Theory of Computation

Homework 2
Due: 2010/10/26

Problem 1. Derive a disjunctive normal form of

$$(x_1 \vee y_1) \wedge (x_2 \vee y_2) \wedge \cdots \wedge (x_n \vee y_n).$$

Problem 2. Prove that $\mathbf{NP} \neq \mathbf{SPACE}(n)$.

(Hint: You don't need to show $\mathbf{NP} \subsetneq \mathbf{SPACE}(n)$ or $\mathbf{SPACE}(n) \subsetneq \mathbf{NP}$ since they are open questions so far as we know. All you need to do is to prove these two sets are **unequal**.

A log-space reduction from language L_1 to language L_2 is a function R which can be computed by a deterministic log-space Turing machine such that $x \in L_1$ iff $R(x) \in L_2$. In the proof, you can treat log space and polynomial time interchangeably. So as long as your reduction R runs in polynomial time, it is fine.

A complexity class $\mathbf{C}$ is closed under log-space reduction if for any log-space reduction R from L_1 to L_2, $L_1 \in \mathbf{C}$ if $L_2 \in \mathbf{C}$. Show first that $\mathbf{NP}$ is closed under log-space reduction. Then show that $\mathbf{SPACE}(n)$ is not closed under log-space reduction by the Space Hierarchy Theorem (the version in the textbook is sufficient). For this, suppose $L_1 \in \mathbf{SPACE}(n^2)$ but $L_1 \notin \mathbf{SPACE}(n)$. Now ask yourself what is the space complexity of deciding "$x \in L_2$?", where L_2 consists of those strings $x \in L_1$ padded with $n^2 - n$ redundant symbols after x with $|x| = n$.)