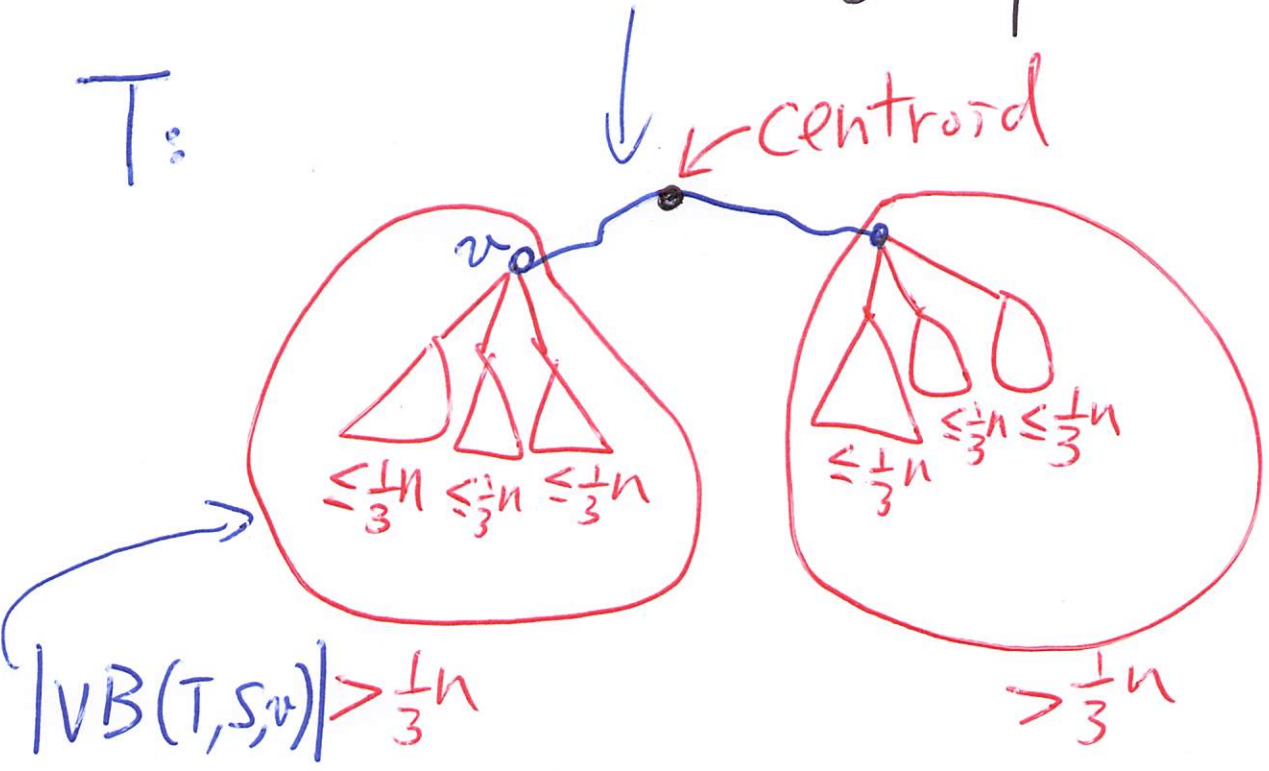


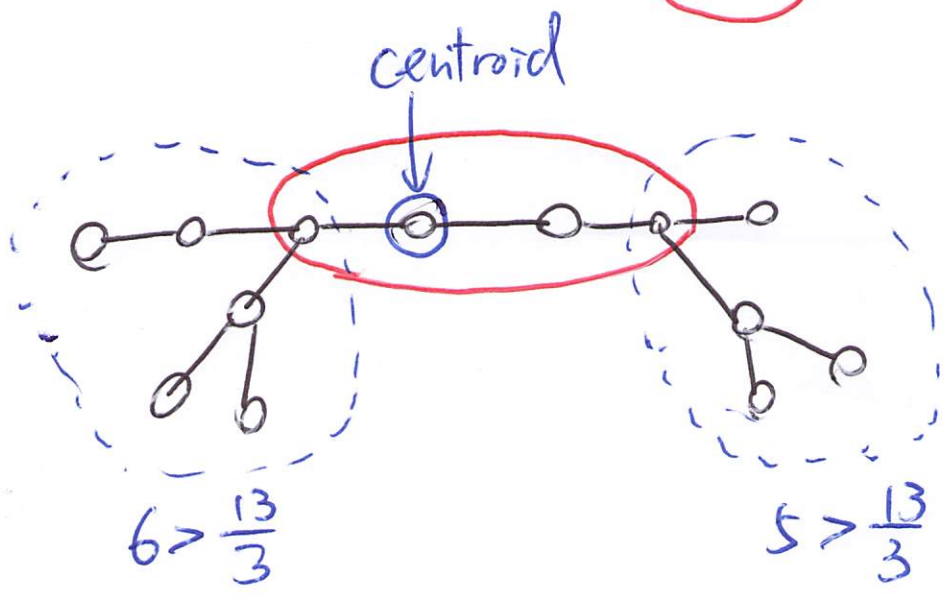
(S)

a minimal  $\frac{1}{3}$ -separator  $v$

$T$ :



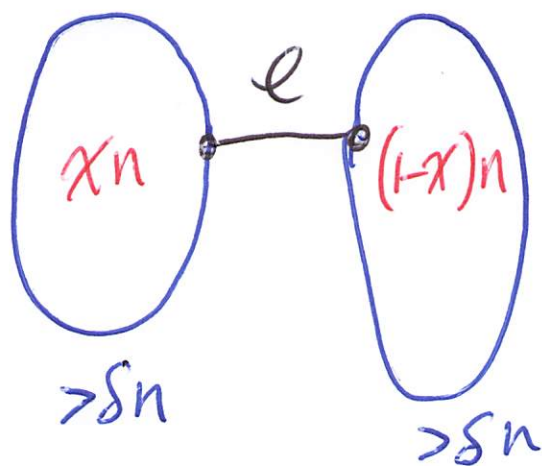
( If  $v$  is a leaf of a minimal  $\delta$ -separator, then  $|VB(T, S, v)| > \delta n$ . )



Kun-Mao Chaw @2021

$S$ : a minimal  $\delta$ -separator ( $0 < \delta \leq \frac{1}{2}$ )

$e$ : an edge in the edge set of  $S$



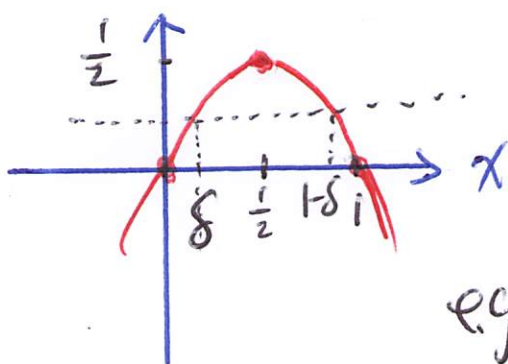
$$\begin{aligned} x > \delta &\Rightarrow \delta < x < 1 - \delta \\ 1 - x > \delta &\Rightarrow \delta < x < 1 - \delta \end{aligned}$$

The routing load of  $e$  is

$$\begin{aligned} 2x(1-x)n^2 &= 2\left(-x^2 + x - \frac{1}{4} + \frac{1}{4}\right)n^2 \\ &= \left(-2\left(x - \frac{1}{2}\right)^2 + \frac{1}{2}\right)n^2 \end{aligned}$$

$$\leq \frac{1}{2}n^2$$

$$2x(1-x)$$



Furthermore, since  $\delta < x < 1 - \delta$ ,

$$2x(1-x)n^2 > 2\delta(1-\delta)n^2$$

eg.  $\delta = \frac{1}{3} \Rightarrow$  the routing load  $> 2 \cdot \frac{1}{3} \cdot \frac{2}{3} n^2 = \frac{4}{9} n^2$

(2)