Theory of Computation

Homework 4

Due: 2010/12/14

Problem 1. Let $a, b \in \mathbb{N}$ and p be a prime. Show that $(a + b)^p = a^p + b^p \bmod p$.

Problem 2. The **permanent** of an $n \times n$ integer matrix A is defined as

$$\mathrm{perm}(A) = \sum_{\pi} \prod_{i=1}^{n} A_{i,\pi(i)}.$$

Above, π ranges over all permutations of n elements. (It is similar to determinant but without the sign.) Show that if A is the adjacency matrix (hence a 0/1 matrix) of a bipartitle graph G, then $\mathrm{perm}(A)$ equals the number of perfect matchings of G.