

Equilibrium Term Structure Models

8. What's your problem? Any moron
can understand bond pricing models.
— *Top Ten Lies Finance Professors
Tell Their Students*

Introduction

- This chapter surveys equilibrium models.
- Since the spot rates satisfy

$$r(t, T) = -\frac{\ln P(t, T)}{T - t},$$

the discount function $P(t, T)$ suffices to establish the spot rate curve.

- All models to follow are short rate models.
- Unless stated otherwise, the processes are risk-neutral.

The Vasicek Model^a

- The short rate follows

$$dr = \beta(\mu - r) dt + \sigma dW.$$

- The short rate is pulled to the long-term mean level μ at rate β .
- Superimposed on this “pull” is a normally distributed stochastic term σdW .
- Since the process is an Ornstein-Uhlenbeck process,

$$E[r(T) | r(t) = r] = \mu + (r - \mu) e^{-\beta(T-t)}$$

from Eq. (53) on p. 494.

^aVasicek (1977).

The Vasicek Model (continued)

- The price of a zero-coupon bond paying one dollar at maturity can be shown to be

$$P(t, T) = A(t, T) e^{-B(t, T) r(t)}, \quad (97)$$

where

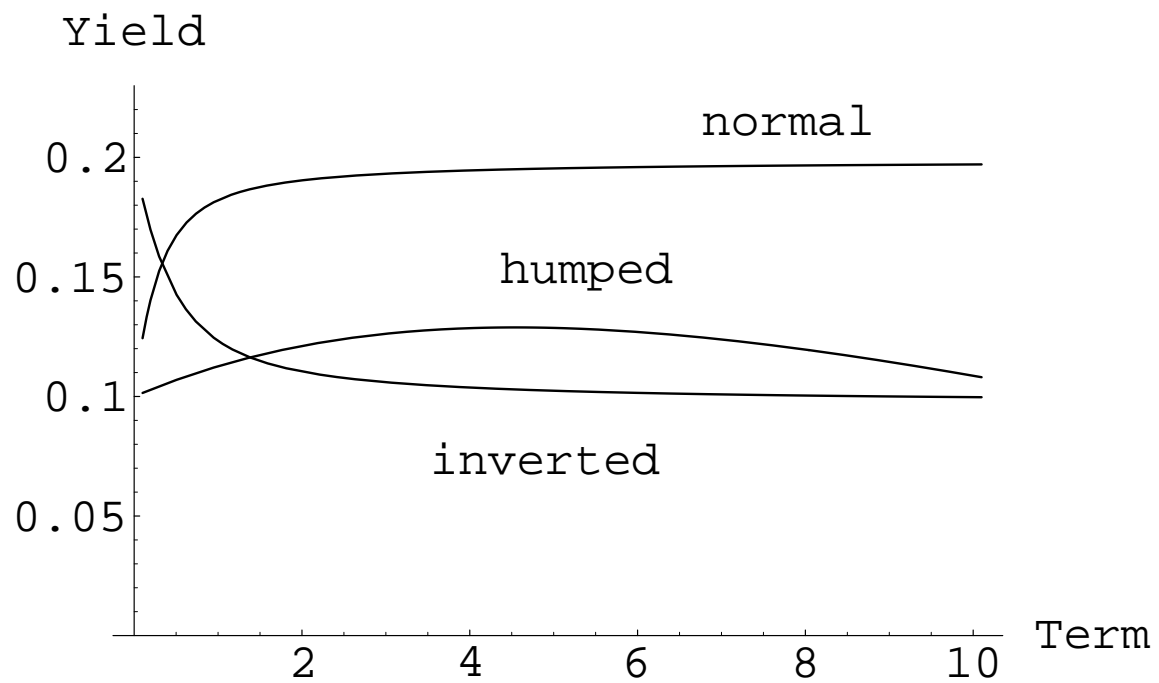
$$A(t, T) = \begin{cases} \exp \left[\frac{(B(t, T) - T + t)(\beta^2 \mu - \sigma^2 / 2)}{\beta^2} - \frac{\sigma^2 B(t, T)^2}{4\beta} \right] & \text{if } \beta \neq 0, \\ \exp \left[\frac{\sigma^2 (T - t)^3}{6} \right] & \text{if } \beta = 0. \end{cases}$$

and

$$B(t, T) = \begin{cases} \frac{1 - e^{-\beta(T-t)}}{\beta} & \text{if } \beta \neq 0, \\ T - t & \text{if } \beta = 0. \end{cases}$$

The Vasicek Model (concluded)

- If $\beta = 0$, then P goes to infinity as $T \rightarrow \infty$.
- Sensibly, P goes to zero as $T \rightarrow \infty$ if $\beta \neq 0$.
- Even if $\beta \neq 0$, P may exceed one for a finite T .
- The spot rate volatility structure is the curve $(\partial r(t, T)/\partial r) \sigma = \sigma B(t, T)/(T - t)$.
- When $\beta > 0$, the curve tends to decline with maturity.
- The speed of mean reversion, β , controls the shape of the curve.
- Indeed, higher β leads to greater attenuation of volatility with maturity.



The Vasicek Model: Options on Zeros^a

- Consider a European call with strike price X expiring at time T on a zero-coupon bond with par value \$1 and maturing at time $s > T$.
- Its price is given by

$$P(t, s) N(x) - X P(t, T) N(x - \sigma_v).$$

^aJamshidian (1989).

The Vasicek Model: Options on Zeros (concluded)

- Above

$$\begin{aligned}x &\equiv \frac{1}{\sigma_v} \ln \left(\frac{P(t, s)}{P(t, T) X} \right) + \frac{\sigma_v}{2}, \\ \sigma_v &\equiv v(t, T) B(T, s), \\ v(t, T)^2 &\equiv \begin{cases} \frac{\sigma^2 [1 - e^{-2\beta(T-t)}]}{2\beta}, & \text{if } \beta \neq 0 \\ \sigma^2 (T - t), & \text{if } \beta = 0 \end{cases}.\end{aligned}$$

- By the put-call parity, the price of a European put is

$$XP(t, T) N(-x + \sigma_v) - P(t, s) N(-x).$$

Binomial Vasicek

- Consider a binomial model for the short rate in the time interval $[0, T]$ divided into n identical pieces.
- Let $\Delta t \equiv T/n$ and

$$p(r) \equiv \frac{1}{2} + \frac{\beta(\mu - r) \sqrt{\Delta t}}{2\sigma}.$$

- The following binomial model converges to the Vasicek model,^a

$$r(k+1) = r(k) + \sigma \sqrt{\Delta t} \xi(k), \quad 0 \leq k < n.$$

^aNelson and Ramaswamy (1990).

Binomial Vasicek (continued)

- Above, $\xi(k) = \pm 1$ with

$$\text{Prob}[\xi(k) = 1] = \begin{cases} p(r(k)) & \text{if } 0 \leq p(r(k)) \leq 1 \\ 0 & \text{if } p(r(k)) < 0 \\ 1 & \text{if } 1 < p(r(k)) \end{cases} .$$

- Observe that the probability of an up move, p , is a decreasing function of the interest rate r .
- This is consistent with mean reversion.

Binomial Vasicek (concluded)

- The rate is the same whether it is the result of an up move followed by a down move or a down move followed by an up move.
- The binomial tree combines.
- The key feature of the model that makes it happen is its *constant* volatility, σ .
- For a general process Y with nonconstant volatility, the resulting binomial tree may not combine.

The Cox-Ingersoll-Ross Model^a

- It is the following square-root short rate model:

$$dr = \beta(\mu - r) dt + \sigma\sqrt{r} dW. \quad (98)$$

- The diffusion differs from the Vasicek model by a multiplicative factor \sqrt{r} .
- The parameter β determines the speed of adjustment.
- The short rate can reach zero only if $2\beta\mu < \sigma^2$.
- See text for the bond pricing formula.

^aCox, Ingersoll, and Ross (1985).

Binomial CIR

- We want to approximate the short rate process in the time interval $[0, T]$.
- Divide it into n periods of duration $\Delta t \equiv T/n$.
- Assume $\mu, \beta \geq 0$.
- A direct discretization of the process is problematic because the resulting binomial tree will *not* combine.

Binomial CIR (continued)

- Instead, consider the transformed process

$$x(r) \equiv 2\sqrt{r}/\sigma.$$

- It follows

$$dx = m(x) dt + dW,$$

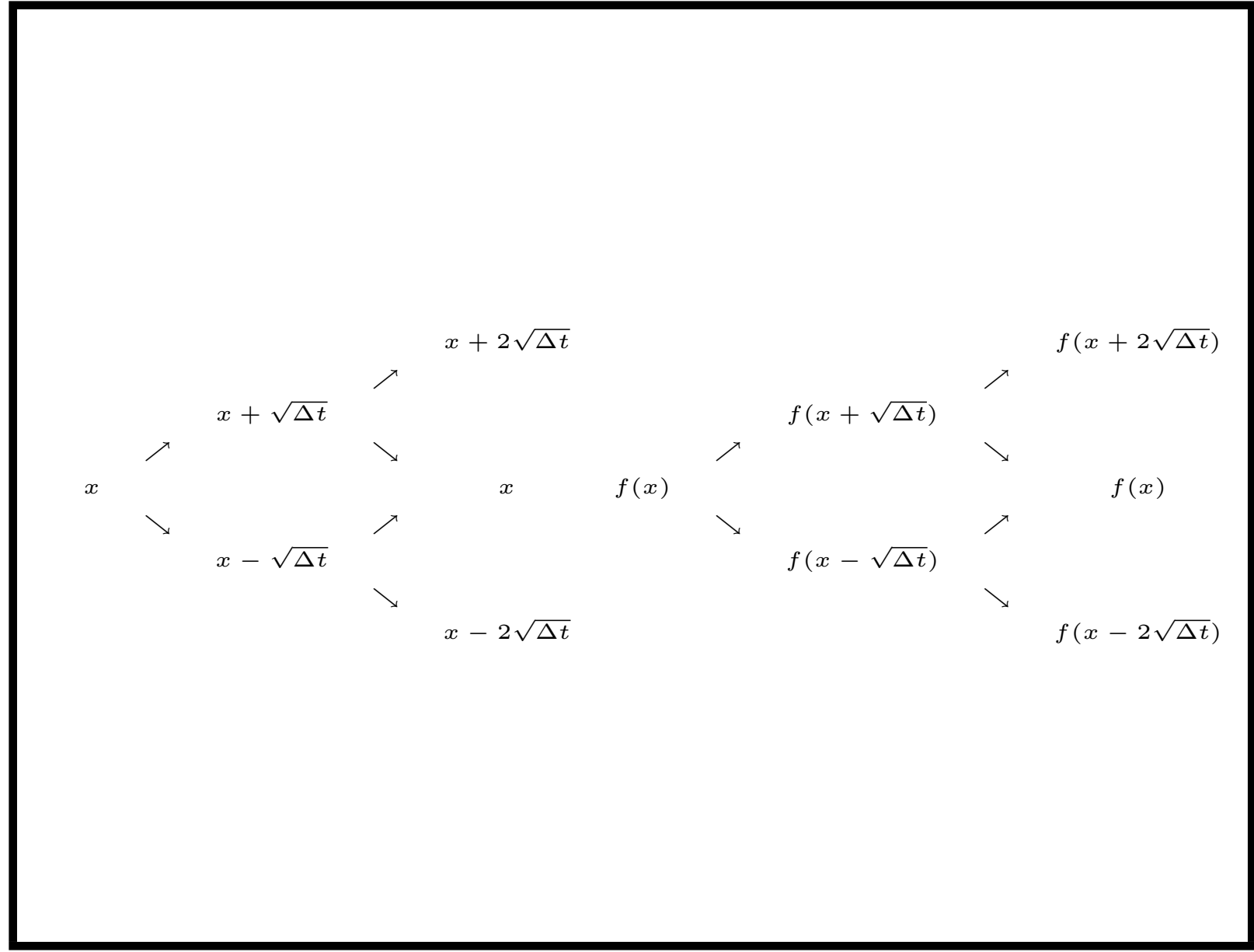
where

$$m(x) \equiv 2\beta\mu/(\sigma^2 x) - (\beta x/2) - 1/(2x).$$

- Since this new process has a constant volatility, its associated binomial tree combines.

Binomial CIR (continued)

- Construct the combining tree for r as follows.
- First, construct a tree for x .
- Then transform each node of the tree into one for r via the inverse transformation $r = f(x) \equiv x^2\sigma^2/4$ (p. 870).



Binomial CIR (concluded)

- The probability of an up move at each node r is

$$p(r) \equiv \frac{\beta(\mu - r) \Delta t + r - r^-}{r^+ - r^-}. \quad (99)$$

- $r^+ \equiv f(x + \sqrt{\Delta t})$ denotes the result of an up move from r .
- $r^- \equiv f(x - \sqrt{\Delta t})$ the result of a down move.
- Finally, set the probability $p(r)$ to one as r goes to zero to make the probability stay between zero and one.

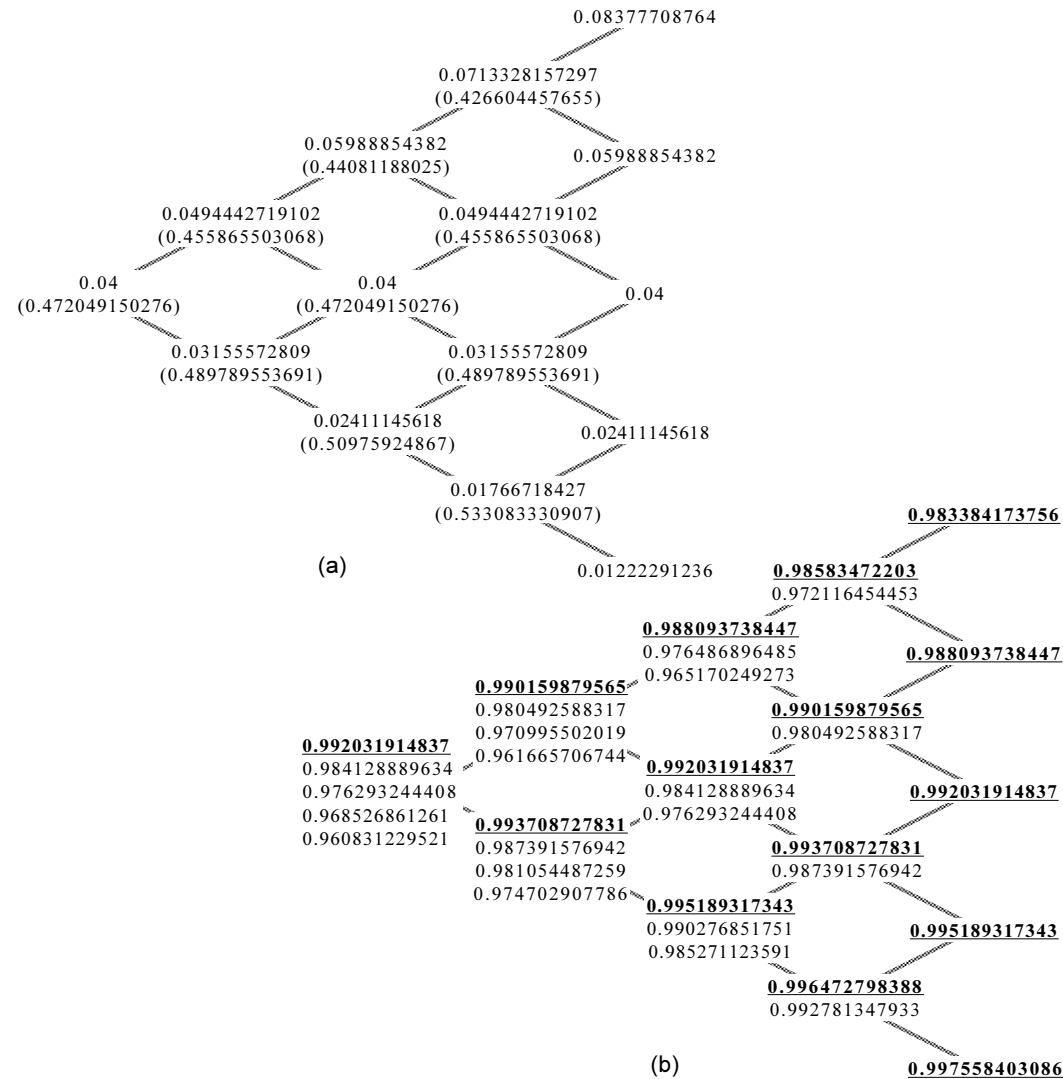
Numerical Examples

- Consider the process,

$$0.2 (0.04 - r) dt + 0.1\sqrt{r} dW,$$

for the time interval $[0, 1]$ given the initial rate $r(0) = 0.04$.

- We shall use $\Delta t = 0.2$ (year) for the binomial approximation.
- See p. 873(a) for the resulting binomial short rate tree with the up-move probabilities in parentheses.



Numerical Examples (continued)

- Consider the node which is the result of an up move from the root.
- Since the root has $x = 2\sqrt{r(0)}/\sigma = 4$, this particular node's x value equals $4 + \sqrt{\Delta t} = 4.4472135955$.
- Use the inverse transformation to obtain the short rate $x^2 \times (0.1)^2/4 \approx 0.0494442719102$.

Numerical Examples (concluded)

- Once the short rates are in place, computing the probabilities is easy.
- Note that the up-move probability decreases as interest rates increase and increases as interest rates decline.
- This phenomenon agrees with mean reversion.
- Convergence is quite good (see text).

A General Method for Constructing Binomial Models^a

- We are given a continuous-time process
 $dy = \alpha(y, t) dt + \sigma(y, t) dW$.
- Make sure the binomial model's drift and diffusion converge to the above process by setting the probability of an up move to

$$\frac{\alpha(y, t) \Delta t + y - y_d}{y_u - y_d}.$$

- Here $y_u \equiv y + \sigma(y, t)\sqrt{\Delta t}$ and $y_d \equiv y - \sigma(y, t)\sqrt{\Delta t}$ represent the two rates that follow the current rate y .
- The displacements are identical, at $\sigma(y, t)\sqrt{\Delta t}$.

^aNelson and Ramaswamy (1990).

A General Method (continued)

- But the binomial tree may not combine:

$$\sigma(y, t)\sqrt{\Delta t} - \sigma(y_u, t)\sqrt{\Delta t} \neq -\sigma(y, t)\sqrt{\Delta t} + \sigma(y_d, t)\sqrt{\Delta t}$$

in general.

- When $\sigma(y, t)$ is a constant independent of y , equality holds and the tree combines.
- To achieve this, define the transformation

$$x(y, t) \equiv \int^y \sigma(z, t)^{-1} dz.$$

- Then x follows $dx = m(y, t) dt + dW$ for some $m(y, t)$ (see text).

A General Method (continued)

- The key is that the diffusion term is now a constant, and the binomial tree for x combines.
- The probability of an up move remains

$$\frac{\alpha(y(x, t), t) \Delta t + y(x, t) - y_d(x, t)}{y_u(x, t) - y_d(x, t)},$$

where $y(x, t)$ is the inverse transformation of $x(y, t)$ from x back to y .

- Note that $y_u(x, t) \equiv y(x + \sqrt{\Delta t}, t + \Delta t)$ and $y_d(x, t) \equiv y(x - \sqrt{\Delta t}, t + \Delta t)$.

A General Method (concluded)

- The transformation is

$$\int^r (\sigma \sqrt{z})^{-1} dz = 2\sqrt{r}/\sigma$$

for the CIR model.

- The transformation is

$$\int^S (\sigma z)^{-1} dz = (1/\sigma) \ln S$$

for the Black-Scholes model.

- The familiar binomial option pricing model in fact discretizes $\ln S$ not S .

Model Calibration

- In the time-series approach, the time series of short rates is used to estimate the parameters of the process.
- This approach may help in validating the proposed interest rate process.
- But it alone cannot be used to estimate the risk premium parameter λ .
- The model prices based on the estimated parameters may also deviate a lot from those in the market.

Model Calibration (concluded)

- The cross-sectional approach uses a cross section of bond prices observed at the same time.
- The parameters are to be such that the model prices closely match those in the market.
- After this procedure, the calibrated model can be used to price interest rate derivatives.
- Unlike the time-series approach, the cross-sectional approach is unable to separate out the interest rate risk premium from the model parameters.

On One-Factor Short Rate Models

- By using only the short rate, they ignore other rates on the yield curve.
- Such models also restrict the volatility to be a function of interest rate *levels* only.
- The prices of all bonds move in the same direction at the same time (their magnitudes may differ).
- The returns on all bonds thus become highly correlated.
- In reality, there seems to be a certain amount of independence between short- and long-term rates.

On One-Factor Short Rate Models (continued)

- One-factor models therefore cannot accommodate nondegenerate correlation structures across maturities.
- Derivatives whose values depend on the correlation structure will be mispriced.
- The calibrated models may not generate term structures as concave as the data suggest.
- The term structure empirically changes in slope and curvature as well as makes parallel moves.
- This is inconsistent with the restriction that all segments of the term structure be perfectly correlated.

On One-Factor Short Rate Models (concluded)

- Multi-factor models lead to families of yield curves that can take a greater variety of shapes and can better represent reality.
- But they are much harder to think about and work with.
- They also take much more computer time—the curse of dimensionality.
- These practical concerns limit the use of multifactor models to two-factor ones.

Options on Coupon Bonds^a

- The price of a European option on a coupon bond can be calculated from those on zero-coupon bonds.
- Consider a European call expiring at time T on a bond with par value \$1.
- Let X denote the strike price.
- The bond has cash flows c_1, c_2, \dots, c_n at times t_1, t_2, \dots, t_n , where $t_i > T$ for all i .
- The payoff for the option is

$$\max \left(\sum_{i=1}^n c_i P(r(T), T, t_i) - X, 0 \right).$$

^aJamshidian (1989).

Options on Coupon Bonds (continued)

- At time T , there is a unique value r^* for $r(T)$ that renders the coupon bond's price equal the strike price X .
- This r^* can be obtained by solving $X = \sum_{i=1}^n c_i P(r, T, t_i)$ numerically for r .
- The solution is also unique for one-factor models whose bond price is a monotonically decreasing function of r .
- Let $X_i \equiv P(r^*, T, t_i)$, the value at time T of a zero-coupon bond with par value \$1 and maturing at time t_i if $r(T) = r^*$.

Options on Coupon Bonds (concluded)

- Note that $P(r(T), T, t_i) \geq X_i$ if and only if $r(T) \leq r^*$.
- As $X = \sum_i c_i X_i$, the option's payoff equals

$$\sum_{i=1}^n c_i \times \max(P(r(T), T, t_i) - X_i, 0).$$

- Thus the call is a package of n options on the underlying zero-coupon bond.

No-Arbitrage Term Structure Models

How much of the structure of our theories
really tells us about things in nature,
and how much do we contribute ourselves?
— Arthur Eddington (1882–1944)

Motivations

- Recall the difficulties facing equilibrium models mentioned earlier.
 - They usually require the estimation of the market price of risk.
 - They cannot fit the market term structure.
 - But consistency with the market is often mandatory in practice.

No-Arbitrage Models^a

- No-arbitrage models utilize the full information of the term structure.
- They accept the observed term structure as consistent with an unobserved and unspecified equilibrium.
- From there, arbitrage-free movements of interest rates or bond prices over time are modeled.
- By definition, the market price of risk must be reflected in the current term structure; hence the resulting interest rate process is risk-neutral.

^aHo and Lee (1986).

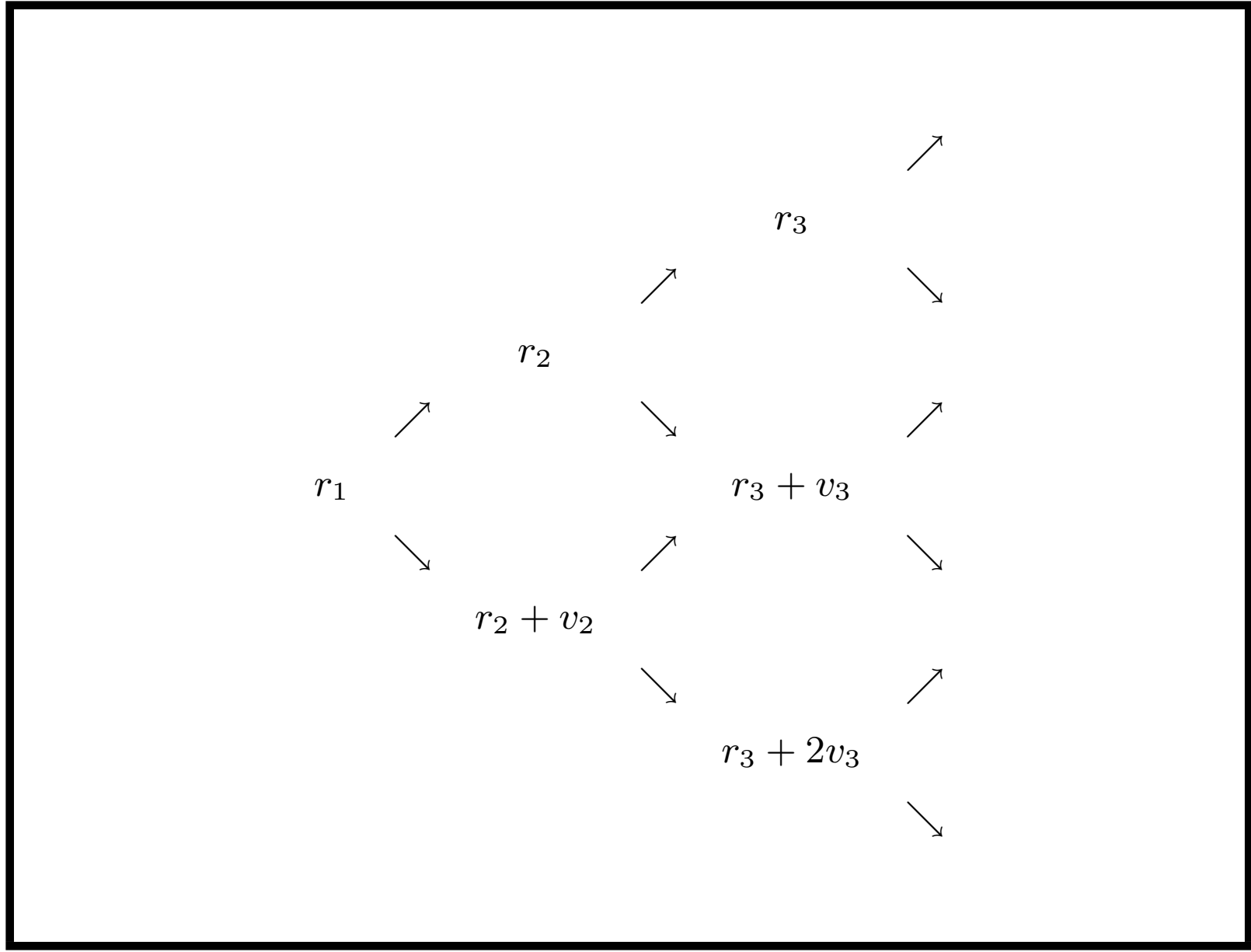
No-Arbitrage Models (concluded)

- No-arbitrage models can specify the dynamics of zero-coupon bond prices, forward rates, or the short rate.
- Bond price and forward rate models are usually non-Markovian (path dependent).
- In contrast, short rate models are generally constructed to be explicitly Markovian (path independent).
- Markovian models are easier to handle computationally.

The Ho-Lee Model^a

- The short rates at any given time are evenly spaced.
- Let p denote the risk-neutral probability that the short rate makes an up move.
- We shall adopt continuous compounding.

^aHo and Lee (1986).



The Ho-Lee Model (continued)

- The Ho-Lee model starts with zero-coupon bond prices $P(t, t+1), P(t, t+2), \dots$ at time t identified with the root of the tree.

- Let the discount factors in the next period be

$$P_d(t+1, t+2), P_d(t+1, t+3), \dots \quad \text{if short rate moves down}$$

$$P_u(t+1, t+2), P_u(t+1, t+3), \dots \quad \text{if short rate moves up}$$

- By backward induction, it is not hard to see that for $n \geq 2$,

$$P_u(t+1, t+n) = P_d(t+1, t+n) e^{-(v_2 + \dots + v_n)} \quad (100)$$

(see text).

The Ho-Lee Model (continued)

- It is also not hard to check that the n -period zero-coupon bond has yields

$$y_d(n) \equiv -\frac{\ln P_d(t+1, t+n)}{n-1}$$

$$y_u(n) \equiv -\frac{\ln P_u(t+1, t+n)}{n-1} = y_d(n) + \frac{v_2 + \cdots + v_n}{n-1}$$

- The volatility of the yield to maturity for this bond is therefore

$$\begin{aligned}\kappa_n &\equiv \sqrt{py_u(n)^2 + (1-p)y_d(n)^2 - [py_u(n) + (1-p)y_d(n)]^2} \\ &= \sqrt{p(1-p)} (y_u(n) - y_d(n)) \\ &= \sqrt{p(1-p)} \frac{v_2 + \cdots + v_n}{n-1}.\end{aligned}$$

The Ho-Lee Model (concluded)

- In particular, the short rate volatility is determined by taking $n = 2$:

$$\sigma = \sqrt{p(1-p)} v_2. \quad (101)$$

- The variance of the short rate therefore equals $p(1-p)(r_u - r_d)^2$, where r_u and r_d are the two successor rates.^a

^aContrast this with the lognormal model .

The Ho-Lee Model: Volatility Term Structure

- The volatility term structure is composed of $\kappa_2, \kappa_3, \dots$
 - It is independent of the r_i .
- It is easy to compute the v_i s from the volatility structure, and vice versa.
- The r_i s can be computed by forward induction.
- The volatility structure is supplied by the market.

The Ho-Lee Model: Bond Price Process

- In a risk-neutral economy, the initial discount factors satisfy

$$P(t, t+n) = (pP_u(t+1, t+n) + (1-p)P_d(t+1, t+n))P(t, t+1)$$

- Combine the above with Eq. (100) on p. 895 and assume $p = 1/2$ to obtain^a

$$P_d(t+1, t+n) = \frac{P(t, t+n)}{P(t, t+1)} \frac{2 \times \exp[v_2 + \cdots + v_n]}{1 + \exp[v_2 + \cdots + v_n]}, \quad (102)$$

$$P_u(t+1, t+n) = \frac{P(t, t+n)}{P(t, t+1)} \frac{2}{1 + \exp[v_2 + \cdots + v_n]}. \quad (102')$$

^aIn the limit, only the volatility matters.

The Ho-Lee Model: Bond Price Process (concluded)

- The bond price tree combines.
- Suppose all v_i equal some constant v and $\delta \equiv e^v > 0$.
- Then

$$P_d(t+1, t+n) = \frac{P(t, t+n)}{P(t, t+1)} \frac{2\delta^{n-1}}{1 + \delta^{n-1}},$$
$$P_u(t+1, t+n) = \frac{P(t, t+n)}{P(t, t+1)} \frac{2}{1 + \delta^{n-1}}.$$

- Short rate volatility σ equals $v/2$ by Eq. (101) on p. 897.
- Price derivatives by taking expectations under the risk-neutral probability.

The Ho-Lee Model: Yields and Their Covariances

- The one-period rate of return of an n -period zero-coupon bond is

$$r(t, t + n) \equiv \ln \left(\frac{P(t + 1, t + n)}{P(t, t + n)} \right).$$

- Its value is either $\ln \frac{P_d(t+1, t+n)}{P(t, t+n)}$ or $\ln \frac{P_u(t+1, t+n)}{P(t, t+n)}$.
- Thus the variance of return is

$$\text{Var}[r(t, t + n)] = p(1 - p)((n - 1) v)^2 = (n - 1)^2 \sigma^2.$$

The Ho-Lee Model: Yields and Their Covariances (concluded)

- The covariance between $r(t, t + n)$ and $r(t, t + m)$ is $(n - 1)(m - 1) \sigma^2$ (see text).
- As a result, the correlation between any two one-period rates of return is unity.
- Strong correlation between rates is inherent in all one-factor Markovian models.

The Ho-Lee Model: Short Rate Process

- The continuous-time limit of the Ho-Lee model is

$$dr = \theta(t) dt + \sigma dW.$$

- This is Vasicek's model with the mean-reverting drift replaced by a deterministic, time-dependent drift.
- A nonflat term structure of volatilities can be achieved if the short rate volatility is also made time varying, i.e.,
$$dr = \theta(t) dt + \sigma(t) dW.$$
- This corresponds to the discrete-time model in which v_i are not all identical.

The Ho-Lee Model: Some Problems

- Future (nominal) interest rates may be negative.
- The short rate volatility is independent of the rate level.

Problems with No-Arbitrage Models in General

- Interest rate movements should reflect shifts in the model's state variables (factors) not its parameters.
- Model *parameters*, such as the drift $\theta(t)$ in the continuous-time Ho-Lee model, should be stable over time.
- But in practice, no-arbitrage models capture yield curve shifts through the recalibration of parameters.
 - A new model is thus born everyday.

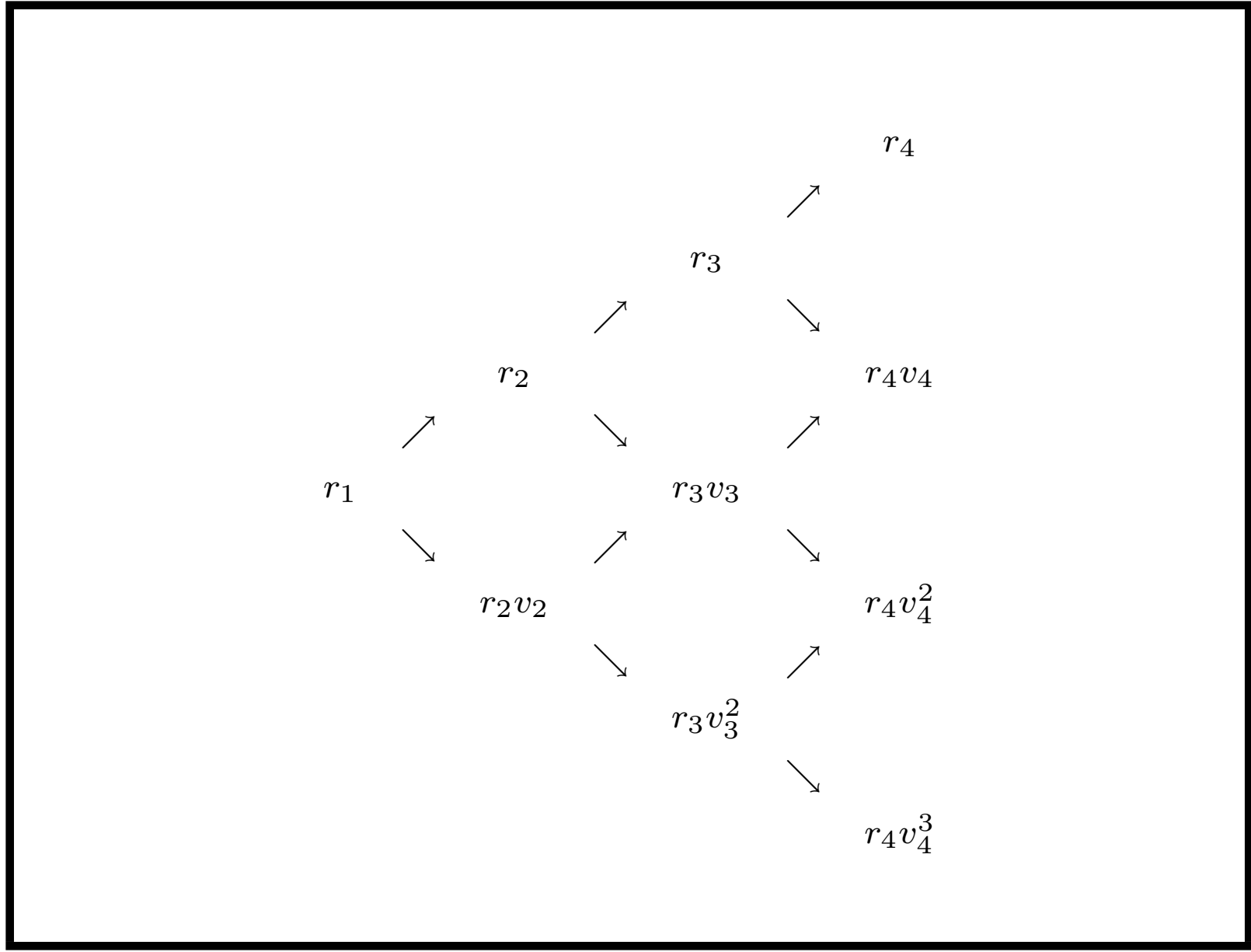
Problems with No-Arbitrage Models in General (concluded)

- This in effect says the model estimated at some time does not describe the term structure of interest rates and their volatilities at other times.
- Consequently, a model's intertemporal behavior is suspect, and using it for hedging and risk management may be unreliable.

The Black-Derman-Toy Model^a

- This model is extensively used by practitioners.
- The BDT short rate process is the lognormal binomial interest rate process described on pp. 747ff (repeated on next page).
- The volatility structure is given by the market.
- From it, the short rate volatilities (thus v_i) are determined together with r_i .

^aBlack, Derman, and Toy (BDT) (1990).



The Black-Derman-Toy Model (concluded)

- Our earlier binomial interest rate tree, in contrast, assumes v_i are given a priori.
 - A related model of Salomon Brothers takes v_i to be constants.
- Lognormal models preclude negative short rates.

The BDT Model: Volatility Structure

- The volatility structure defines the yield volatilities of zero-coupon bonds of various maturities.
- Let the yield volatility of the i -period zero-coupon bond be denoted by κ_i .
- P_u is the price of the i -period zero-coupon bond one period from now if the short rate makes an up move.

The BDT Model: Volatility Structure (concluded)

- P_d is the price of the i -period zero-coupon bond one period from now if the short rate makes a down move.
- Corresponding to these two prices are the following yields to maturity,

$$\begin{aligned}y_u &\equiv P_u^{-1/(i-1)} - 1, \\y_d &\equiv P_d^{-1/(i-1)} - 1.\end{aligned}$$

- The yield volatility is defined as $\kappa_i \equiv (1/2) \ln(y_u/y_d)$.

The BDT Model: Calibration

- The inputs to the BDT model are riskless zero-coupon bond yields and their volatilities.
- For economy of expression, all numbers are period based.
- Suppose inductively that we have calculated

$$r_1, v_1, r_2, v_2, \dots, r_{i-1}, v_{i-1}.$$

- They define the binomial tree up to period $i - 1$.
- We now proceed to calculate r_i and v_i to extend the tree to period i .

The BDT Model: Calibration (continued)

- Assume the price of the i -period zero can move to P_u or P_d one period from now.
- Let y denote the current i -period spot rate, which is known.
- In a risk-neutral economy,

$$\frac{P_u + P_d}{2(1 + r_1)} = \frac{1}{(1 + y)^i}. \quad (103)$$

- Obviously, P_u and P_d are functions of the unknown r_i and v_i .

The BDT Model: Calibration (continued)

- Viewed from now, the future $(i - 1)$ -period spot rate at time one is uncertain.
- Let y_u and y_d represent the spot rates at the up node and the down node, respectively, with κ^2 denoting the variance, or

$$\kappa_i = \frac{1}{2} \ln \left(\frac{P_u^{-1/(i-1)} - 1}{P_d^{-1/(i-1)} - 1} \right). \quad (104)$$

The BDT Model: Calibration (continued)

- We will employ forward induction to derive a quadratic-time calibration algorithm.^a
- Recall that forward induction inductively figures out, by moving forward in time, how much \$1 at a node contributes to the price (review p. 772(a)).
- This number is called the state price and is the price of the claim that pays \$1 at that node and zero elsewhere.

^aChen and Lyuu (1997); Lyuu (1999).

The BDT Model: Calibration (continued)

- Let the unknown baseline rate for period i be $r_i = r$.
- Let the unknown multiplicative ratio be $v_i = v$.
- Let the state prices at time $i - 1$ be P_1, P_2, \dots, P_i , corresponding to rates r, rv, \dots, rv^{i-1} , respectively.
- One dollar at time i has a present value of

$$f(r, v) \equiv \frac{P_1}{1 + r} + \frac{P_2}{1 + rv} + \frac{P_3}{1 + rv^2} + \dots + \frac{P_i}{1 + rv^{i-1}}.$$

The BDT Model: Calibration (continued)

- The yield volatility is

$$g(r, v) \equiv \frac{1}{2} \ln \left(\frac{\left(\frac{P_{u,1}}{1+rv} + \frac{P_{u,2}}{1+rv^2} + \cdots + \frac{P_{u,i-1}}{1+rv^{i-1}} \right)^{-1/(i-1)} - 1}{\left(\frac{P_{d,1}}{1+r} + \frac{P_{d,2}}{1+rv} + \cdots + \frac{P_{d,i-1}}{1+rv^{i-2}} \right)^{-1/(i-1)} - 1} \right).$$

- Above, $P_{u,1}, P_{u,2}, \dots$ denote the state prices at time $i - 1$ of the subtree rooted at the up node (like $r_2 v_2$ on p. 908).
- And $P_{d,1}, P_{d,2}, \dots$ denote the state prices at time $i - 1$ of the subtree rooted at the down node (like r_2 on p. 908).

The BDT Model: Calibration (concluded)

- Now solve

$$f(r, v) = \frac{1}{(1 + y)^i} \quad \text{and} \quad g(r, v) = \kappa_i$$

for $r = r_i$ and $v = v_i$.

- This $O(n^2)$ -time algorithm appears in the text.

The BDT Model: Continuous-Time Limit

- The continuous-time limit of the BDT model is

$$d \ln r = \left(\theta(t) + \frac{\sigma'(t)}{\sigma(t)} \ln r \right) dt + \sigma(t) dW.$$

- The short rate volatility clearly should be a declining function of time for the model to display mean reversion.
- In particular, constant volatility will not attain mean reversion.