Toward the Black-Scholes Formula

- The binomial model seems to suffer from two unrealistic assumptions.
  - The stock price takes on only two values in a period.
  - Trading occurs at discrete points in time.
- As n increases, the stock price ranges over ever larger numbers of possible values, and trading takes place nearly continuously.
- Any proper calibration of the model parameters makes the BOPM converge to the continuous-time model.
- We now skim through the proof.

Toward the Black-Scholes Formula (continued)

- Let τ denote the time to expiration of the option measured in years.
- Let r be the continuously compounded annual rate.
- With n periods during the option's life, each period represents a time interval of τ/n.
- Need to adjust the period-based u, d, and interest rate $\hat{r}$ to match the empirical results as n goes to infinity.
- First, $\hat{r} = r\tau/n$.
  - The period gross return $R = e^{\hat{r}}$.

Toward the Black-Scholes Formula (continued)

- Use

$$\widehat{\mu} \equiv \frac{1}{n}\, E\left[\ln \frac{S_\tau}{S} \right] \quad \text{and} \quad \widehat{\sigma}^2 \equiv \frac{1}{n}\, \text{Var}\left[\ln \frac{S_\tau}{S} \right]$$

  to denote, resp., the expected value and variance of the continuously compounded rate of return per period.

- Under the BOPM, it is not hard to show that

$$\begin{aligned} \widehat{\mu} &= q \ln(u/d) + \ln d, \\ \widehat{\sigma}^2 &= q(1-q) \ln^2(u/d). \end{aligned}$$

Toward the Black-Scholes Formula (continued)

- Assume the stock's true continuously compounded rate of return over τ years has mean $\mu\tau$ and variance $\sigma^2\tau$.
  - Call σ the stock's (annualized) volatility.
- The BOPM converges to the distribution only if

$$
\begin{aligned}
n\widehat{\mu} &= n(q\ln(u/d) + \ln d) \to \mu\tau, \\
n\widehat{\sigma}^2 &= nq(1-q)\ln^2(u/d) \to \sigma^2\tau.
\end{aligned}
$$

- Impose $ud = 1$ to make nodes at the same horizontal level of the tree have identical price (review p. 233).
  - Other choices are possible (see text).

Toward the Black-Scholes Formula (continued)

- The above requirements can be satisfied by

$$u = e^{\sigma\sqrt{\tau/n}}, \quad d = e^{-\sigma\sqrt{\tau/n}}, \quad q = \frac{1}{2} + \frac{1}{2}\,\frac{\mu}{\sigma}\sqrt{\frac{\tau}{n}}\,. \quad (24)$$

- With Eqs. (24),

$$\begin{aligned} n\widehat{\mu} &= \mu\tau, \\ n\widehat{\sigma}^2 &= \left[1 - \left(\frac{\mu}{\sigma} \right)^2 \frac{\tau}{n} \right] \sigma^2\tau \to \sigma^2\tau. \end{aligned}$$

Toward the Black-Scholes Formula (continued)

- The no-arbitrage inequalities $u > R > d$ may not hold under Eqs. (24) on p. 242.
  - If this happens, the risk-neutral probability may lie outside $[\,0, 1\,]$.
- The problem disappears when n satisfies

$$e^{\sigma \sqrt{\tau/n}} > e^{r\tau/n},$$

or when $n > r^2\tau/\sigma^2$ (check it).

  - So it goes away if n is large enough.
  - Other solutions will be presented later.

Toward the Black-Scholes Formula (continued)

- What is the limiting probabilistic distribution of the continuously compounded rate of return $\ln(S_\tau/S)$?
- The central limit theorem says $\ln(S_\tau/S)$ converges to the normal distribution with mean $\mu\tau$ and variance $\sigma^2\tau$.
- So $\ln S_\tau$ approaches the normal distribution with mean $\mu\tau + \ln S$ and variance $\sigma^2\tau$.
- S_τ has a lognormal distribution in the limit.

Toward the Black-Scholes Formula (continued)

Lemma 7 *The continuously compounded rate of return* $\ln(S_\tau/S)$ *approaches the normal distribution with mean* $(r-\sigma^2/2)\,\tau$ *and variance* $\sigma^2\tau$ *in a risk-neutral economy.*

- Let q equal the risk-neutral probability $p \equiv (e^{r\tau/n}-d)/(u-d)$.
- Let $n \to \infty$.

Toward the Black-Scholes Formula (continued)

- By Lemma 7 (p. 245) and Eq. (18) on p. 149, the expected stock price at expiration in a risk-neutral economy is $Se^{r\tau}$.
- The stock's expected annual rate of return[a] is thus the riskless rate r.

---

[a] In the sense of $(1/\tau) \ln E[\, S_\tau / S\,]$ (arithmetic average rate of return) not $(1/\tau)E[\,\ln(S_\tau/S)\,]$ (geometric average rate of return).

Toward the Black-Scholes Formula (concluded)

Theorem 8 (The Black-Scholes Formula)

$$\begin{aligned} C &= SN(x) - Xe^{-r\tau} N(x - \sigma\sqrt{\tau}), \\ P &= Xe^{-r\tau} N(-x + \sigma\sqrt{\tau}) - SN(-x), \end{aligned}$$

where

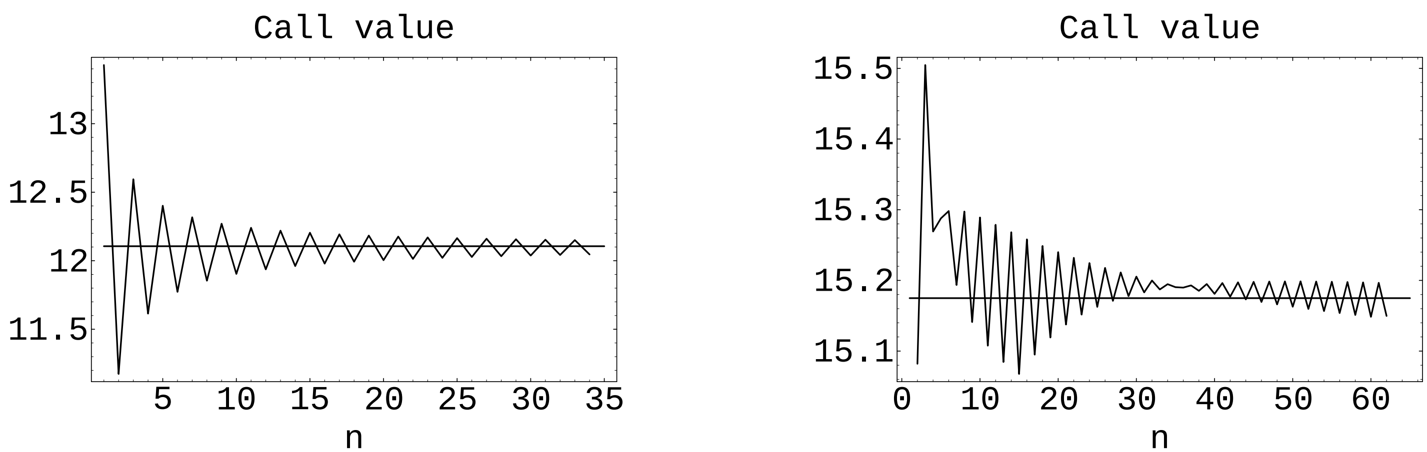
$$x \equiv \frac{\ln(S/X) + \left(r + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}.$$

BOPM and Black-Scholes Model

- The Black-Scholes formula needs five parameters: S, X, σ, τ, and r.
- Binomial tree algorithms take six inputs: S, X, u, d, $\hat{r}$, and n.
- The connections are

$$u = e^{\sigma\sqrt{\tau/n}},\ d = e^{-\sigma\sqrt{\tau/n}},\ \hat{r} = r\tau/n.$$

- The binomial tree algorithms converge reasonably fast.
- Oscillations can be dealt with by the judicious choices of u and d (see text).



$S = 100$, $X = 100$ (left), and $X = 95$ (right).

Implied Volatility

- Volatility is the sole parameter not directly observable.
- The Black-Scholes formula can be used to compute the market's opinion of the volatility.
  - Solve for σ given the option price, S, X, τ, and r with numerical methods.
  - How about American options?
- This volatility is called the implied volatility.
- Implied volatility is often preferred to historical volatility in practice.[a]

[a] It is like driving a car with your eyes on the rearview mirror?

Problems; the Smile

- Options written on the same underlying asset usually do not produce the same implied volatility.
- A typical pattern is a "smile" in relation to the strike price.
  - The implied volatility is lowest for at-the-money options.
  - It becomes higher the further the option is in- or out-of-the-money.

Problems; the Smile (concluded)

- To address this issue, volatilities are often combined to produce a composite implied volatility.
- This practice is not sound theoretically.
- The existence of different implied volatilities for options on the same underlying asset shows the Black-Scholes model cannot be literally true.

Trading Days and Calendar Days

- Interest accrues based on the calendar day.
- But σ is usually calculated based on trading days only.
  - Stock price seems to have lower volatilities when the exchange is closed.[a]
  - σ measures the volatility of stock price one year from now (regardless of what happens in between).
- How to incorporate these two different ways of day count into the Black-Scholes formula and binomial tree algorithms?

[a]Fama (1965); French (1980); French and Roll (1986).

Trading Days and Calendar Days (concluded)

- Suppose a year has 260 trading days.
- A quick and dirty way is to replace σ with[a]

$$\sigma \sqrt{\frac{365}{260} \, \frac{\text{number of trading days to expiration}}{\text{number of calendar days to expiration}}}\,.$$

- How about binomial tree algorithms?

[a] French (1984).

Binomial Tree Algorithms for American Puts

- Early exercise has to be considered.
- The binomial tree algorithm starts with the terminal payoffs
$$\max(0, X - Su^j d^{n-j})$$
and applies backward induction.
- At each intermediate node, it checks for early exercise by comparing the payoff if exercised with the continuation value.

Bermudan Options

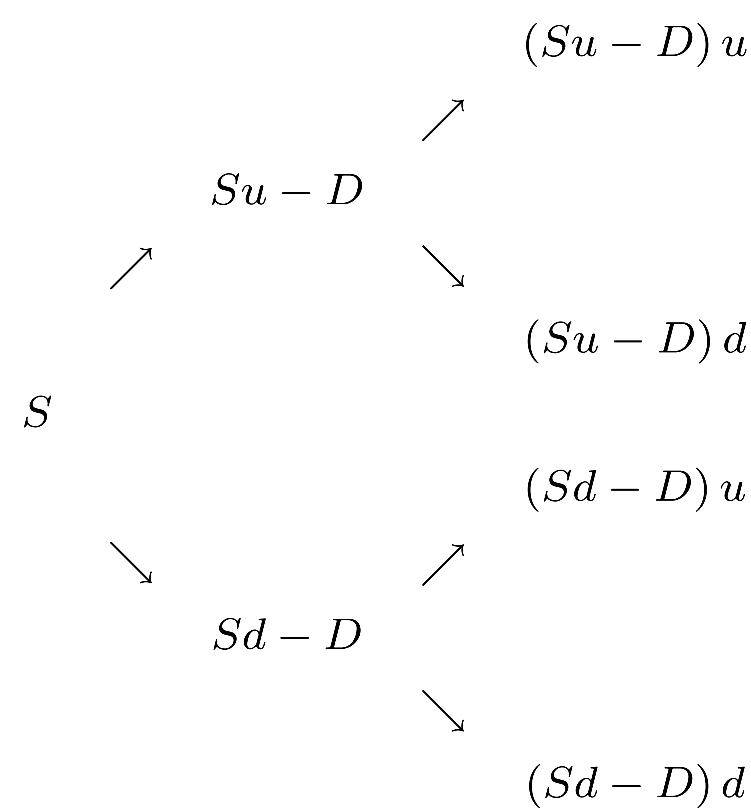
- Some American options can be exercised only at discrete time points instead of continuously.
- They are called Bermudan options.
- Their pricing algorithm is identical to that for American options.
- The only exception is early exercise is considered for only those nodes when early exercise is permitted.

Options on a Stock That Pays Dividends

- Early exercise must be considered.
- Proportional dividend payout model is tractable (see text).
  - The dividend amount is a constant proportion of the prevailing stock price.
- In general, the corporate dividend policy is a complex issue.

Known Dividends

- Constant dividends introduce complications.
- Use D to denote the amount of the dividend.
- Suppose an ex-dividend date falls in the first period.
- At the end of that period, the possible stock prices are $Su - D$ and $Sd - D$.
- Follow the stock price one more period.
- The number of possible stock prices is not three but four: $(Su - D)\,u$, $(Su - D)\,d$, $(Sd - D)\,u$, $(Sd - D)\,d$.
  - The binomial tree no longer combines.


S
$Su - D$
$Sd - D$
$(Su - D)\,u$
$(Su - D)\,d$
$(Sd - D)\,u$
$(Sd - D)\,d$

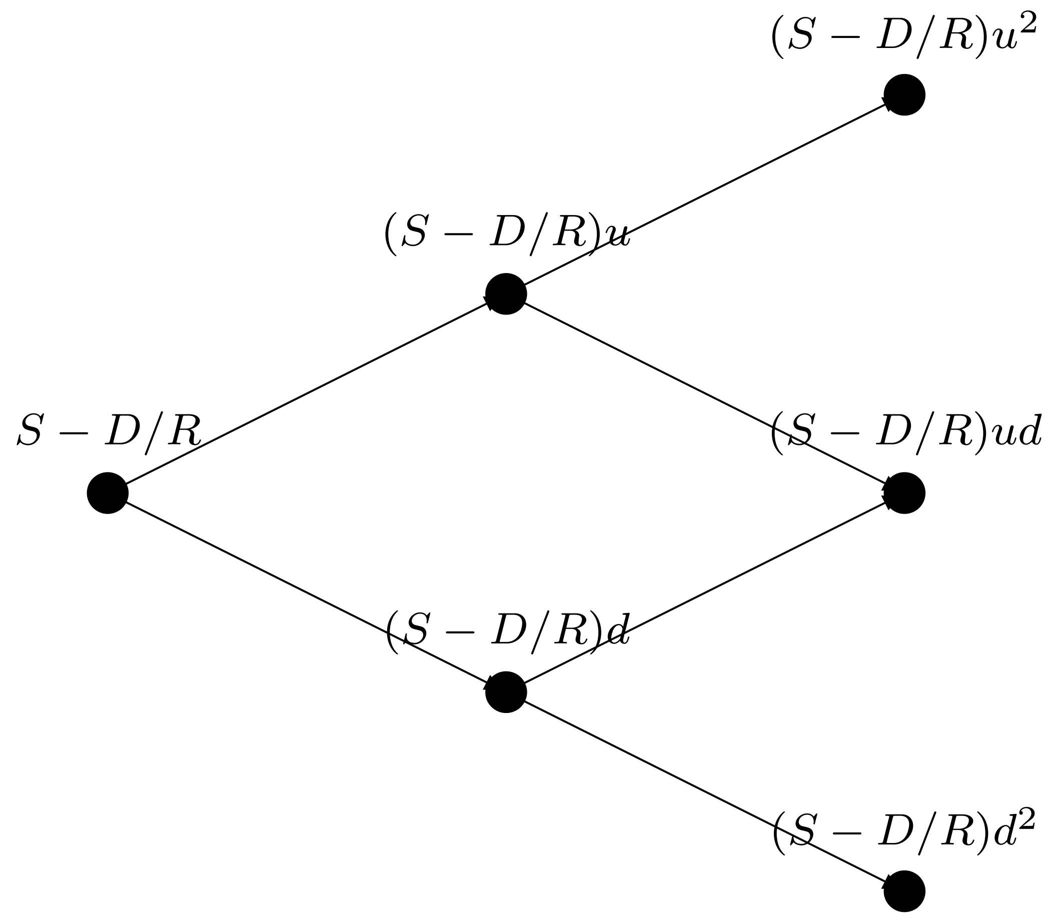
An Ad-Hoc Approximation

- Use the Black-Scholes formula with the stock price reduced by the PV of the dividends (Roll, 1977).
- This essentially decomposes the stock price into a riskless one paying known dividends and a risky one.
- The riskless component at any time is the PV of future dividends during the life of the option.
  - σ equal to the volatility of the process followed by the risky component.
- The stock price, between two adjacent ex-dividend dates, follows the same lognormal distribution.

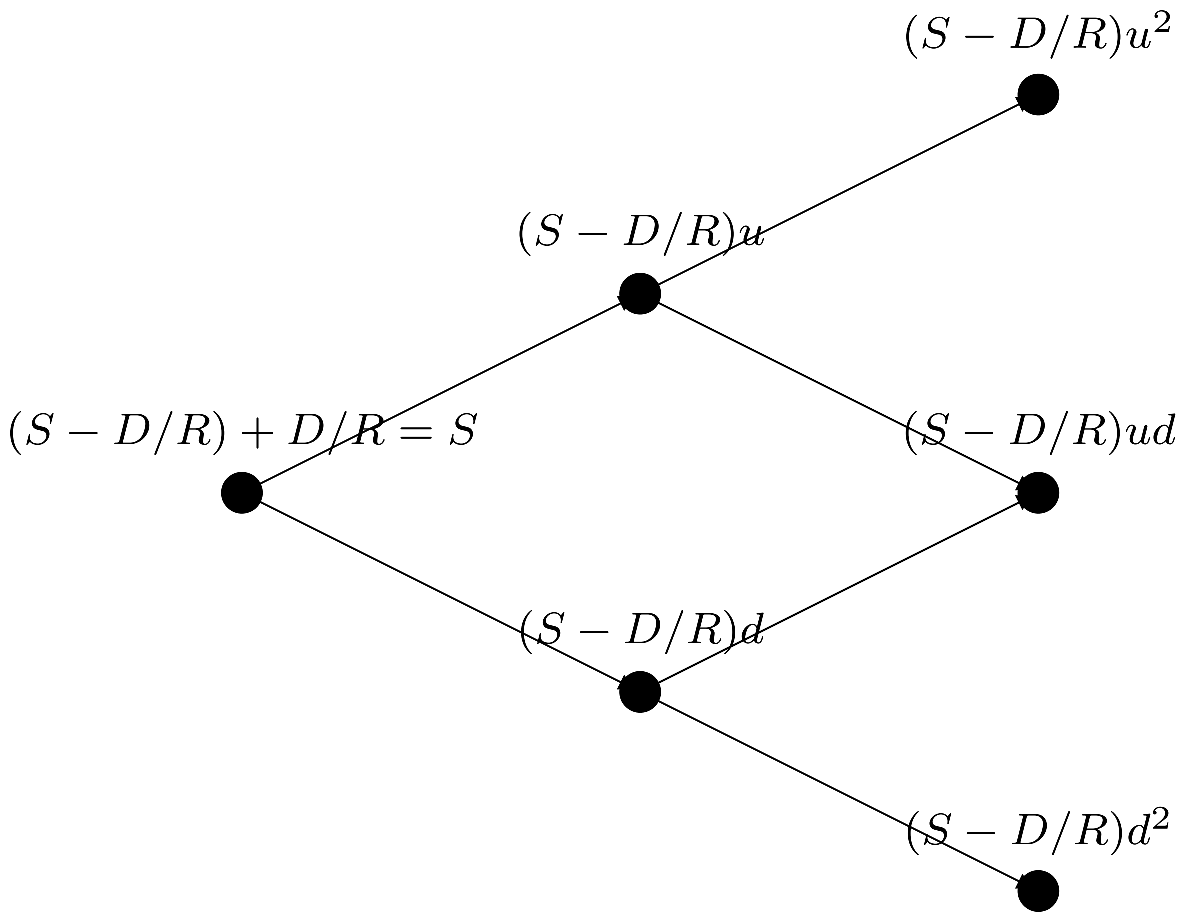
An Ad-Hoc Approximation (concluded)

- Start with the current stock price minus the PV of future dividends before expiration.
- Develop the binomial tree for the new stock price as if there were no dividends.
- Then add to each stock price on the tree the PV of all *future* dividends before expiration.
- American option prices can be computed as before on this tree of stock prices.

An Ad-Hoc Approximation vs. P. 259 (Step 1)



An Ad-Hoc Approximation vs. P. 259 (Step 2)



An Ad-Hoc Approximation vs. P. 259[a]

- The trees are different.
- The stock prices at maturity are also different.
  - $(Su - D)\,u, (Su - D)\,d, (Sd - D)\,u, (Sd - D)\,d$ (p. 259) vs. $(S - D/R)u^2, (S - D/R)ud, (S - D/R)d^2$ (ad hoc).
- Note that $(Su - D)\,u > (S - D/R)u^2$ and $(Sd - D)\,d < (S - D/R)d^2$ as $d < R < u$.
- So the ad hoc approximation has a smaller dynamic range.
- This explains why in practice the volatility is usually increased when using the ad hoc approximation.

[a] Thanks to a lively class discussion on March 18, 2009.

A General Approach[a]

- A new tree structure.
- No approximation assumptions are made.
- A mathematical proof that the tree can always be constructed.
- The actual performance is quadratic except in pathological cases.
- Other approaches include adjusting σ and approximating the known dividend with a dividend yield.

[a]Dai and Lyuu (2004).

Continuous Dividend Yields

- Dividends are paid continuously.
  - Approximates a broad-based stock market portfolio.
- The payment of a continuous dividend yield at rate q reduces the growth rate of the stock price by q.
  - A stock that grows from S to S_τ with a continuous dividend yield of q would grow from S to $S_\tau e^{q\tau}$ without the dividends.
- A European option has the same value as one on a stock with price $Se^{-q\tau}$ that pays no dividends.

Continuous Dividend Yields (continued)

- The Black-Scholes formulas hold with S replaced by $Se^{-q\tau}$ (Merton, 1973):

$$C = Se^{-q\tau} N(x) - Xe^{-r\tau} N(x - \sigma\sqrt{\tau}), \tag{25}$$

$$P = Xe^{-r\tau} N(-x + \sigma\sqrt{\tau}) - Se^{-q\tau} N(-x), \tag{25'}$$

where

$$x \equiv \frac{\ln(S/X) + \left(r - q + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}.$$

- Formulas (25) and (25') remain valid as long as the dividend yield is predictable.

Continuous Dividend Yields (continued)

- To run binomial tree algorithms, replace u with $ue^{-q\Delta t}$ and d with $de^{-q\Delta t}$, where $\Delta t \equiv \tau/n$.
  - The reason: The stock price grows at an expected rate of $r - q$ in a risk-neutral economy.
- Other than the changes, binomial tree algorithms stay the same.
  - In particular, p should use the *original* u and d.[a]

[a] Contributed by Ms. Chuan-Ju Wang (`R95922018`) on May 2, 2007.

Continuous Dividend Yields (concluded)

- Alternatively, pick the risk-neutral probability as

$$\frac{e^{(r-q)\,\Delta t} - d}{u - d}, \tag{26}$$

  where $\Delta t \equiv \tau/n$.

  – The reason: The stock price grows at an expected rate of $r - q$ in a risk-neutral economy.

- The u and d remain unchanged.

- Other than the change in Eq. (26), binomial tree algorithms stay the same.

Sensitivity Analysis of Options

Cleopatra's nose, had it been shorter,
the whole face of the world
would have been changed.
— Blaise Pascal (1623–1662)

Sensitivity Measures ("The Greeks")

- How the value of a security changes relative to changes in a given parameter is key to hedging.
  - Duration, for instance.
- Let $x \equiv \frac{\ln(S/X)+(r+\sigma^2/2)\,\tau}{\sigma\sqrt{\tau}}$ (recall p. 247).
- Note that

$$N'(y) = \frac{e^{-y^2/2}}{\sqrt{2\pi}} > 0,$$

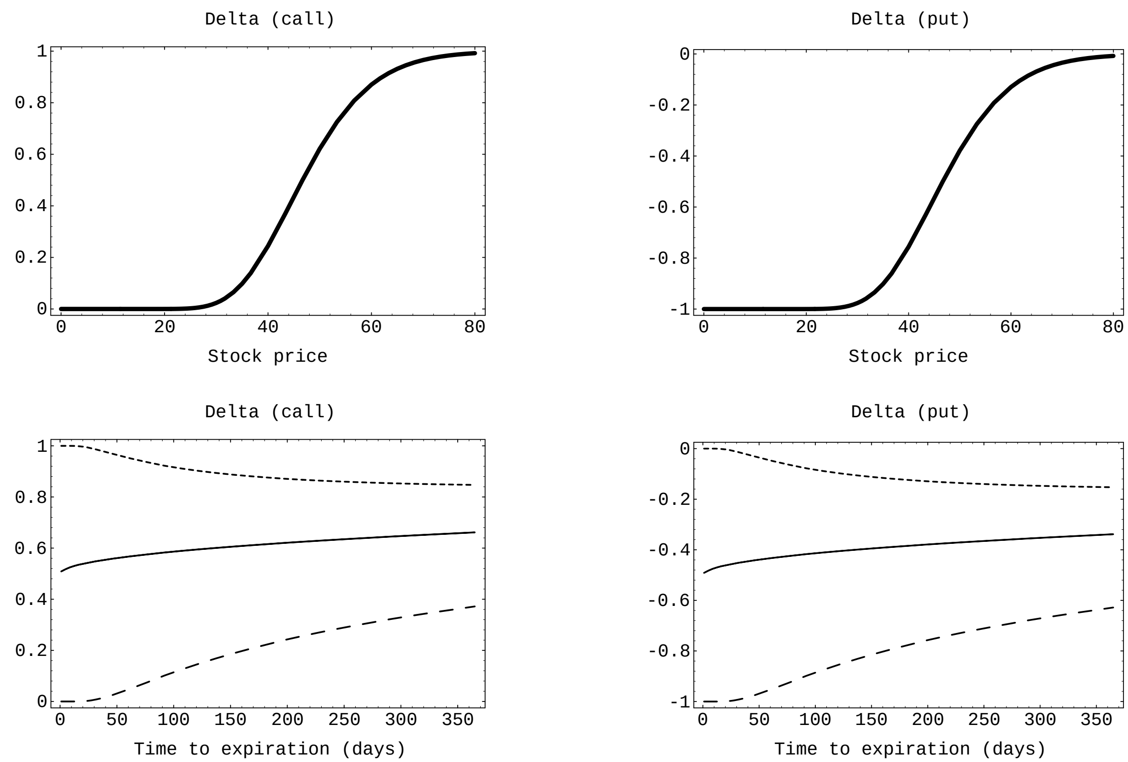
the density function of standard normal distribution.

Delta

- Defined as $\Delta \equiv \partial f / \partial S$.
  - f is the price of the derivative.
  - S is the price of the underlying asset.
- The delta of a portfolio of derivatives on the same underlying asset is the sum of their individual deltas.
  - Elementary calculus.
- The delta used in the BOPM is the discrete analog.

Delta (concluded)

- The delta of a European call on a non-dividend-paying stock equals
$$\frac{\partial C}{\partial S} = N(x) > 0.$$
- The delta of a European put equals
$$\frac{\partial P}{\partial S} = N(x) - 1 < 0.$$
- The delta of a long stock is 1.



Solid curves: at-the-money options.

Dashed curves: out-of-the-money calls or in-the-money puts.

Delta Neutrality

- A position with a total delta equal to 0 is delta-neutral.
  - A delta-neutral portfolio is immune to *small* price changes in the underlying asset.
- Creating one serves for hedging purposes.
  - A portfolio consisting of a call and $-\Delta$ shares of stock is delta-neutral.
  - Short Δ shares of stock to hedge a long call.
- In general, hedge a position in a security with delta Δ_1 by shorting Δ_1/Δ_2 units of a security with delta Δ_2.

Theta (Time Decay)

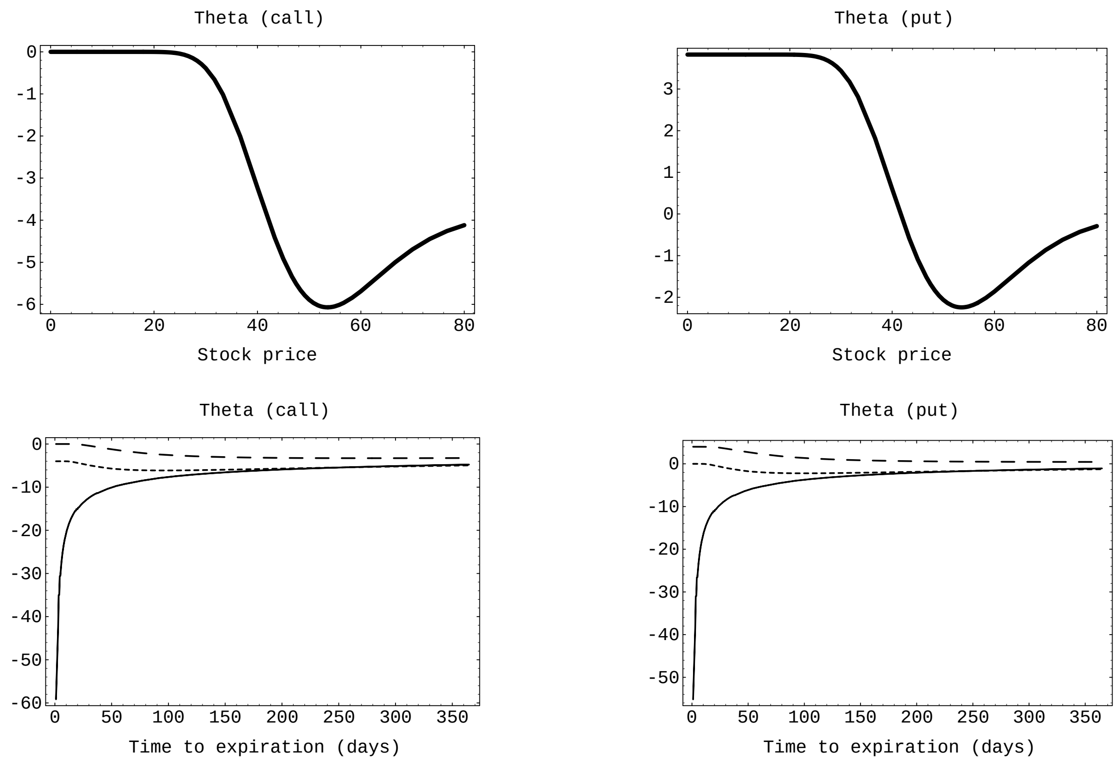
- Defined as the rate of change of a security's value with respect to time, or $\Theta \equiv -\partial f / \partial \tau = \partial f / \partial t$.
- For a European call on a non-dividend-paying stock,

$$\Theta = -\frac{SN'(x)\,\sigma}{2\sqrt{\tau}} - rXe^{-r\tau} N(x - \sigma\sqrt{\tau}) < 0.$$

  - The call loses value with the passage of time.
- For a European put,

$$\Theta = -\frac{SN'(x)\,\sigma}{2\sqrt{\tau}} + rXe^{-r\tau} N(-x + \sigma\sqrt{\tau}).$$

  - Can be negative or positive.



Dotted curve: in-the-money call or out-of-the-money put.

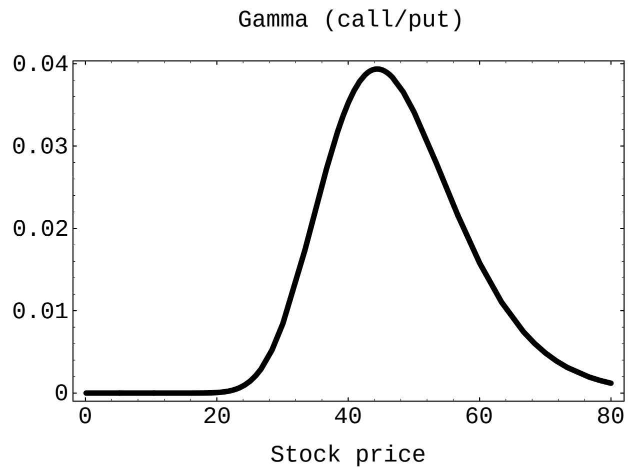
Solid curves: at-the-money options.

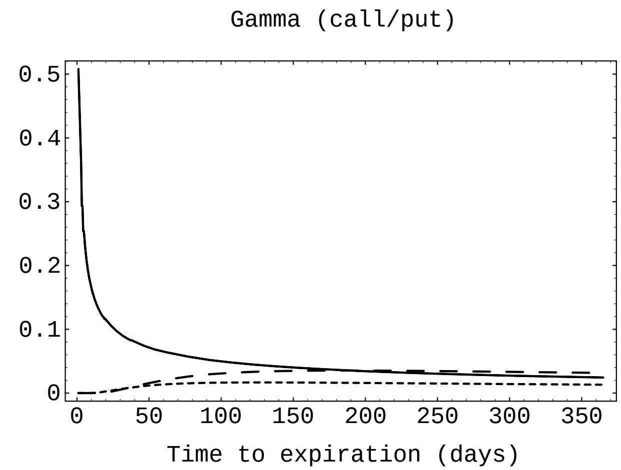
Dashed curve: out-of-the-money call or in-the-money put.

Gamma

- Defined as the rate of change of its delta with respect to the price of the underlying asset, or $\Gamma \equiv \partial^2 \Pi / \partial S^2$.
- Measures how sensitive delta is to changes in the price of the underlying asset.
- In practice, a portfolio with a high gamma needs be rebalanced more often to maintain delta neutrality.
- Roughly, delta $\sim$ duration, and gamma $\sim$ convexity.
- The gamma of a European call or put on a non-dividend-paying stock is

$$N'(x)/(S\sigma\sqrt{\tau}) > 0.$$





Dotted lines: in-the-money call or out-of-the-money put.

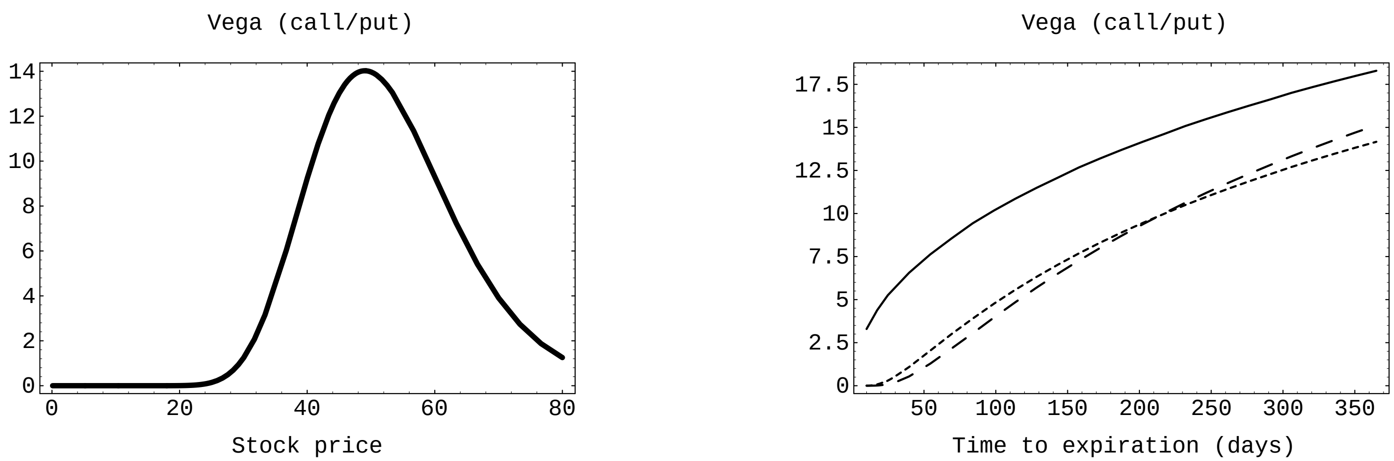
Solid lines: at-the-money option.

Dashed lines: out-of-the-money call or in-the-money put.

Vega[a] (Lambda, Kappa, Sigma)

- Defined as the rate of change of its value with respect to the volatility of the underlying asset $\Lambda \equiv \partial \Pi / \partial \sigma$.
- Volatility often changes over time.
- A security with a high vega is very sensitive to small changes or estimation error in volatility.
- The vega of a European call or put on a non-dividend-paying stock is $S \sqrt{\tau}\, N'(x) > 0$.
  - So higher volatility increases option value.

[a] Vega is not Greek.



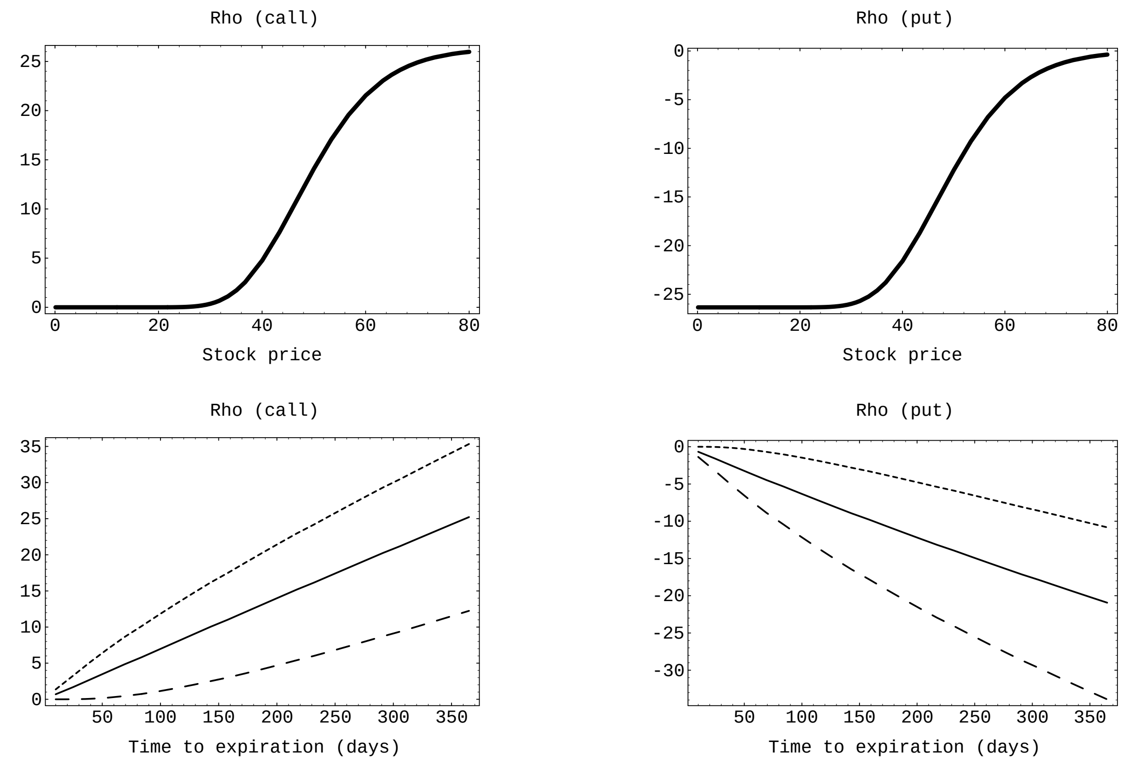
Dotted curve: in-the-money call or out-of-the-money put.

Solid curves: at-the-money option.

Dashed curve: out-of-the-money call or in-the-money put.

Rho

- Defined as the rate of change in its value with respect to interest rates $\rho \equiv \partial \Pi / \partial r$.
- The rho of a European call on a non-dividend-paying stock is
$$X \tau e^{-r\tau} N(x - \sigma\sqrt{\tau}) > 0.$$
- The rho of a European put on a non-dividend-paying stock is
$$-X \tau e^{-r\tau} N(-x + \sigma\sqrt{\tau}) < 0.$$



Dotted curves: in-the-money call or out-of-the-money put.

Solid curves: at-the-money option.

Dashed curves: out-of-the-money call or in-the-money put.

Numerical Greeks

- Needed when closed-form formulas do not exist.
- Take delta as an example.
- A standard method computes the finite difference,

$$\frac{f(S + \Delta S) - f(S - \Delta S)}{2\Delta S}.$$

- The computation time roughly doubles that for evaluating the derivative security itself.

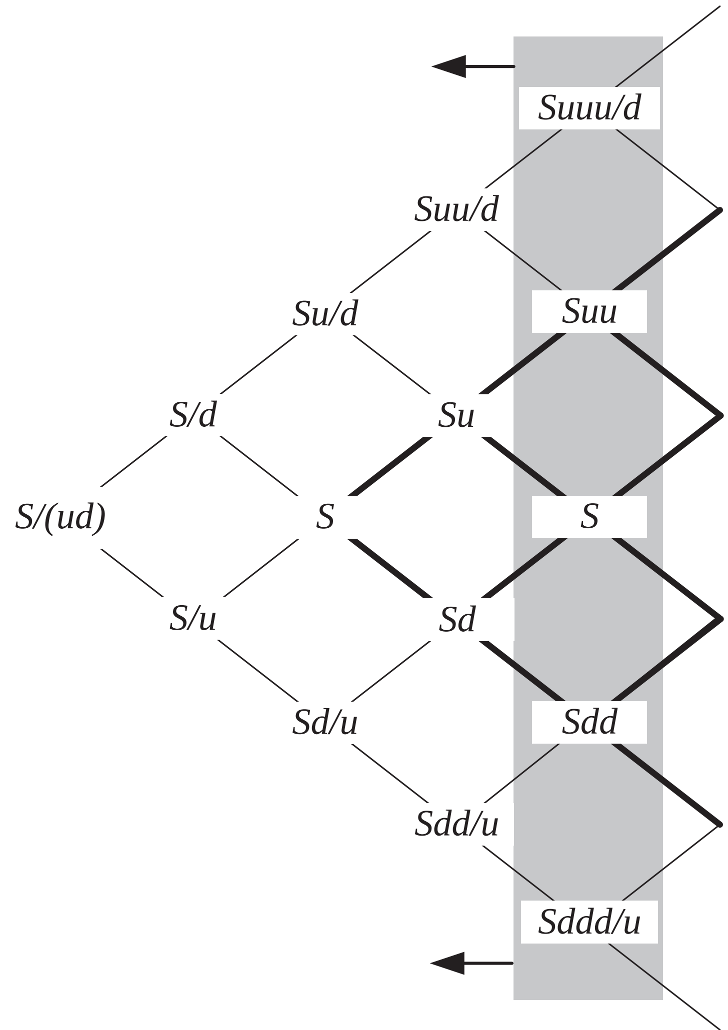
An Alternative Numerical Delta[a]

- Use intermediate results of the binomial tree algorithm.
- When the algorithm reaches the end of the first period, f_u and f_d are computed.
- These values correspond to derivative values at stock prices Su and Sd, respectively.
- Delta is approximated by

$$\frac{f_u - f_d}{Su - Sd}.$$

- Almost zero extra computational effort.

[a]Pelsser and Vorst (1994).


Suuu/d
Suu/d
Su/d
Suu
S/d
Su
S/(ud)
S
S
S/u
Sd
Sd/u
Sdd
Sdd/u
Sddd/u

Numerical Gamma

- At the stock price $(Suu + Sud)/2$, delta is approximately $(f_{uu} - f_{ud})/(Suu - Sud)$.
- At the stock price $(Sud + Sdd)/2$, delta is approximately $(f_{ud} - f_{dd})/(Sud - Sdd)$.
- Gamma is the rate of change in deltas between $(Suu + Sud)/2$ and $(Sud + Sdd)/2$, that is,

$$\frac{\frac{f_{uu}-f_{ud}}{Suu-Sud} - \frac{f_{ud}-f_{dd}}{Sud-Sdd}}{(Suu - Sdd)/2}.$$

- Alternative formulas exist.

Finite Difference Fails for Numerical Gamma

- Numerical differentiation gives

$$\frac{f(S+\Delta S)-2f(S)+f(S-\Delta S)}{(\Delta S)^2}.$$

- It does not work (see text).
- But why did the binomial tree version work?

Other Numerical Greeks

- The theta can be computed as

$$\frac{f_{ud} - f}{2(\tau/n)}.$$

  - In fact, the theta of a European option can be derived from delta and gamma (p. 510).

- For vega and rho, there is no alternative but to run the binomial tree algorithm twice.

Extensions of Options Theory

As I never learnt mathematics,
so I have had to think.
— Joan Robinson (1903–1983)

Pricing Corporate Securities[a]

- Interpret the underlying asset as the total value of the firm.
- The option pricing methodology can be applied to pricing corporate securities.
- Assume:
  - A firm can finance payouts by the sale of assets.
  - If a promised payment to an obligation other than stock is missed, the claim holders take ownership of the firm and the stockholders get nothing.

[a]Black and Scholes (1973).

Risky Zero-Coupon Bonds and Stock

- Consider XYZ.com.
- Capital structure:
  - n shares of its own common stock, S.
  - Zero-coupon bonds with an aggregate par value of X.
- What is the value of the bonds, B?
- What is the value of the XYZ.com stock?

Risky Zero-Coupon Bonds and Stock (continued)

- On the bonds' maturity date, suppose the total value of the firm V^* is less than the bondholders' claim X.
- Then the firm declares bankruptcy, and the stock becomes worthless.
- If $V^* > X$, then the bondholders obtain X and the stockholders $V^* - X$.

	$V^* \leq X$	$V^* > X$
Bonds	V^*	X
Stock	0	$V^* - X$

Risky Zero-Coupon Bonds and Stock (continued)

- The stock is a call on the total value of the firm with a strike price of X and an expiration date equal to the bonds'.
  - This call provides the limited liability for the stockholders.
- The bonds are a covered call on the total value of the firm.
- Let V stand for the total value of the firm.
- Let C stand for a call on V.

Risky Zero-Coupon Bonds and Stock (continued)

- Thus $nS = C$ and $B = V - C$.
- Knowing C amounts to knowing how the value of the firm is divided between stockholders and bondholders.
- Whatever the value of C, the total value of the stock and bonds at maturity remains V^*.
- The relative size of debt and equity is irrelevant to the firm's current value V.

Risky Zero-Coupon Bonds and Stock (continued)

- From Theorem 8 (p. 247) and the put-call parity,

$$\begin{aligned} nS &= VN(x) - Xe^{-r\tau} N(x - \sigma\sqrt{\tau}), \\ B &= VN(-x) + Xe^{-r\tau} N(x - \sigma\sqrt{\tau}). \end{aligned}$$

  - Above,

$$x \equiv \frac{\ln(V/X) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}.$$

- The continuously compounded yield to maturity of the firm's bond is

$$\frac{\ln(X/B)}{\tau}.$$

Risky Zero-Coupon Bonds and Stock (concluded)

- Define the credit spread or default premium as the yield difference between risky and riskless bonds,

$$\begin{aligned}&\frac{\ln(X/B)}{\tau} - r\\ = \quad & -\frac{1}{\tau} \ln\left(N(-z) + \frac{1}{\omega} N(z - \sigma\sqrt{\tau}) \right).\end{aligned}$$

  - $\omega \equiv Xe^{-r\tau}/V$.
  - $z \equiv (\ln \omega)/(\sigma\sqrt{\tau}) + (1/2)\,\sigma\sqrt{\tau} = -x + \sigma\sqrt{\tau}$.
  - Note that ω is the debt-to-total-value ratio.

A Numerical Example

- XYZ.com's assets consist of 1,000 shares of Merck as of March 20, 1995.
  - Merck's market value per share is \$44.5.
- XYZ.com's securities consist of 1,000 shares of common stock and 30 zero-coupon bonds maturing on July 21, 1995.
- Each bond promises to pay \$1,000 at maturity.
- $n = 1000$, $V = 44.5 \times n = 44500$, and $X = 30 \times 1000 = 30000$.

			—Call—		—Put—	
Option	Strike	Exp.	Vol.	Last	Vol.	Last
Merck	30	Jul	328	15 1/4	...	...
44 1/2	35	Jul	150	9 1/2	10	1/16
44 1/2	40	Apr	887	4 3/4	136	1/16
44 1/2	40	Jul	220	5 1/2	297	1/4
44 1/2	40	Oct	58	6	10	1/2
44 1/2	45	Apr	3050	7/8	100	1 1/8
44 1/2	45	May	462	1 3/8	50	1 3/8
44 1/2	45	Jul	883	1 15/16	147	1 3/4
44 1/2	45	Oct	367	2 3/4	188	2 1/16

A Numerical Example (continued)

- The Merck option relevant for pricing is the July call with a strike price of $X/n = 30$ dollars.
- Such a call is selling for $15.25.
- So XYZ.com's stock is worth $15.25 \times n = 15250$ dollars.
- The entire bond issue is worth $B = 44500 - 15250 = 29250$ dollars.
  - Or $975 per bond.

A Numerical Example (continued)

- The XYZ.com bonds are equivalent to a default-free zero-coupon bond with $\$X$ par value plus n written European puts on Merck at a strike price of $30.
  - By the put-call parity.
- The difference between B and the price of the default-free bond is the value of these puts.
- The next table shows the total market values of the XYZ.com stock and bonds under various debt amounts X.

Promised payment to bondholders X	Current market value of bonds B	Current market value of stock nS	Current total value of firm V
30,000	29,250.0	15,250.0	44,500
35,000	35,000.0	9,500.0	44,500
40,000	39,000.0	5,500.0	44,500
45,000	42,562.5	1,937.5	44,500

A Numerical Example (continued)

- Suppose the promised payment to bondholders is \$45,000.
- Then the relevant option is the July call with a strike price of $45000/n = 45$ dollars.
- Since that option is selling for \$1 15/16, the market value of the XYZ.com stock is $(1 + 15/16) \times n = 1937.5$ dollars.
- The market value of the stock decreases as the debt-equity ratio increases.

A Numerical Example (continued)

- There are conflicts between stockholders and bondholders.
- An option's terms cannot be changed after issuance.
- But a firm can change its capital structure.
- There lies one key difference between options and corporate securities.
  - Parameters such volatility, dividend, and strike price are under partial control of the stockholders.

A Numerical Example (continued)

- Suppose XYZ.com issues 15 more bonds with the same terms to buy back stock.
- The total debt is now $X = 45{,}000$ dollars.
- The table on p. 304 says the total market value of the bonds should be $42,562.5.
- The *new* bondholders pay $42562.5 \times (15/45) = 14187.5$ dollars.
- The remaining stock is worth $1,937.5.

A Numerical Example (continued)

- The stockholders therefore gain

$$14187.5 + 1937.5 - 15250 = 875$$

dollars.

- The *original* bondholders lose an equal amount,

$$29250 - \frac{30}{45} \times 42562.5 = 875. \tag{27}$$

A Numerical Example (continued)

- Suppose the stockholders sell $(1/3) \times n$ Merck shares to fund a \$14,833.3 cash dividend.
- They now have \$14,833.3 in cash plus a call on $(2/3) \times n$ Merck shares.
- The strike price remains $X = 30000$.
- This is equivalent to owning 2/3 of a call on n Merck shares with a total strike price of \$45,000.
- n such calls are worth \$1,937.5 (p. 304).
- So the total market value of the XYZ.com stock is $(2/3) \times 1937.5 = 1291.67$ dollars.

A Numerical Example (concluded)

- The market value of the XYZ.com bonds is hence $(2/3) \times n \times 44.5 - 1291.67 = 28375$ dollars.
- Hence the stockholders gain

$$14833.3 + 1291.67 - 15250 \approx 875$$

dollars.

- The bondholders watch their value drop from \$29,250 to \$28,375, a loss of \$875.

Other Examples

- Subordinated debts as bull call spreads.
- Warrants as calls.
- Callable bonds as American calls with 2 strike prices.
- Convertible bonds.