Theory of Computation

Homework 2

Due: 2015/10/20

Problem 1 Assume L is recursive. Prove that $\bar{L}$ is recursively enumerable. (You cannot use Lemma 14.)

Proof: By Proposition 2 on p. 52, it suffices to show that $\bar{L}$ is recursive. Let TM M decide L. We now construct a TM M' which accepts $\bar{L}$ as follows. Let x be any input of M'. If $M(x) =$ Yes, then $M'(x) = \nearrow$; otherwise, $M'(x) =$ Yes. It is clear that M' accepts $\bar{L}$. Consequently, $\bar{L}$ is recursively enumerable. ■

Problem 2 Let A and B be two sets and $A \subseteq B$. Prove that $|A| \leq |B|$.

Proof: Since $A \subseteq B$, the identity mapping $f(x) = x$ between A and $A \subseteq B$. Hence, $|A| \leq |B|$ by our definition. ■