

# Theory of Computation

## Homework 4

Due: 2009/12/15

**Problem 1.** Let  $a, b \in \mathbb{N}$  and  $p$  be a prime. Show that  $(a + b)^p = a^p + b^p \pmod{p}$ .

**Problem 2.** Let  $d$  be a positive integer. Show that

$$\left| \left\{ x \in \mathbb{R} \mid \exists a_0, \dots, a_d \in \{1, 2, 3\}, \sum_{i=0}^d a_i x^i = 0 \right\} \right| \leq d 3^{d+1},$$

i.e., degree- $d$  polynomials with coefficients in  $\{1, 2, 3\}$  have at most  $d 3^{d+1}$  distinct roots altogether.