ad hoc).
- Note that $(Su - D)u > (S - D/R)u^2$ and $(Sd - D)d < (S - D/R)d^2$ as $d < R < u$.

^aContributed by Mr. Yang, Jui-Chung (D97723002) on March 18, 2009.

An Ad-Hoc Approximation vs. P. 273 (concluded)

- So the ad hoc approximation has a smaller dynamic range.
- This explains why in practice the volatility is usually increased when using the ad hoc approximation.

A General Approach^a

- A new tree structure.
- No approximation assumptions are made.
- A mathematical proof that the tree can always be constructed.
- The actual performance is quadratic except in pathological cases (see pp. 639ff).
- Other approaches include adjusting σ and approximating the known dividend with a dividend yield.

^aDai (R86526008, D8852600) and Lyuu (2004).

Continuous Dividend Yields

- Dividends are paid continuously.
 - Approximates a broad-based stock market portfolio.
- The payment of a continuous dividend yield at rate q reduces the growth rate of the stock price by q .
 - A stock that grows from S to S_τ with a continuous dividend yield of q would grow from S to $S_\tau e^{q\tau}$ without the dividends.
- A European option has the same value as one on a stock with price $Se^{-q\tau}$ that pays no dividends.^a

^aIn pricing European options, we care only about the distribution of S_τ .

Continuous Dividend Yields (continued)

- The Black-Scholes formulas hold with S replaced by $Se^{-q\tau}$.^a

$$C = Se^{-q\tau} N(x) - Xe^{-r\tau} N(x - \sigma\sqrt{\tau}), \quad (28)$$

$$P = Xe^{-r\tau} N(-x + \sigma\sqrt{\tau}) - Se^{-q\tau} N(-x), \quad (28')$$

where

$$x \equiv \frac{\ln(S/X) + (r - q + \sigma^2/2) \tau}{\sigma\sqrt{\tau}}.$$

- Formulas (28) and (28') remain valid as long as the dividend yield is predictable.

^aMerton (1973).

Continuous Dividend Yields (continued)

- To run binomial tree algorithms, replace u with $ue^{-q\Delta t}$ and d with $de^{-q\Delta t}$, where $\Delta t \equiv \tau/n$.
 - The reason: The stock price grows at an expected rate of $r - q$ in a risk-neutral economy.
- Other than the changes, binomial tree algorithms stay the same.
 - In particular, p should use the *original* u and d !^a

^aContributed by Ms. Wang, Chuan-Ju (F95922018) on May 2, 2007.

Continuous Dividend Yields (concluded)

- Alternatively, pick the risk-neutral probability as

$$\frac{e^{(r-q) \Delta t} - d}{u - d}, \quad (29)$$

where $\Delta t \equiv \tau/n$.

- The reason: The stock price grows at an expected rate of $r - q$ in a risk-neutral economy.
- The u and d remain unchanged.
- Other than the change in Eq. (29), binomial tree algorithms stay the same *as if there were no dividends*.

Sensitivity Analysis of Options

Cleopatra's nose, had it been shorter,
the whole face of the world
would have been changed.
— Blaise Pascal (1623–1662)

Sensitivity Measures (“The Greeks”)

- How the value of a security changes relative to changes in a given parameter is key to hedging.
 - Duration, for instance.
- Let $x \equiv \frac{\ln(S/X) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}$ (recall p. 261).
- Recall that

$$N'(y) = \frac{e^{-y^2/2}}{\sqrt{2\pi}} > 0,$$

the density function of standard normal distribution.

Delta

- Defined as $\Delta \equiv \partial f / \partial S$.
 - f is the price of the derivative.
 - S is the price of the underlying asset.
- The delta of a portfolio of derivatives on the same underlying asset is the sum of their individual deltas.
 - Elementary calculus.
- The delta used in the BOPM (p. 213) is the discrete analog.

Delta (concluded)

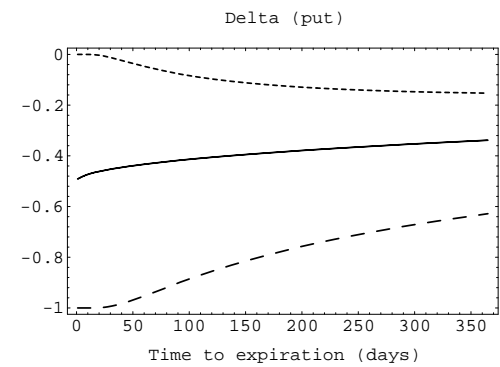
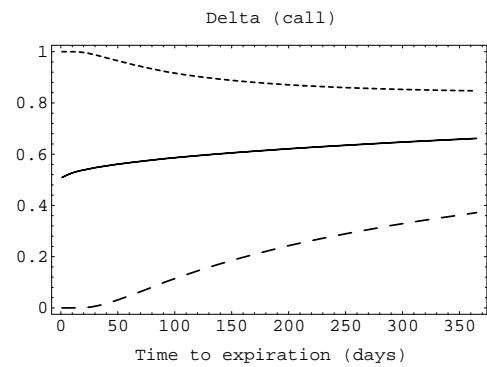
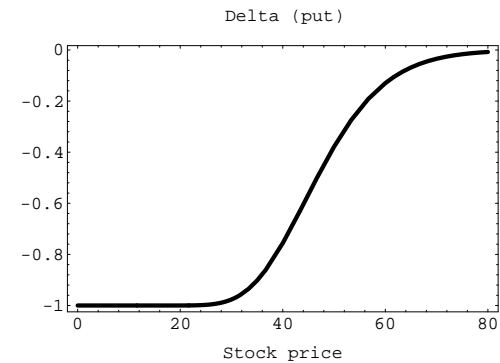
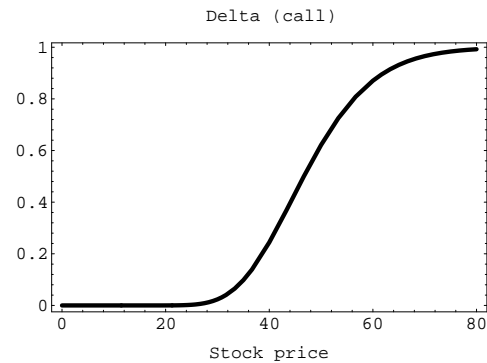
- The delta of a European call on a non-dividend-paying stock equals

$$\frac{\partial C}{\partial S} = N(x) > 0.$$

- The delta of a European put equals

$$\frac{\partial P}{\partial S} = N(x) - 1 < 0.$$

- The delta of a long stock is apparently 1.



Solid curves: at-the-money options.

Dashed curves: out-of-the-money calls or in-the-money puts.

Delta Neutrality

- A position with a total delta equal to 0 is delta-neutral.
 - A delta-neutral portfolio is immune to *small* price changes in the underlying asset.
- Creating one serves for hedging purposes.
 - A portfolio consisting of a call and $-\Delta$ shares of stock is delta-neutral.
 - Short Δ shares of stock to hedge a long call.
 - Long Δ shares of stock to hedge a short call.
- In general, hedge a position in a security with delta Δ_1 by shorting Δ_1/Δ_2 units of a security with delta Δ_2 .

Theta (Time Decay)

- Defined as the rate of change of a security's value with respect to time, or $\Theta \equiv -\partial f / \partial \tau = \partial f / \partial t$.
- For a European call on a non-dividend-paying stock,

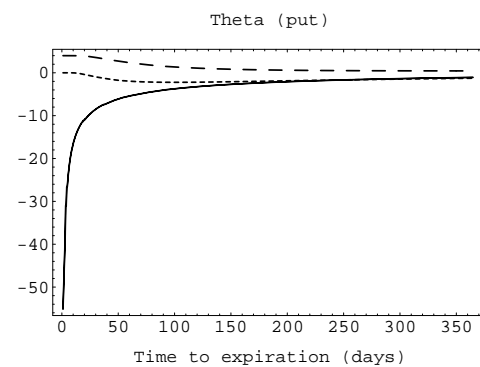
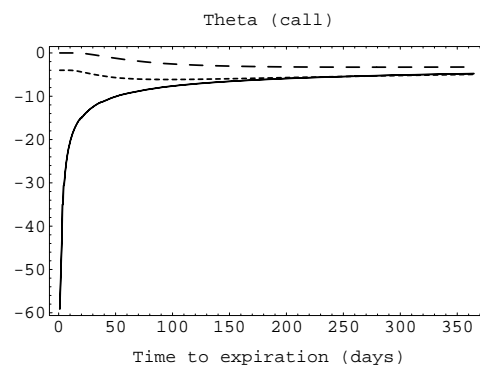
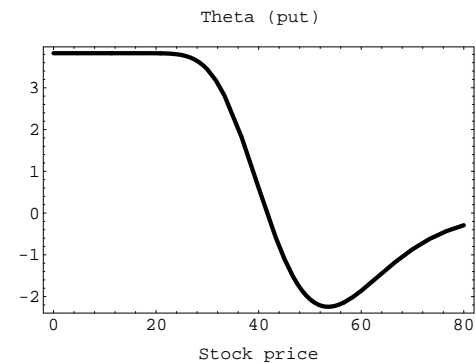
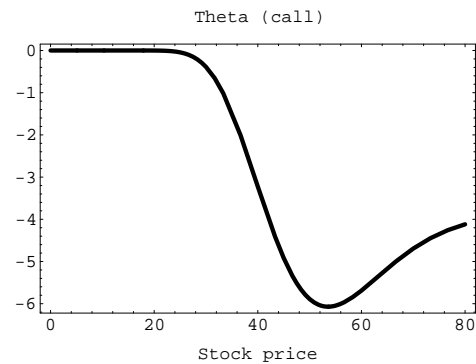
$$\Theta = -\frac{SN'(x)\sigma}{2\sqrt{\tau}} - rXe^{-r\tau}N(x - \sigma\sqrt{\tau}) < 0.$$

– The call loses value with the passage of time.

- For a European put,

$$\Theta = -\frac{SN'(x)\sigma}{2\sqrt{\tau}} + rXe^{-r\tau}N(-x + \sigma\sqrt{\tau}).$$

– Can be negative or positive.

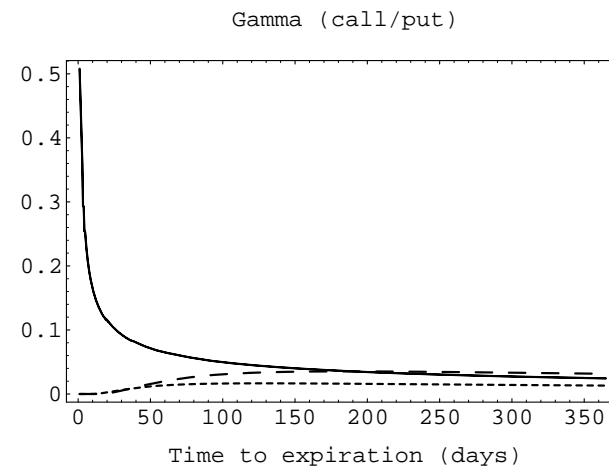
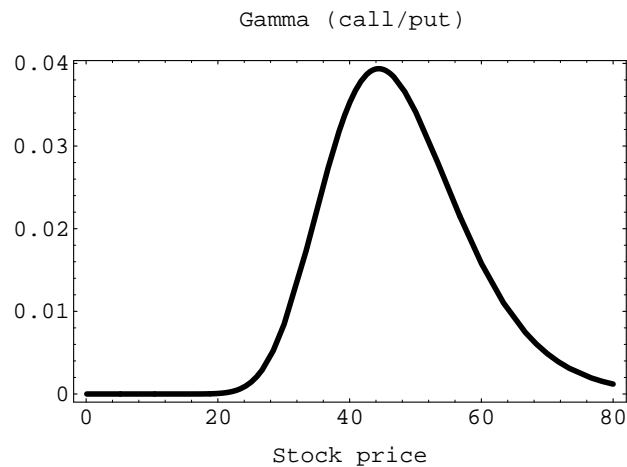


Dotted curve: in-the-money call or out-of-the-money put.
Solid curves: at-the-money options.
Dashed curve: out-of-the-money call or in-the-money put.

Gamma

- Defined as the rate of change of its delta with respect to the price of the underlying asset, or $\Gamma \equiv \partial^2 \Pi / \partial S^2$.
- Measures how sensitive delta is to changes in the price of the underlying asset.
- In practice, a portfolio with a high gamma needs be rebalanced more often to maintain delta neutrality.
- Roughly, delta \sim duration, and gamma \sim convexity.
- The gamma of a European call or put on a non-dividend-paying stock is

$$N'(x)/(S\sigma\sqrt{\tau}) > 0.$$



Dotted lines: in-the-money call or out-of-the-money put.

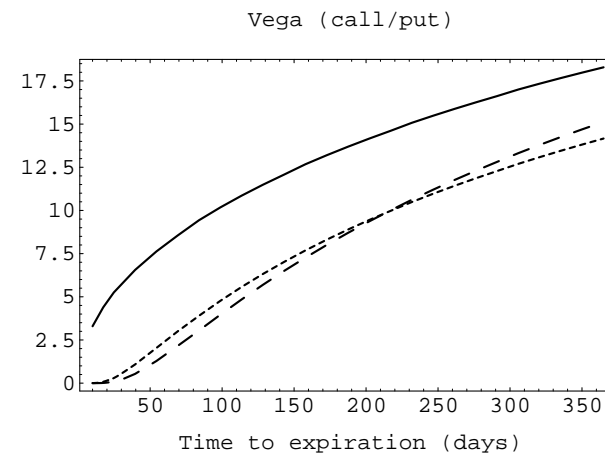
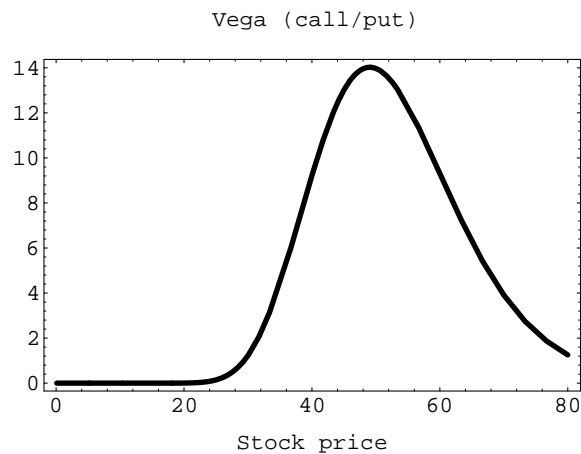
Solid lines: at-the-money option.

Dashed lines: out-of-the-money call or in-the-money put.

Vega^a (Lambda, Kappa, Sigma)

- Defined as the rate of change of its value with respect to the volatility of the underlying asset $\Lambda \equiv \partial\Pi/\partial\sigma$.
- Volatility often changes over time.
- A security with a high vega is very sensitive to small changes or estimation error in volatility.
- The vega of a European call or put on a non-dividend-paying stock is $S\sqrt{\tau} N'(x) > 0$.
 - So higher volatility always increases the option value.

^aVega is not Greek.



Dotted curve: in-the-money call or out-of-the-money put.
Solid curves: at-the-money option.
Dashed curve: out-of-the-money call or in-the-money put.

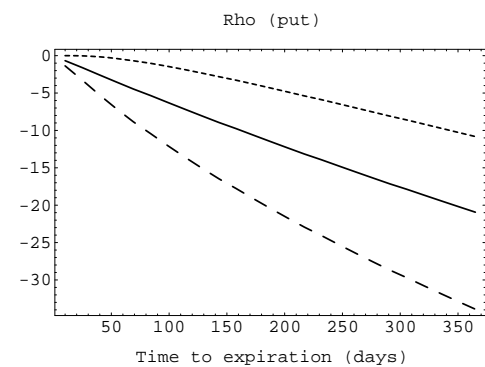
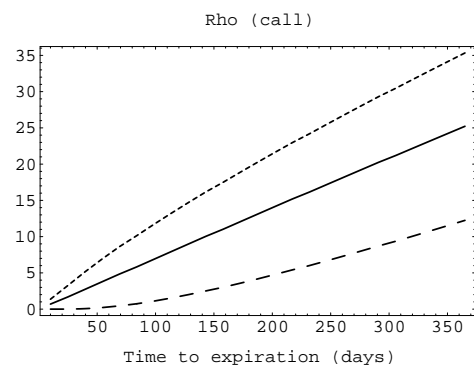
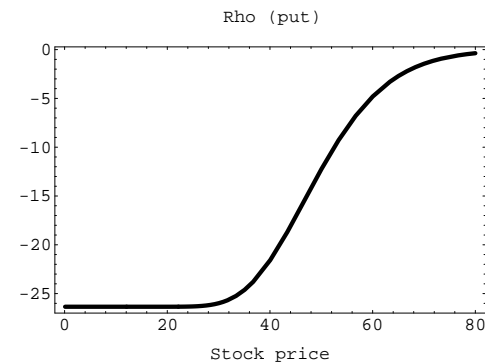
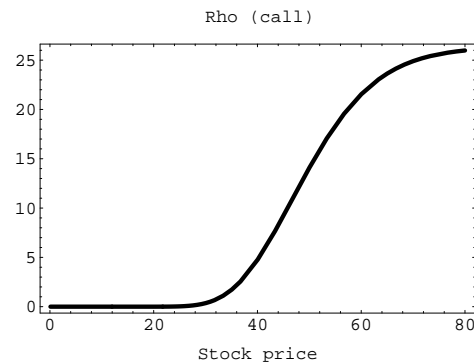
Rho

- Defined as the rate of change in its value with respect to interest rates $\rho \equiv \partial\Pi/\partial r$.
- The rho of a European call on a non-dividend-paying stock is

$$X\tau e^{-r\tau} N(x - \sigma\sqrt{\tau}) > 0.$$

- The rho of a European put on a non-dividend-paying stock is

$$-X\tau e^{-r\tau} N(-x + \sigma\sqrt{\tau}) < 0.$$



Dotted curves: in-the-money call or out-of-the-money put.

Solid curves: at-the-money option.

Dashed curves: out-of-the-money call or in-the-money put.

Numerical Greeks

- Needed when closed-form formulas do not exist.
- Take delta as an example.
- A standard method computes the finite difference,

$$\frac{f(S + \Delta S) - f(S - \Delta S)}{2\Delta S}.$$

- The computation time roughly doubles that for evaluating the derivative security itself.

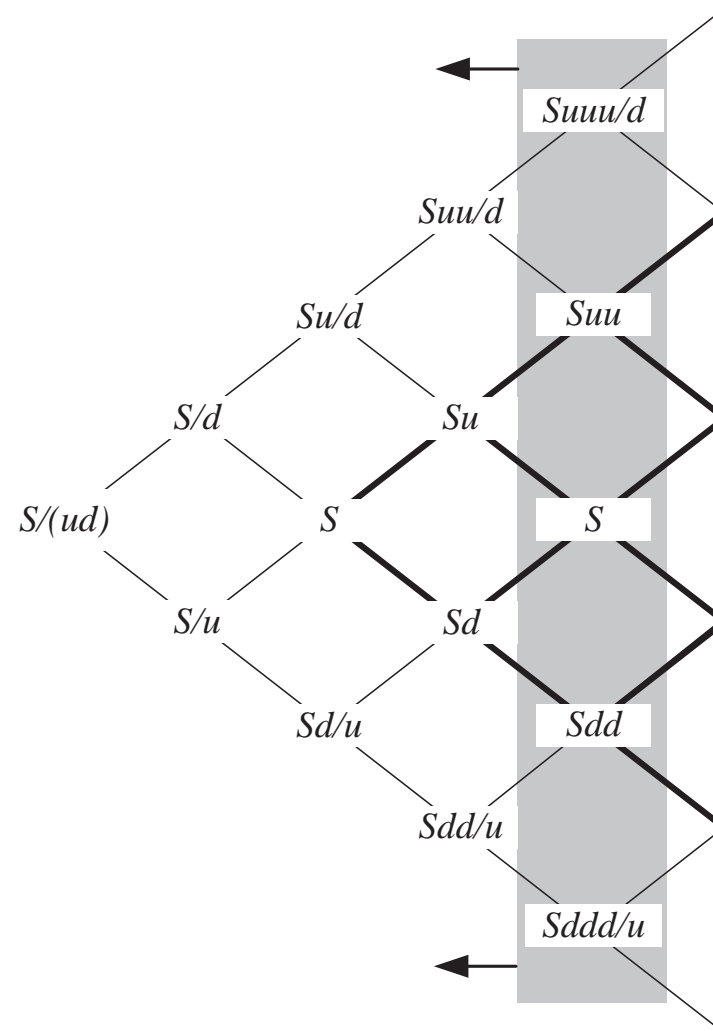
An Alternative Numerical Delta^a

- Use intermediate results of the binomial tree algorithm.
- When the algorithm reaches the end of the first period, f_u and f_d are computed.
- These values correspond to derivative values at stock prices S_u and S_d , respectively.
- Delta is approximated by

$$\frac{f_u - f_d}{S_u - S_d}.$$

- Almost zero extra computational effort.

^aPelsser and Vorst (1994).



Numerical Gamma

- At the stock price $(S_{uu} + S_{ud})/2$, delta is approximately $(f_{uu} - f_{ud})/(S_{uu} - S_{ud})$.
- At the stock price $(S_{ud} + S_{dd})/2$, delta is approximately $(f_{ud} - f_{dd})/(S_{ud} - S_{dd})$.
- Gamma is the rate of change in deltas between $(S_{uu} + S_{ud})/2$ and $(S_{ud} + S_{dd})/2$, that is,

$$\frac{\frac{f_{uu} - f_{ud}}{S_{uu} - S_{ud}} - \frac{f_{ud} - f_{dd}}{S_{ud} - S_{dd}}}{(S_{uu} - S_{dd})/2}.$$

- Alternative formulas exist.

Finite Difference Fails for Numerical Gamma

- Numerical differentiation gives

$$\frac{f(S + \Delta S) - 2f(S) + f(S - \Delta S)}{(\Delta S)^2}.$$

- It does not work (see text).
- But why did the binomial tree version work?

Other Numerical Greeks

- The theta can be computed as

$$\frac{f_{ud} - f}{2(\tau/n)}.$$

- In fact, the theta of a European option can be derived from delta and gamma (p. 540).
- For vega and rho, there seems no alternative but to run the binomial tree algorithm twice.^a

^aBut see pp. 868ff.

Extensions of Options Theory

As I never learnt mathematics,
so I have had to think.
— Joan Robinson (1903–1983)

Pricing Corporate Securities^a

- Interpret the underlying asset as the total value of the firm.
- The option pricing methodology can be applied to pricing corporate securities.
 - The result is called the structural model.
- Assumptions:
 - A firm can finance payouts by the sale of assets.
 - If a promised payment to an obligation other than stock is missed, the claim holders take ownership of the firm and the stockholders get nothing.

^aBlack and Scholes (1973).

Risky Zero-Coupon Bonds and Stock

- Consider XYZ.com.
- Capital structure:
 - n shares of its own common stock, S .
 - Zero-coupon bonds with an aggregate par value of X .
- What is the value of the bonds, B ?
- What is the value of the XYZ.com stock?

Risky Zero-Coupon Bonds and Stock (continued)

- On the bonds' maturity date, suppose the total value of the firm V^* is less than the bondholders' claim X .
- Then the firm declares bankruptcy, and the stock becomes worthless.
- If $V^* > X$, then the bondholders obtain X and the stockholders $V^* - X$.

	$V^* \leq X$	$V^* > X$
Bonds	V^*	X
Stock	0	$V^* - X$

Risky Zero-Coupon Bonds and Stock (continued)

- The stock is a call on the total value of the firm with a strike price of X and an expiration date equal to the bonds'.
 - This call provides the limited liability for the stockholders.
- The bonds are a covered call on the total value of the firm.
- Let V stand for the total value of the firm.
- Let C stand for a call on V .

Risky Zero-Coupon Bonds and Stock (continued)

- Thus $nS = C$ and $B = V - C$.
- Knowing C amounts to knowing how the value of the firm is divided between stockholders and bondholders.
- Whatever the value of C , the total value of the stock and bonds at maturity remains V^* .
- The relative size of debt and equity is irrelevant to the firm's current value V .

Risky Zero-Coupon Bonds and Stock (continued)

- From Theorem 10 (p. 261) and the put-call parity,

$$\begin{aligned}nS &= VN(x) - Xe^{-r\tau}N(x - \sigma\sqrt{\tau}), \\ B &= VN(-x) + Xe^{-r\tau}N(x - \sigma\sqrt{\tau}).\end{aligned}$$

– Above,

$$x \equiv \frac{\ln(V/X) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}.$$

- The continuously compounded yield to maturity of the firm's bond is

$$\frac{\ln(X/B)}{\tau}.$$

Risky Zero-Coupon Bonds and Stock (concluded)

- Define the credit spread or default premium as the yield difference between risky and riskless bonds,

$$\begin{aligned} & \frac{\ln(X/B)}{\tau} - r \\ &= -\frac{1}{\tau} \ln \left(N(-z) + \frac{1}{\omega} N(z - \sigma\sqrt{\tau}) \right). \end{aligned}$$

- $\omega \equiv Xe^{-r\tau}/V$.
- $z \equiv (\ln \omega)/(\sigma\sqrt{\tau}) + (1/2)\sigma\sqrt{\tau} = -x + \sigma\sqrt{\tau}$.
- Note that ω is the debt-to-total-value ratio.

A Numerical Example

- XYZ.com's assets consist of 1,000 shares of Merck as of March 20, 1995.
 - Merck's market value per share is \$44.5.
- XYZ.com's securities consist of 1,000 shares of common stock and 30 zero-coupon bonds maturing on July 21, 1995.
- Each bond promises to pay \$1,000 at maturity.
- $n = 1,000$, $V = 44.5 \times n = 44,500$, and $X = 30 \times 1,000 = 30,000$.

Option	Strike	Exp.	—Call—		—Put—	
			Vol.	Last	Vol.	Last
Merck	30	Jul	328	15 ¹ / ₄
44 ¹ / ₂	35	Jul	150	9 ¹ / ₂	10	1/16
44 ¹ / ₂	40	Apr	887	43/4	136	1/16
44 ¹ / ₂	40	Jul	220	5 ¹ / ₂	297	1/4
44 ¹ / ₂	40	Oct	58	6	10	1/2
44 ¹ / ₂	45	Apr	3050	7/8	100	11/8
44 ¹ / ₂	45	May	462	13/8	50	13/8
44 ¹ / ₂	45	Jul	883	115/16	147	13/4
44 ¹ / ₂	45	Oct	367	23/4	188	21/16

A Numerical Example (continued)

- The Merck option relevant for pricing is the July call with a strike price of $X/n = 30$ dollars.
- Such a call is selling for \$15.25.
- So XYZ.com's stock is worth $15.25 \times n = 15,250$ dollars.
- The entire bond issue is worth
 $B = 44,500 - 15,250 = 29,250$ dollars.
 - Or \$975 per bond.

A Numerical Example (continued)

- The XYZ.com bonds are equivalent to a default-free zero-coupon bond with $\$X$ par value plus n written European puts on Merck at a strike price of \$30.
 - By the put-call parity.
- The difference between B and the price of the default-free bond is the value of these puts.
- The next table shows the total market values of the XYZ.com stock and bonds under various debt amounts X .

Promised payment to bondholders	Current market value of bonds	Current market value of stock	Current total value of firm
X	B	nS	V
30,000	29,250.0	15,250.0	44,500
35,000	35,000.0	9,500.0	44,500
40,000	39,000.0	5,500.0	44,500
45,000	42,562.5	1,937.5	44,500

A Numerical Example (continued)

- Suppose the promised payment to bondholders is \$45,000.
- Then the relevant option is the July call with a strike price of $45,000/n = 45$ dollars.
- Since that option is selling for $\$1_{15/16}$, the market value of the XYZ.com stock is $(1 + 15/16) \times n = 1,937.5$ dollars.
- The market value of the stock decreases as the debt-equity ratio increases.

A Numerical Example (continued)

- There are conflicts between stockholders and bondholders.
- An option's terms cannot be changed after issuance.
- But a firm can change its capital structure.
- There lies one key difference between options and corporate securities.
 - Parameters such volatility, dividend, and strike price are under partial control of the stockholders.

A Numerical Example (continued)

- Suppose XYZ.com issues 15 more bonds with the same terms to buy back stock.
- The total debt is now $X = 45,000$ dollars.
- The table on p. 319 says the total market value of the bonds should be \$42,562.5.
- The *new* bondholders pay
 $42,562.5 \times (15/45) = 14,187.5$ dollars.
- The remaining stock is worth \$1,937.5.

A Numerical Example (continued)

- The stockholders therefore gain

$$14,187.5 + 1,937.5 - 15,250 = 875$$

dollars.

- The *original* bondholders lose an equal amount,

$$29,250 - \frac{30}{45} \times 42,562.5 = 875.$$

A Numerical Example (continued)

- Suppose the stockholders sell $(1/3) \times n$ Merck shares to fund a \$14,833.3 cash dividend.
- They now have \$14,833.3 in cash plus a call on $(2/3) \times n$ Merck shares.
- The strike price remains $X = 30,000$.
- This is equivalent to owning $2/3$ of a call on n Merck shares with a total strike price of \$45,000.
- n such calls are worth \$1,937.5 (p. 319).
- So the total market value of the XYZ.com stock is $(2/3) \times 1,937.5 = 1,291.67$ dollars.

A Numerical Example (concluded)

- The market value of the XYZ.com bonds is hence $(2/3) \times n \times 44.5 - 1,291.67 = 28,375$ dollars.
- Hence the stockholders gain

$$14,833.3 + 1,291.67 - 15,250 \approx 875$$

dollars.

- The bondholders watch their value drop from \$29,250 to \$28,375, a loss of \$875.

Further Topics

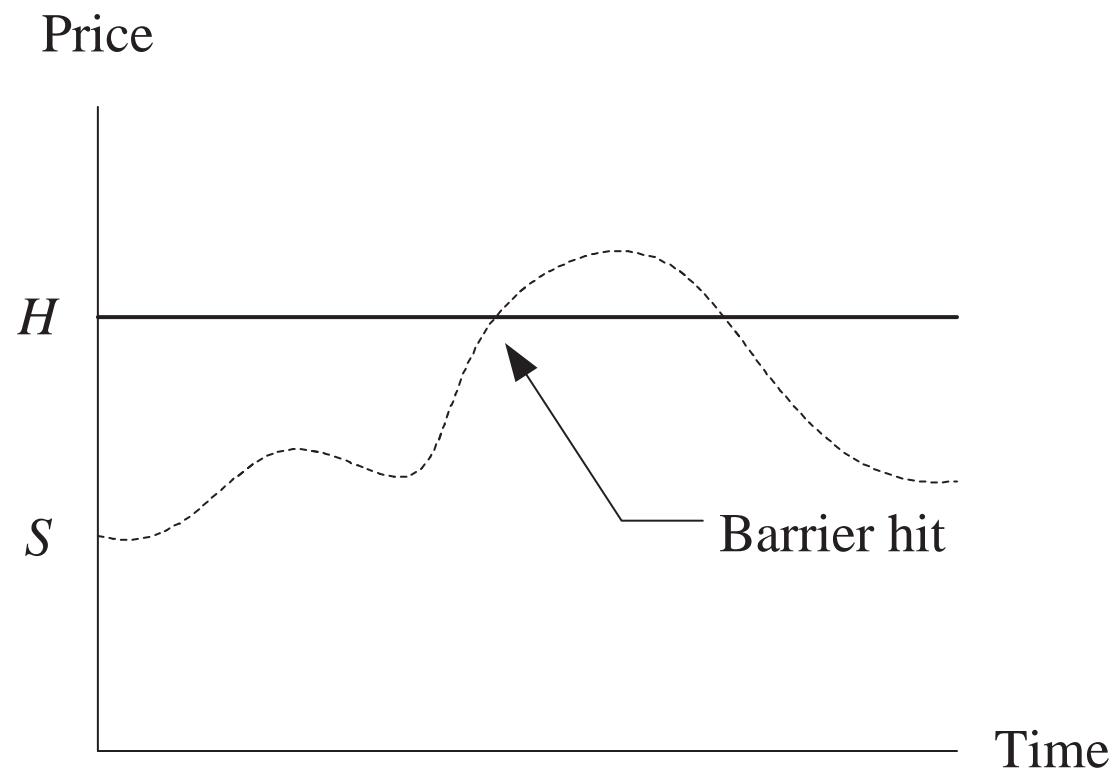
- Other Examples:
 - Subordinated debts as bull call spreads.
 - Warrants as calls.
 - Callable bonds as American calls with 2 strike prices.
 - Convertible bonds.
- Securities with a complex liability structure must be solved by trees.^a

^aDai (R86526008, D8852600), Lyuu, and Wang (F95922018) (2010).

Barrier Options^a

- Their payoff depends on whether the underlying asset's price reaches a certain price level H .
- A knock-out option is an ordinary European option which ceases to exist if the barrier H is reached by the price of its underlying asset.
- A call knock-out option is sometimes called a down-and-out option if $H < S$.
- A put knock-out option is sometimes called an up-and-out option when $H > S$.

^aA former MBA student in finance told me on March 26, 2004, that she did not understand why I covered barrier options until she started working in a bank. She was working for Lehman Brothers in HK as of April, 2006.



Barrier Options (concluded)

- A knock-in option comes into existence if a certain barrier is reached.
- A down-and-in option is a call knock-in option that comes into existence only when the barrier is reached and $H < S$.
- An up-and-in is a put knock-in option that comes into existence only when the barrier is reached and $H > S$.
- Formulas exist for all the possible barrier options mentioned above.^a

^aHaug (2006).

A Formula for Down-and-In Calls^a

- Assume $X \geq H$.
- The value of a European down-and-in call on a stock paying a dividend yield of q is

$$Se^{-q\tau} \left(\frac{H}{S}\right)^{2\lambda} N(x) - Xe^{-r\tau} \left(\frac{H}{S}\right)^{2\lambda-2} N(x - \sigma\sqrt{\tau}), \quad (30)$$

$$- x \equiv \frac{\ln(H^2/(SX)) + (r - q + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}.$$

$$- \lambda \equiv (r - q + \sigma^2/2)/\sigma^2.$$

- A European down-and-out call can be priced via the in-out parity (see text).

^aMerton (1973).

A Formula for Up-and-In Puts^a

- Assume $X \leq H$.
- The value of a European up-and-in put is

$$Xe^{-r\tau} \left(\frac{H}{S}\right)^{2\lambda-2} N(-x + \sigma\sqrt{\tau}) - Se^{-q\tau} \left(\frac{H}{S}\right)^{2\lambda} N(-x).$$

- Again, a European up-and-out put can be priced via the in-out parity.

^aMerton (1973).

Are American Options Barrier Options?^a

- American options are barrier options with the exercise boundary as the barrier and the payoff as the rebate?
- One salient difference is that the exercise boundary must be derived during backward induction.
- But the barrier in a barrier option is given a priori.

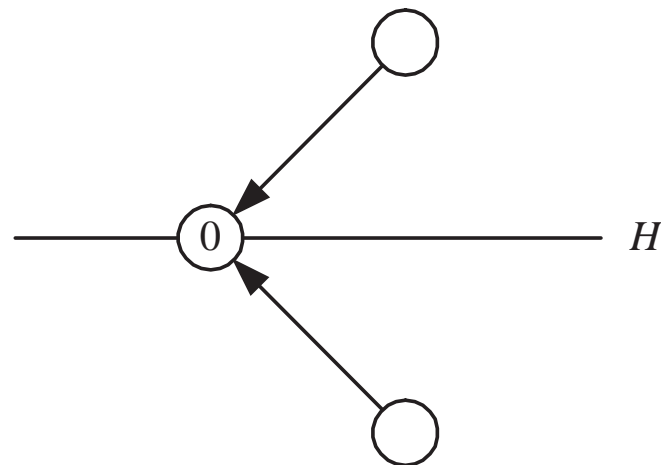
^aContributed by Mr. Yang, Jui-Chung (D97723002) on March 25, 2009.

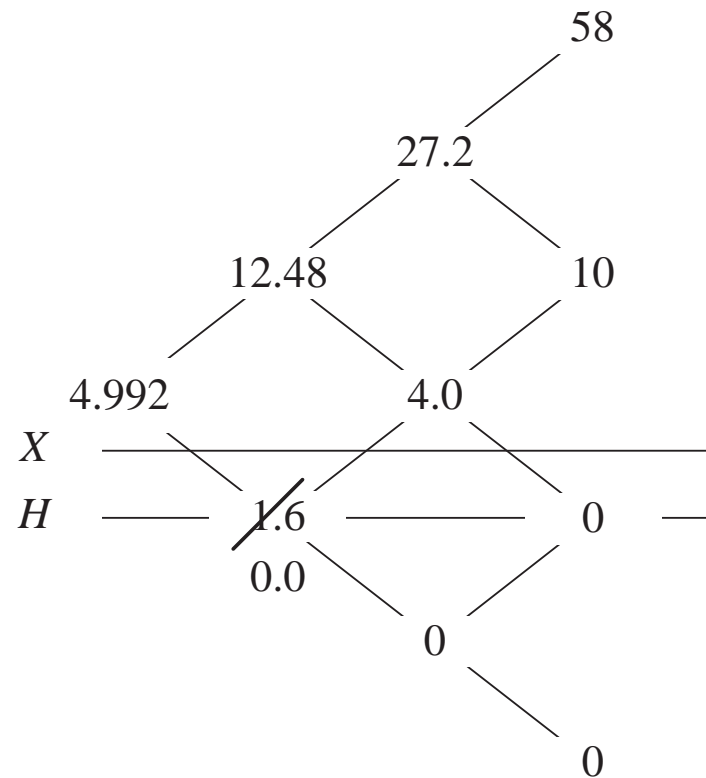
Interesting Observations

- Assume $H < X$.
- Replace S in the pricing formula for the down-and-in call, Eq. (30) on p. 330, with H^2/S .
- Equation (30) becomes Eq. (28) on p. 282 when $r - q = \sigma^2/2$.
- Equation (30) becomes S/H times Eq. (28) on p. 282 when $r - q = 0$.
- Why?

Binomial Tree Algorithms

- Barrier options can be priced by binomial tree algorithms.
- Below is for the down-and-out option.



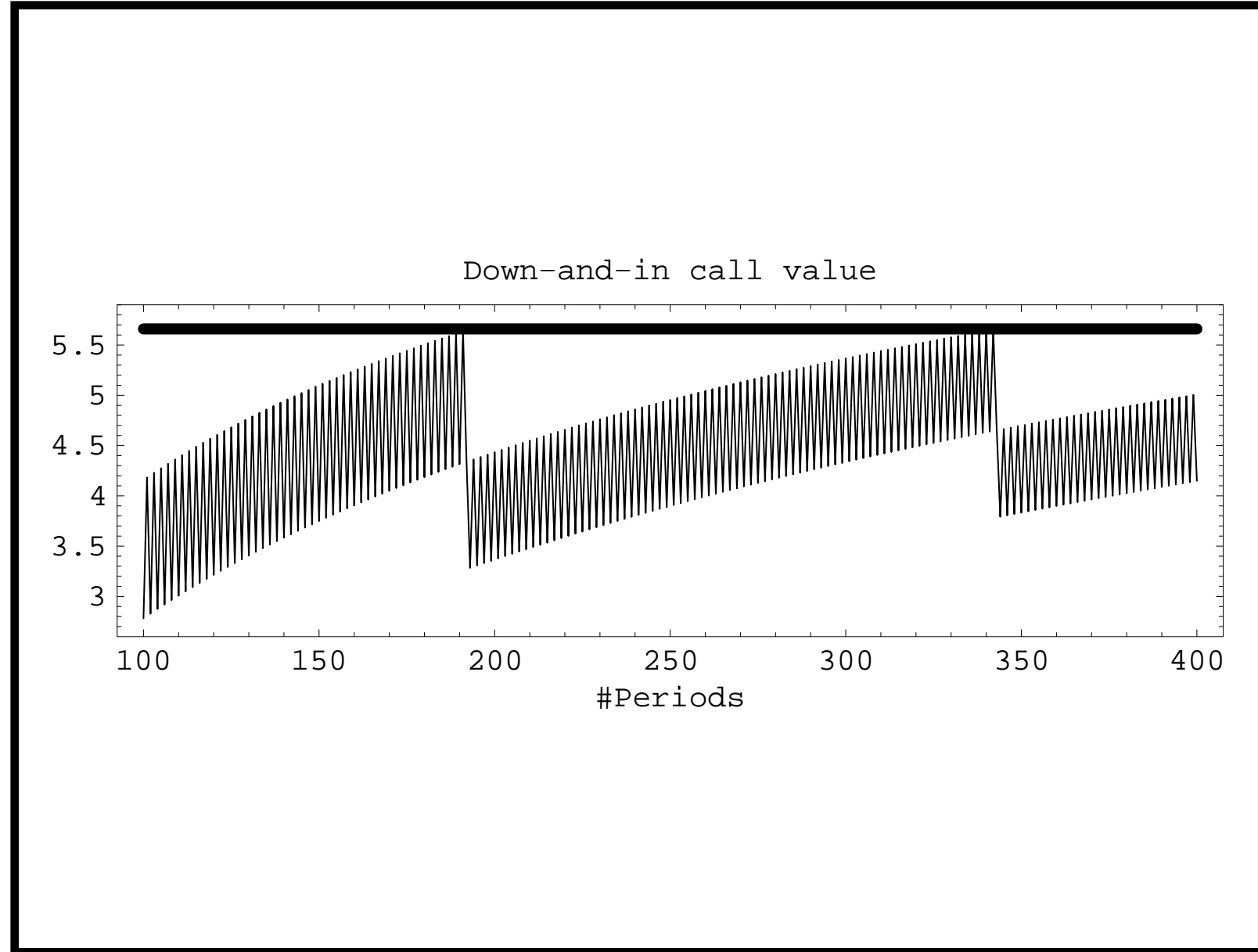


Backward-induction: $C = (0.5 \times C_u + 0.5 \times C_d)/1.25$.

Binomial Tree Algorithms (concluded)

- But convergence is erratic because H is not at a price level on the tree (see plot on next page).
 - The barrier has to be adjusted to be at a price level.
 - The “effective barrier” changes as n increases.
- In fact, the binomial tree is $O(1/\sqrt{n})$ convergent.^a
- Solutions will be presented later.

^aLin (R95221010) (2008).



Daily Monitoring

- Almost all barrier options monitor the barrier only for daily closing prices.
- If so, only nodes at the end of a day need to check for the barrier condition.
- We can even remove intraday nodes to create a multinomial tree.
 - A node is then followed by $d + 1$ nodes if each day is partitioned into d periods.
- Does this save time or space?^a

^aContributed by Ms. Chen, Tzu-Chun (R94922003) and others on April 12, 2006.

A Heptanomial Tree (6 Periods Per Day)

