Theory of Computation

Midterm Examination on November 10, 2015
Fall Semester, 2015

Problem 1 (20 points) Prove that the halting problem H is complete for RE (the set of recursively enumerable languages). (Recall that a problem A is complete for RE if every language in RE can be reduced to A.)

Problem 2 (20 points) Let $P(x, y)$ be a binary predicate, and let Q be the unary predicate defined by $Q(a) \Leftrightarrow \neg P(a, a)$. Show that Q is distinct from all the predicates P_b, defined by $P_b(a) \Leftrightarrow P(a, b)$.

Problem 3 (20 points) If the following language L is decidable, please give an algorithm; otherwise, prove that it is undecidable by reduction:

$$\begin{aligned} L = \{ & M \mid M \text{ is a Turing machine and there exists an input whose length} \\ & \text{is less than } |M| \text{ on which } M \text{ halts}\}. \end{aligned}$$

Problem 4 (20 points)

1. **(10 points)** Give the definitions of
   (a) The complement of a complexity class.
   (b) Being closed under complements.
2. **(10 points)** Show that if NP $\neq$ coNP, then P $\neq$ NP. (Half of the grade will be deducted if any of (a) and (b) above is wrongly answered.)

Problem 5 (20 points) Recall that NL = NSPACE (log n) and REACHABILITY $\in$ NL. Prove that REACHABILITY is NL-complete.