Theory of Computation

homework 3
Due: 11/19/2013

Problem 1 Prove that the following language is coNP-complete.

$$L_{\text{coNP}} = \{\phi\text{: a Boolean formula that is satisfied by every assignment}\}.$$

Ans: It is clear that L_{coNP} is in coNP by its definition. We then prove that every $L \in \text{coNP}$ can be reduced to L_{coNP}. First, we know that $\bar{L}$ (which is in NP) can reduce to SAT (an NP-complete problem). For every input $x \in \{0,1\}^*$ that reduction produces a formula ϕ_x that is satisfiable iff $x \in \bar{L}$. On p. 424 of the lecture notes, we know that L' is coNP-complete iff $\bar{L}'$ is NP-complete. Hence SAT COMPLEMENT is coNP-complete and $L \in \text{coNP}$ can reduce to SAT COMPLEMENT. As ϕ_x is unsatisfiable iff $x \in L$, we can readily see that the *same* reduction shows that L_{coNP} is coNP-complete.

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Problem 2 Given a set $S = \{a_1, a_2, ..., a_n\}$ and a number T, we ask if there exists a subset $S' \subseteq S$ such that $\sum_{a_i \in S'} a_i = T$. Prove that this problem is NP-complete.

Ans: An instance of KNAPSACK contains n items with values v_1, ..., v_n and weights w_1, ..., w_n, a weight limit W, and a goal K. KNAPSACK asks if there exists a subset $S \subseteq \{1, 2, ..., n\}$ such that $\sum_{i \in S} w_i \leq W$ and $\sum_{i \in S} v_i \geq K$. We now reduce KNAPSACK to our problem by simply letting $x_i = 0, 1$, $w_i = v_i$ and $W = K$ to give us the equation $\sum_{i \in S} w_i x_i = K$. Clearly, a solution to this instance exists if and only if a solution S exists such that $\sum_{a_i \in S'} a_i = T$. Since this version of KNAPSACK is NP-complete (refers to slide p. 393), our problem is hence NP-complete.

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