Theory of Computation

Homework 3
Due: 2008/05/01

Problem 1. Show that there exist a constant $c > 0$ and a language $L \notin \mathrm{NTIME}(n^c)$ such that L is logspace reducible to a language in $\mathrm{NTIME}(n^c)$. You may use the nondeterministic time hierarchy theorem, proved by Cook in 1972, which implies $\mathrm{NTIME}(n^a) \subsetneq \mathrm{NTIME}(n^b)$ for all $b > a > 1$. (Hint: The Cook-Levin theorem states that every language in NP is logspace reducible to SAT, which lies in $\mathrm{NTIME}(n^c)$ for some constant $c > 0$. The nondeterministic time hierarchy theorem guarantees the nonemptiness of $\mathrm{NP} \setminus \mathrm{NTIME}(n^c)$.)

Problem 2. Prove that

$$\left\{ x_1, \ldots, x_n, w \in \mathbb{N} \mid \exists S \subseteq \{1, \ldots, n\} \text{ such that } \sum_{i \in S} x_i = w \geq \frac{\sum_{i=1}^{n} x_i}{2} \right\}$$

is NP-complete. You may use reductions from any problem shown to be NP-complete in class or in the textbook. For example, the following problem is shown to be NP-complete on pages 349–355 of the slides:

> Given positive integers $v_1, \ldots, v_n, K$, does there exist a subset of $\{v_1, \ldots, v_n\}$ that adds up to exactly K?