Theory of Computation

homework 1
Due: 9/29/2015

Problem 1 The TM on p. 30 of the slides halts with a "yes" if the input string contains two consecutive 1's; otherwise, it halts at "no". That program assumes the input alphabet $\Sigma = \{\, 0, 1, \bigsqcup, \rhd \,\}$. Now, write a TM program for the same problem when $\Sigma = \{\, 0, 1, 2, \bigsqcup, \rhd \,\}$.

Ans: Assume $M = (K, \Sigma, \delta, s)$, where $K = (s, s_1, h)$, $\Sigma = \{0, 1, 2, \bigsqcup, \rhd\}$. Then

$p \in K$	$\sigma \in \Sigma$	$\delta(p, \sigma)$
s	$\rhd$	$(s, \rhd, \rightarrow)$
s	0	$(s, 0, \rightarrow)$
s	1	$(s_1, 1, \rightarrow)$
s	2	$(s, 2, \rightarrow)$
s_1	0	$(s, 0, \rightarrow)$
s_1	1	("yes", $1, -$)
s_1	2	$(s, 2, \rightarrow)$
s	$\sqcup$	("no", $\sqcup, -$)
s_1	$\sqcup$	("no", $\sqcup, -$)

■

Problem 2 Explain why the following Turing machine does not decide the language of polynomials with integer coefficients which have integer roots: The input represents a polynomial over variables $x_1, \ldots, x_n$ with integer coefficients.

1. Examine all possible integer values of $x_1, \ldots, x_n$.
2. Evaluate the polynomial on all of them.
3. If any of them evaluates to 0, accept; else reject.

Ans: The variables $x_1, \ldots, x_n$ have infinitely many possible integer values. A Turing machine would required infinite time to try them all. But we require that every stage in the Turing machine description be completed in a finite number of steps. ■