Theory of Computation

Homework 1
Due: 2008/10/14

Problem 1. Briefly describe a Turing machine that accepts a string $x \in \{0,1\}^*$ if and only if x contains at least one 0. You do not need to specify the exact states and state transitions of the Turing machine. Just sketch the idea.

Problem 2. Let M be a one-string deterministic Turing machine and denote by $T(M, \epsilon)$ the number of configurations that M goes through on the empty input ϵ. Denote by $(q^{(n)}, w^{(n)}, u^{(n)})$ the n-th configuration of M on ϵ, $1 \leq n \leq T(M, \epsilon)$, where we adhere to the representation of configurations in the slides. For $1 \leq n \leq T(M, \epsilon)$ and $i \geq 1$, write $A^{(n)}[i]$ for the i-th symbol, counting from left to right, of the concatenation of $w^{(n)}, u^{(n)}$ and an infinite string of $\sqcup$s. That is, $A^{(n)}[i]$ is the i-th symbol of M's string at the n-th configuration starting from the input ϵ. Briefly argue why $A^{(n)}[k+1]$ is uniquely determined given $q^{(n-1)}$, $A^{(n-1)}[k]$, $A^{(n-1)}[k+1]$, $A^{(n-1)}[k+2]$ and the length of $w^{(n-1)}$, for each $2 \leq n \leq T(M, \epsilon)$ and $k \geq 1$.