Pricing Sequential-Pay CMOs: Methodology

- Suppose we have the interest rate path and the prepayment vector for that interest rate path.
- Then a CMO's cash flow can be calculated and the CMO priced.
- Unfortunately, the remaining principal of a CMO under prepayments is path dependent.
  - For example, a period of high rates before dropping to the current level is not likely to result in the same remaining principal as a period of low rates before rising to the current level.

Pricing Sequential-Pay CMOs: Methodology (concluded)

- If we try to price a 30-year CMO on a binomial interest rate model, there will be $2^{360} \approx 2.35 \times 10^{108}$ paths!
- Hence Monte Carlo simulation is the method of choice.
- First, one interest rate path is generated.
- Based on that path, the prepayment model is applied to generate the pool's principal, prepayment, and interest cash flows.
- Now, the cash flows of individual tranches can be generated and their present values derived.
- Repeat the above procedure over many interest rate scenarios and average the present values.

MBS Valuation Methodologies

1. Static cash flow yield.
2. Option modeling.
3. Option-adjusted spread (OAS).

Cash Flow Yield

- To price an MBS, one starts with its cash flow: The periodic P&I under a static prepayment assumption as given by a prepayment vector.
- The invoice price is now

$$\sum_{i=1}^{n} C_i/(1+r)^{\omega-1+i}.$$

  - C_i is the cash flow at time i.
  - n is the weighted average maturity (WAM).
  - r is the discount rate.
  - ω is the fraction of period from settlement until the first P&I payment date.

Cash Flow Yield (continued)

- The r that equates the above with the market price is called the (static) cash flow yield.
- The static cash flow yield methodology compares the cash flow yield on an MBS with that on "comparable" bonds.
- The implied PSA is the single PSA speed producing the same cash flow yield.[a]

[a]Fabozzi (1991).

Cash Flow Yield (concluded)

- This simple methodology has obvious weaknesses (some generic).
  - It is static.
  - The projected cash flow may not be reinvested at the cash flow yield.[a]
  - The MBS may not be held until the final payout date.
  - The actual prepayment behavior is likely to deviate from the assumptions.

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[a]This deficiency can be remedied somewhat by adopting the static spread methodology on p. 115.

The Option Pricing Methodology

- Virtually all mortgage loans give the homeowner the right to prepay the mortgage at any time.
- The totality of these rights to prepay constitutes the embedded call option of the pass-through.
- In contrast, the MBS investor is short the embedded call.
- Therefore,

$$\begin{aligned} & \textsf{pass-through price} \\ = \; & \textsf{noncallable pass-through price} - \textsf{call option price.} \end{aligned}$$

The Option Pricing Methodology (continued)

- The option pricing methodology prices the call option by an option pricing model.
- It then estimates the market price of the noncallable pass-through by

$$\begin{aligned} & \textsf{noncallable pass-through price} \\ = \; & \textsf{pass-through price} + \textsf{call option price.} \end{aligned}$$

- The above price is finally used to compute the yield on this theoretical bond which does not prepay.
- This yield is called the option-adjusted yield.

The Option Pricing Methodology (continued)

- The option pricing methodology suffers from several difficulties (some generic).
  - The Black-Scholes model is not satisfactory for pricing fixed-income securities.[a]
  - There may not exist a benchmark to compare the option-adjusted yield with to obtain the yield spread.
  - This methodology does not incorporate the shape of the yield curve.

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[a] See Section 24.7 of the textbook.

The Option Pricing Methodology (concluded)

- (continued)
  - Prepayment options are often "irrationally" exercised.
  - A partial exercise is possible as the homeowner can prepay a portion of the loan.
    * There is not one option but many, one per homeowner.
  - Valuation of the call option becomes very complicated for CMO bonds.

The OAS Methodology[a]

- The OAS methodology has four major components.
- The interest rate model is the first component.
- The second component is the prepayment model.[b]
  - Deterministic models that are accurate on average seem good enough for pass-throughs, IOs, and POs.[c]
- The OAS methodology is meant to identify investments with the best potential for excess returns.

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[a] The idea can be traced to Waldman and Modzelewski (1985), if not earlier.

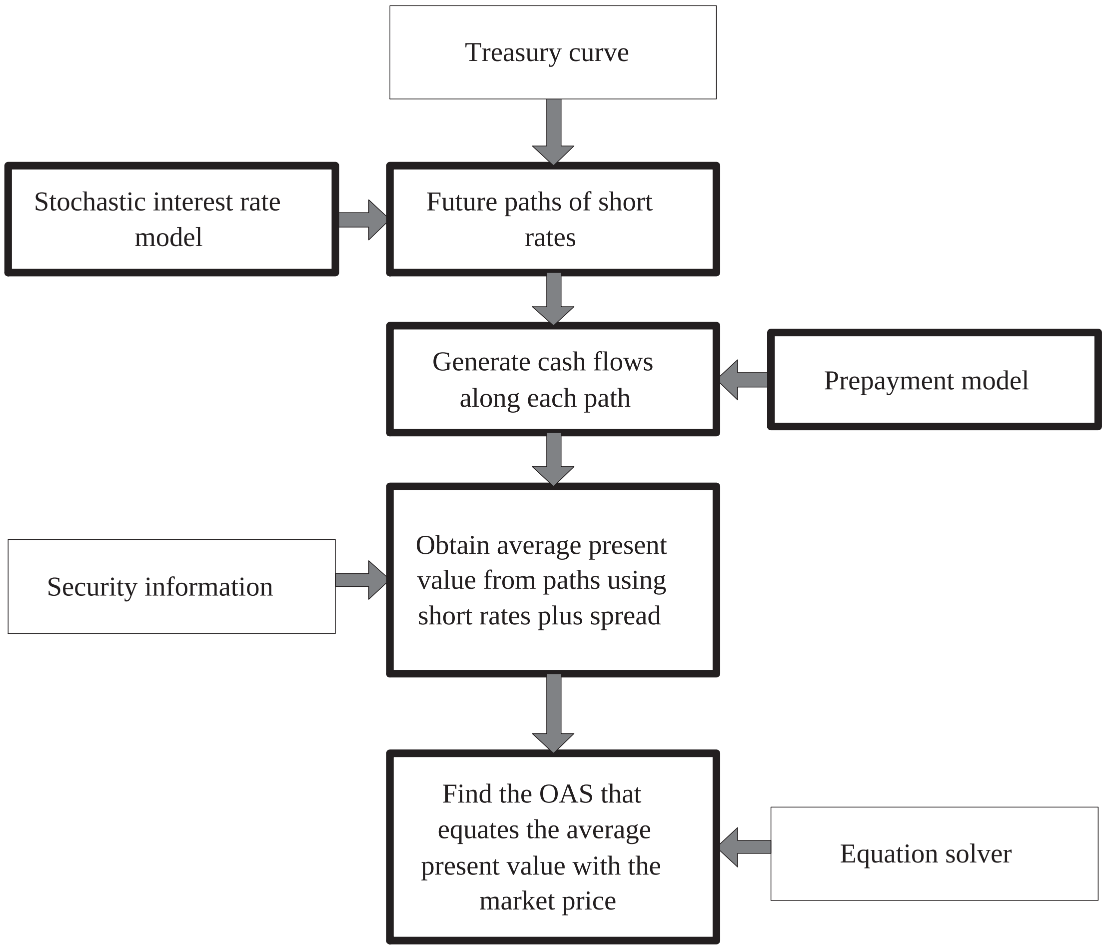
[b] May be the single most important component.

[c] Hayre (1997) and Hayre and Rajan (1995).

The OAS Methodology (continued)

- The cash flow generator is the third component.
  - It calculates the current coupon rates for the interest rate paths given by the interest rate model.
  - It then generates the P&I cash flows for the pool as well as allocating them for individual securities based on the prepayment model and security information.
  - Note that the pool cash flow drives many securities.
- Finally, the equation solver calculates the OAS.

The OAS Methodology (continued)



The OAS Methodology (continued)

- The general valuation formula for uncertain cash flows can be written as

$$\text{PV} = \lim_{N \to \infty} \frac{1}{N} \sum_{N \text{ paths } r^*} \sum_n \frac{C_n^*}{(1+r_1^*)(1+r_2^*)\cdots(1+r_n^*)}. \tag{139}$$

  – r^* denotes a risk-neutral interest rate path for which r_i^* is the ith one-period rate.

  – C_n^* is the cash flow at time n under this scenario.

The OAS Methodology (continued)

- The Monte Carlo calculation of OAS is closely related to Eq. (139) on p. 1149.
- The interest rate model randomly produces a set of risk-neutral rate paths.
- The cash flow is then generated for each path.
- Finally, we solve for the spread s that makes the average discounted cash flow equal the market price,

$$P = \lim_{N \to \infty} \frac{1}{N} \sum_{N \text{ paths } r^*} \sum_{n} \frac{C_n^*}{(1 + r_1^* + s)(1 + r_2^* + s) \cdots (1 + r_n^* + s)}.$$

- This spread s is the OAS.

The OAS Methodology (continued)

- A common alternative averages the cash flows first and then calculates the OAS as the spread that equates this average cash flow with the market price.
- Although this approach is more efficient, it will generally give a different spread.

The OAS Methodology (continued)

- OAS calculation is very time consuming.
- The majority of the cost lies in generating the cash flows.
- This is because CMOs can become arbitrarily complex in their rules for allocating the cash flows.
- Such complexity requires special care in software design.

The OAS Methodology (concluded)

- Although extremely popular, the OAS methodology suffers from several difficulties.
  - It may be difficult to interpret the OAS number.
  - The spread must be a constant.
  - It assumes the cash flow can be reinvested with the same OAS.

Collateralized Mortgage Obligations

Capital can be understood only as motion,
not as a thing at rest.
— Karl Marx (1818–1883)

CMOs

- The complexity of a CMO arises from layering different types of payment rules on a prioritized basis.
- In the first-generation CMOs, the sequential-pay CMOs, each class of bond would be retired sequentially.
- A sequential-pay CMO with a large number of tranches will have very narrow cash flow windows for the tranches.
- To further reduce prepayment risk, tranches with a principal repayment schedule were introduced.
- They are called scheduled bonds.

CMOs (continued)

- For example, bonds that guarantee the repayment schedule when the actual prepayment speed lies within a specified range are known as planned amortization class bonds (PACs).[a]
- PACs offer protection against both contraction and extension risks.
- But some investors may desire only protection from one of these risks.
- For them, a bond class known as the targeted amortization class (TAC) was created.

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[a] Introduced in August 1986.

CMOs (continued)

- Scheduled bonds expose certain CMO classes to less prepayment risk.
- However, this can occur only if the redirection in the prepayment risk is absorbed as much as possible by other classes known as the support bonds.
- Pro rata bonds provide another means of layering.
- Principal cash flows to these bonds are divided proportionally, but the bonds can have different interest payment rules.

CMOs (continued)

- Suppose the weighted average coupon (WAC) of the collateral is 10%, tranche B1 receives 40% of the principal, and tranche B2 receives 60% of the principal.
- Given this pro rata structure, many choices of interest payment rules are possible for B1 and B2 as long as the interest payments are nonnegative and the WAC does not exceed 10%.
- The coupon rates can even be floating.

CMOs (continued)

- One possibility is for B1 to have a coupon of 5% and for B2 to have a coupon of 13.33%.
- This works because

$$\frac{40}{100} \times 5\% + \frac{60}{100} \times 13.33\% = 10\%.$$

- Bonds with pass-through coupons that are higher and lower than the collateral coupon have thus been created.
- Bonds like B1 are called synthetic discount securities.
- Bonds like B2 are called synthetic premium securities.

CMOs (continued)

- An extreme case is for B1 to receive 99% of the principal and have a 5% coupon and for B2 to receive only 1% of the principal and have a 505% coupon.[a]
- IOs have either a nominal principal or a notional principal.
- A nominal principal represents actual principal that will be paid.
- It is called "nominal" because it is extremely small, resulting in an extremely high coupon rate.

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[a]First-generation IOs issued by Fannie Mae took the form of B2 in July 1986.

CMOs (concluded)

- A case in point is the B2 class with a 505% coupon above.
- A notional principal, in contrast, is the amount on which interest is calculated.
- An IO holder owns none of the notional principal.
- Once the notional principal amount declines to zero, no further payments are made on the IO.

Floating-Rate Tranches[a]

- A form of pro rata bonds are floaters and inverse floaters whose combined coupon does not exceed the collateral coupon.
- A floater is a class whose coupon rate varies directly with the change in the reference rate.
- An inverse floater is a class whose coupon rate changes in the direction opposite to the change in the reference rate.
- When the coupon on the inverse floater changes by x times the amount of the change in the reference rate, this multiple x is called its slope.

[a]Created in September 1986.

Floating-Rate Tranches (continued)

- Because the interest comes from fixed-rate mortgages, floaters must have a coupon cap.
- Similarly, inverse floaters must have a coupon floor.
- Suppose the floater has a principal of $P_{\rm f}$ and the inverse floater has a principal of $P_{\rm i}$.
- Define

$$
\begin{aligned}
\omega_{\rm f} &\equiv P_{\rm f}/(P_{\rm f}+P_{\rm i}),\\
\omega_{\rm i} &\equiv P_{\rm i}/(P_{\rm f}+P_{\rm i}).
\end{aligned}
$$

Floating-Rate Tranches (concluded)

- The coupon rates of the floater, c_{f}, and the inverse floater, c_{i}, must satisfy $\omega_{\mathrm{f}} \times c_{\mathrm{f}} + \omega_{\mathrm{i}} \times c_{\mathrm{i}} = \mathrm{WAC}$, or

$$c_{\mathrm{i}} = \frac{\mathrm{WAC} - \omega_{\mathrm{f}} \times c_{\mathrm{f}}}{\omega_{\mathrm{i}}}.$$

- The slope is clearly $\omega_{\mathrm{f}}/\omega_{\mathrm{i}}$.
- To make sure that the inverse floater will not encounter a negative coupon, the cap on the floater must be less than $\mathrm{WAC}/\omega_{\mathrm{f}}$.
- In fact, caps and floors are related via

$$\mathsf{floor} = \frac{\mathrm{WAC} - \omega_{\mathrm{f}} \times \mathsf{cap}}{\omega_{\mathrm{i}}}.$$

An Example

- Take a CMO deal that includes a floater with a principal of \$64 million and an inverse floater with a principal of \$16 million.
- The coupon rate for the floating-rate class is $\text{LIBOR} + 0.65$.
- The coupon rate for the inverse floater is $42.4 - 4 \times \text{LIBOR}$.
- The slope is thus four.

An Example (concluded)

- The WAC of the two classes is

$$\frac{64}{80} \times \textsf{floater coupon rate} + \frac{16}{80} \times \textsf{inverse floater coupon rate} = 9\%$$

  regardless of the level of the LIBOR.

- Consequently, the coupon rate on the underlying collateral, 9%, can support the aggregate interest payments that must be made to these two classes.

- If we set a floor of 0% for the inverse floater, the cap on the floater is 11.25%.

Superfloaters

- A variant of the floating-rate CMO is the superfloater introduced in 1987.
- In a conventional floating-rate class, the coupon rate moves up or down on a one-to-one basis with the reference rate.
- A superfloater's coupon rate, in comparison, changes by some multiple of the change in the reference rate.
  - It magnifies any changes in the value of the reference rate.
- Superfloater tranches are bearish because their value generally appreciates with rising interest rates.

An Example

- Suppose the initial LIBOR is 7% and the coupon rate for a superfloater is set by

$$(\text{initial LIBOR} - 40 \text{ basis points}) + 2 \times (\text{change in LIBOR}).$$

- The following table shows how the superfloater changes its coupon rate as LIBOR changes.
  - The coupon rates for a conventional floater of LIBOR plus 50 basis points are also listed for comparison.

LIBOR change (basis points)	−300	−200	−100	0	+100	+200	+300
Superfloater	0.6	2.6	4.6	6.6	8.6	10.6	12.6
Conventional floater	4.5	5.5	6.5	7.5	8.5	9.5	10.5

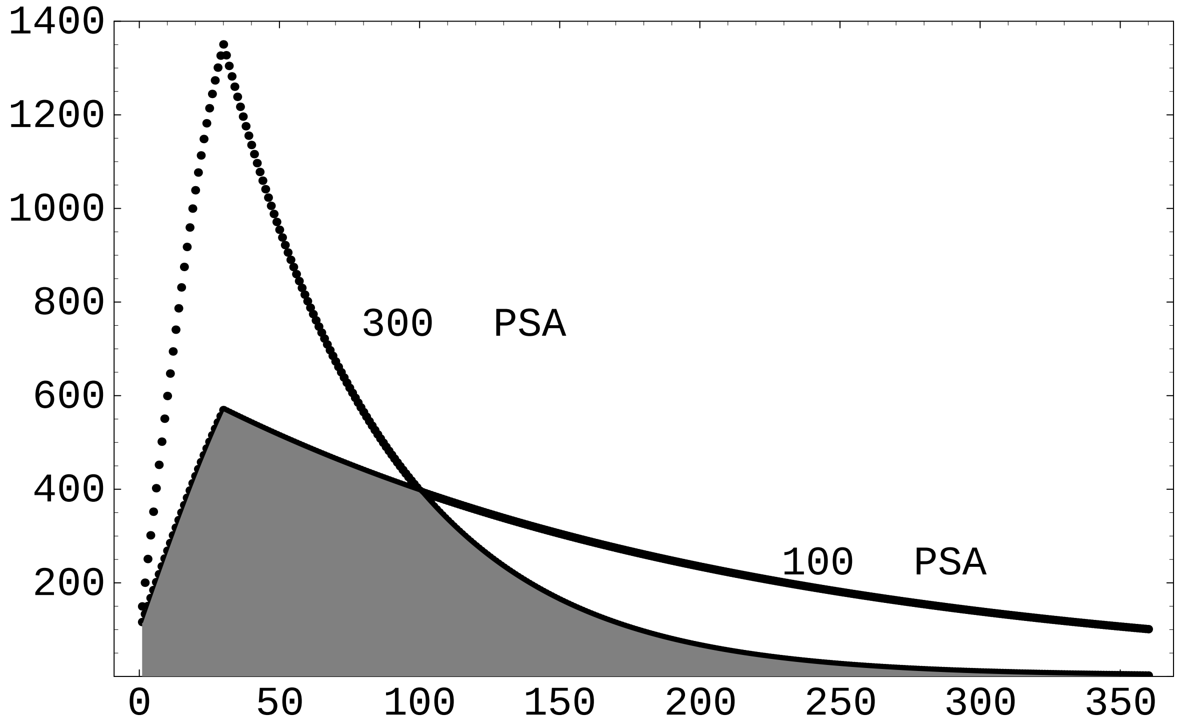
An Example (concluded)

- A superfloater provides a much higher yield than a conventional floater when interest rates rise.
- It provides a much lower yield when interest rates fall or remain stable.
- This can be verified by looking at the above table via spreads in basis points to the LIBOR in the next table.

LIBOR change (basis points)	−300	−200	−100	0	+100	+200	+300
Superfloater	−340	−240	−140	−40	60	160	260
Conventional floater	50	50	50	50	50	50	50

PAC Bonds

- PAC bonds are created by calculating the cash flows from the collateral by use of two prepayment speeds: a fast one and a slow one.
- Consider a PAC band of 100 PSA (the lower collar) to 300 PSA (the upper collar).
- The plot on p. 1172 shows the principal payments at the two collars.
- The principal payments under the higher-speed scenario are higher in the earlier years but lower in later years.



The underlying mortgages are 30-year ones with a total original loan amount of $100,000,000 (the numbers on the y axis are in thousands) and a coupon rate of 6%.

PAC Bonds (continued)

- The shaded area represents the principal payment schedule that is "guaranteed" for every possible prepayment speed between 100% and 300% PSAs.
- It is calculated by taking the minimum of the principal paydowns at the lower and upper collars.
- This schedule is called the PAC schedule.
- See Figure 30.2 in the text for a linear-time cash flow generator.

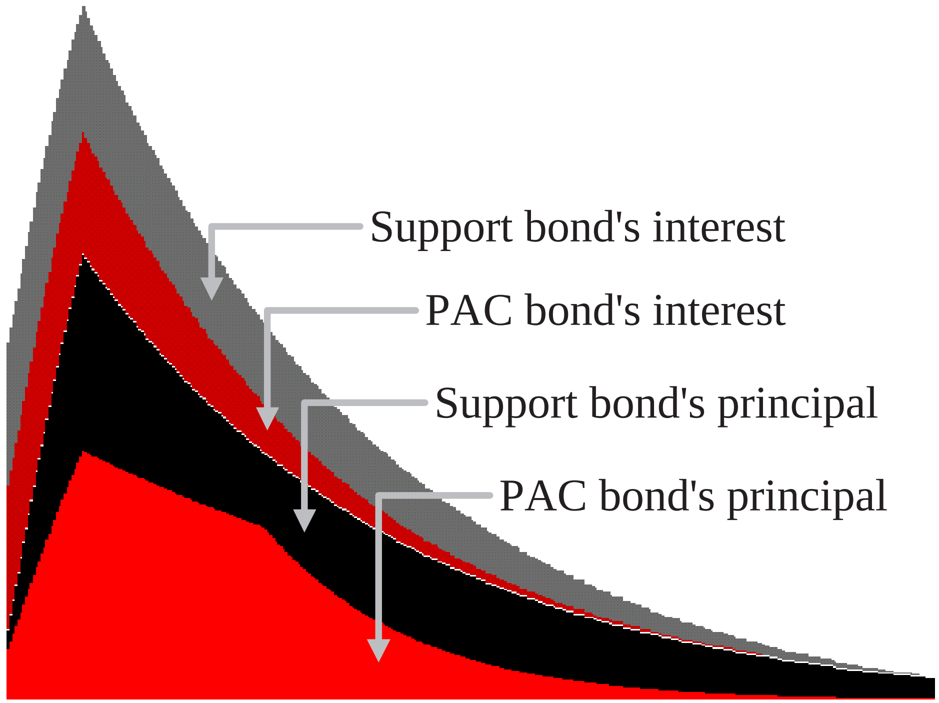
PAC Bonds (continued)

- Adherence to the amortization schedule of the PAC takes priority over those of all other bonds.
- The cash flow of a PAC bond is therefore known as long as its support bonds are not fully paid off.
- Whether this happens depends to a large extent on the CMO structure, such as priority and the relative sizes of PAC and non-PAC classes.
- For example, a relatively small PAC is harder to break than a larger PAC, other things being equal.

PAC Bonds (continued)

- If the actual prepayment speed is 150 PSA, the principal payment pattern of the PAC bond adheres to the PAC schedule.
- The cash flows of the support bond "flow around" the PAC bond (see the plot on p. 1176).
- The cash flows are neither sequential nor pro rata.
- In fact, the support bond pays down *simultaneously* with the PAC bond.
- Because more than one class of bonds may be receiving principal payments at the same time, structures with PAC bonds are simultaneous-pay CMOs.

Cash Flow of a PAC Bond at 150 PSA



The mortgage rate is 6%, the PAC band is 100 PSA to 300 PSA, and the actual prepayment speed is 150 PSA.

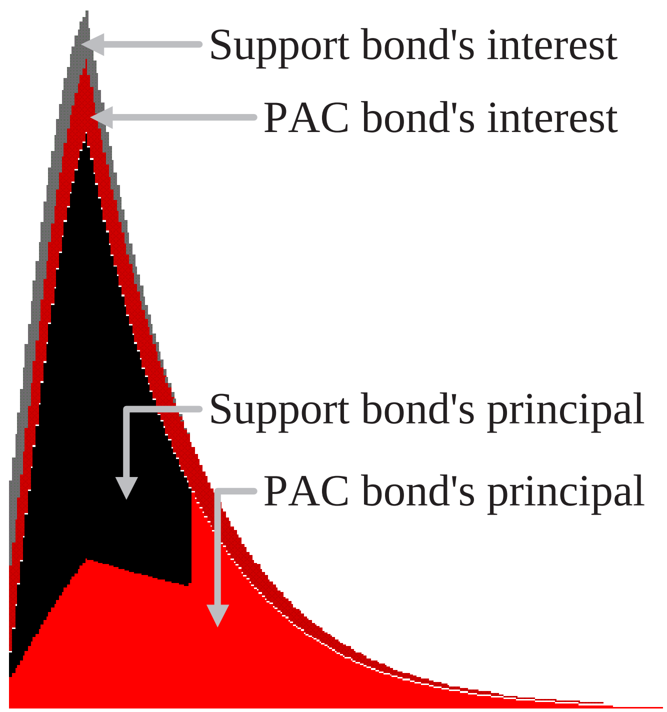
PAC Bonds (continued)

- At the lower prepayment speed of 100 PSA, far less principal cash flow is available in the early years of the CMO.
- As all the principal cash flows go to the PAC bond in the early years, the principal payments on the support bond are deferred and the support bond extends.

PAC Bonds (continued)

- If prepayments move outside the PAC band, the PAC schedule may not be met.
- At 400 PSA, for example, the cash flows to the support bond are accelerated.
- After the support bond is fully paid off, all remaining principal payments go to the PAC bond; its life is shortened.
- See the plot on p. 1179.

Cash Flow of a PAC Bond at 400 PSA



The mortgage rate is 6%, the PAC band is 100 PSA to 300 PSA, and the actual prepayment speed is 400 PSA.

PAC Bonds (concluded)

- The support bond thus absorbs part of the contraction risk.
- Similarly, should the actual prepayment speed fall below the lower collar, then in subsequent periods the PAC bond has priority on the principal payments.
- This reduces the extension risk, which is again absorbed by the support bond.

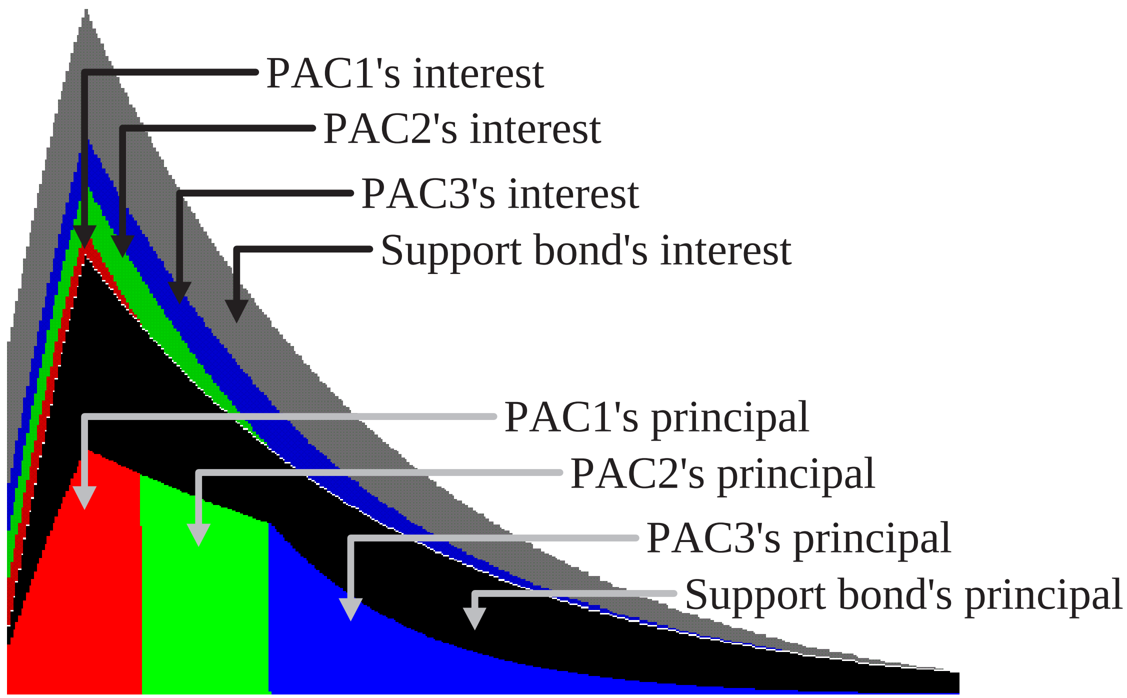
PAC Drift

- The PAC band guarantees that if prepayments occur at any single constant speed within the band *and* stay there, the PAC schedule will be met.
- However, the PAC schedule may not be met even if prepayments on the collateral always vary within the band over time.
- This is because the band that guarantees the original PAC schedule can expand and contract, depending on actual prepayments.
- This phenomenon is known as PAC drift.

Sequential PACs

- PACs can be divided sequentially to provide narrower paydown structures.
- These sequential PACs narrow the range of years over which principal payments occur.
- See the plot on p. 1183.
- Although these bonds are all structured with the same band, the actual range of speeds over which their schedules will be met may differ.

Cash Flow of a Sequential PAC Bond



The mortgage rate is 6%, the PAC band is 100 PSA to 300 PSA, and the actual prepayment speed is 150 PSA. The three PAC bonds have identical original principal amounts.

Sequential PACs (concluded)

- We can take a CMO bond and further structure it.
- For example, the sequential PACs can be split by use of a pro rata structure to create high and low coupon PACs.
- We can also replace the B tranche in a four-tranche ABCZ sequential CMO with a PAC class that amortizes starting in year four, say.
- But note that tranche C may start to receive prepayments that are in excess of the schedule of the PAC bond.
- It may even be retired earlier than tranche B.

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