Theory of Computation

Mid-Term Examination on November 09, 2010

Fall Semester, 2010

Problem 1 (25 points) How many functions from $\{0, 1, 2\}^n$ to $\{0, 1, 2\}$ are there? (Hint: Do not write a^{b^c} as it is not clear whether it means $(a^b)^c$ or $a^{(b^c)}$.)

Ans: $3^{(3^n)}$. ∎

Problem 2 (25 points) Prove that $\text{NSPACE}(\log^2 n) \subseteq \text{TIME}(2^{(\log^4 n)})$. (Hint: You can use Savitch's theorem.)

Ans: Savitch's theorem implies $\text{NSPACE}(\log^2 n) \subseteq \text{SPACE}(\log^4 n)$. Hence $\text{NSPACE}(\log^2 n) \subseteq \text{SPACE}(\log^4 n) \subseteq \text{TIME}(O(1)^{(\log^4 n)})$. ∎

Problem 3 (25 points) Let $\mathbb{N}$ be the set of natural numbers. Does there exist a bijection between $2^{\mathbb{N}}$ and NP?

Ans: NP is countable because there are countably many Turing machines. So no bijections exist between the countable set NP and the uncountable $2^{\mathbb{N}}$. ∎

Problem 4 (25 points) Show that it is NP-hard to determine whether a Boolean expression in 3SAT form has at least two satisfying assignments. (Hint: What is the property of $F \wedge (x \vee y \vee z)$, where F is a 3SAT formula?)

Ans: We describe a logspace reduction from 3SAT to the problem in question. On input is a 3SAT expression F. The reduction outputs $F' = F \wedge (x \vee y \vee z)$, where x, y, z are variables that do not appear in F. Now if F is satisfiable, then F' has at least 7 satisfying assignments because $(x \vee y \vee z)$ is satisfied by seven assignments to x, y and z. Conversely, if F' has at least two satisfying assignments, it is clear that F must have at least one. ∎