Theory of Computation

Mid-Term Examination on November 11, 2008
Fall Semester, 2008

Problem 1 (30 points)**.** Show that REACHABILITY $\in$ NL.

Proof. Given a directed graph $G = (V, E)$ and $x, y \in V$, we nondeterministically guess a path and accepts if and only if it goes from x to y in G. Instead of storing the whole guessed path, we always keep a current node, guess its next node and verify that there is an edge connecting them. Once a new node is guessed and verified, the space used to store other nodes can be recycled. Hence the space usage is logarithmic in the size of the input. $\square$

Problem 2 (20 points)**.** Does there exist a logarithmic-space reduction from PALINDROME to CIRCUIT VALUE? Briefly justify your answer.

Proof. Yes because PALINDROME $\in$ P and CIRCUIT VALUE is P-complete. $\square$

Problem 3 (30 points)**.** Prove or disprove that MAX CUT remains NP-hard for graphs whose number of nodes is a multiple of 3.

Proof. We prove the statement by presenting a logarithmic-space reduction from MAX CUT to the said problem. Given an undirected graph $G = (V, E)$ and a positive integer K, the reduction creates a graph G' by adding at most two isolated nodes to G so that the total number of nodes is a multiple of 3. Then it outputs G' and K. As isolated nodes contribute none to the size of any cut, G has a cut of size at least K if and only if the same holds for G'. $\square$

Problem 4 (20 points)**.** Let L be a recursive language. Prove that it is recursively enumerable.

Proof. Let M be a deterministic Turing machine that accepts every string in L and rejects all other strings. Clearly, it can be modified to accept strings in L and enter an infinite loop on other strings. $\square$

Problem 5 (30 points)**.** Is it possible that exactly one inclusion in the chain

$$\mathrm{NL} \subseteq \mathrm{SPACE}(\log^9 n) \subseteq \mathrm{PSPACE} \subseteq \mathrm{EXP}$$

is proper?

Proof. No.

By Savitch's theorem, $\mathrm{NL} \subseteq \mathrm{SPACE}(\log^2 n)$. Then the space hierarchy theorem gives $\mathrm{NL} \subseteq \mathrm{SPACE}(\log^2 n) \neq \mathrm{SPACE}(\log^9 n)$ and $\mathrm{SPACE}(\log^9 n) \neq \mathrm{PSPACE}$. So at least two inclusions are proper. $\square$