Theory of Computation

Homework 3

Due: 2008/11/25

Problem 1. Define MAJORITY-3-COLORING to be the problem of asking whether the nodes of a given undirected graph $G = (V, E)$ can be colored with $0, 1$ or 2 such that the following two conditions hold:

1. No two adjacent nodes have the same color.

2. At least $|V|/2$ nodes have the color 2.

Find a logarithmic-space reduction from 3-COLORING to MAJORITY-3-COLORING or prove that such reductions cannot exist.

Problem 2. Let p be an odd prime and $\phi(\cdot)$ be Euler's function as in the slides. Prove or disprove that

$$\frac{|\{2i \bmod p \mid 1 \le i \le p\}|}{p} > \frac{\phi(3p)}{3p - 1}.$$