Answers to the Final Examination on January 12, 2005

Problem 1 Answer:

This problem is *not* TSP (D) COMPLEMENT, which asks if every tour has a total distance greater than B. This problem is in NP as it is easy to verify if a tour has the quality. But how hard is it? Let $d_{ij} \geq 0$ be the distance between nodes i and j. Define $M \equiv \max_{i,j} d_{ij}$. It is NP-complete. Here is the reason. We reduce TSP (D) to our problem. Create a new graph with distance $M - d_{ij}$ between nodes i and j. The original graph has a tour at least B if and only if the new graph has a tour at most $nM - B$. Hence this problem is most likely not in coNP.

Problem 2 Answer:

For all $L \in \text{DP}$, there exist NTMs M' and M'' such that if $x \in L$, then $M'(x) =$ "yes" for some computation paths, and if $x \notin L$, then $M''(x) =$ "no" for some computation paths, respectively. Then we could construct a new NTM M that simulates both M' and M''. If M' accepts the input, then M accepts the input, else M halt. If M'' rejects the input, then M rejects the input, else M halt. Clearly the claim follows.

Problem 3 (30 points) Answer:

Add up the relations for $t(1), t(2), t(3), \ldots, t(n-1)$ to obtain

t(1)+t(2)+t(3)+...+t(n-1)$\leq \frac{t(0)+t(1)+2t(2)+\ldots+2t(n-2)+t(n-1)+t(n)}{2} + n - 1$,

$\Rightarrow \frac{t(1)+t(n-1)-t(n)}{2} \leq n - 1$,

$\Rightarrow t(1) + t(n-1) - t(n) \leq 2n - 2$,

$\Rightarrow t(1) + t(n-1) - t(n) + t(n) - t(n-1) \leq 2n - 2 + 1$,

$\Rightarrow t(1) \leq 2n - 1$

Simplify it to yield

$$t(1) \leq 2n - 1.$$

Add up the relations for $t(2), t(3), \ldots, t(n-1)$ to obtain

t(2)+t(3)+...+t(n-1)$\leq \frac{t(1)+2t(2)+\ldots+2t(n-2)+t(n-1)+t(n)}{2} + n - 2$,

$\Rightarrow \frac{t(2)}{2} \leq \frac{t(1)+t(n)-t(n-1)}{2} + n - 2$,

$\Rightarrow t(2) \leq t(1) + t(n-1) + t(n) + 2(n-2),$
$\Rightarrow t(2) \leq t(1) + 2n - 4 + 1,$
$\Rightarrow t(2) \leq 4n - 4$
etc.

Problem 4 (20 points) Answer:

Please refer the page 360 of lecture note.