Theory of Computation

Solutions to Homework 2

Problem 1. We call a boolean function $f : \{0,1\}^k \to \{0,1\}$ symmetric if $f(x_1, x_2, \ldots, x_k)$ depends only on $\sum_{i=1}^{k} x_i$. How many symmetric boolean functions of k variables are there?

Solution. 2^{k+1}. □

Problem 2. It is known that the language

$$\{M : M \text{ halts on all inputs}\}$$

is undecidable. Prove or disprove that the following restricted language

$$L_{1000} = \{M : M \text{ halts on all inputs and } M \text{ is at most 1000 bits long}\}$$

is undecidable.

Proof. There exists a TM that keeps all the $M \in L$ in its states (which is finite in number) and tests if the input is one of them. Therefore, L_{1000} is decidable. □