

A note on optimal communication spanning trees

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The *optimal communication spanning tree* (OCT) problem is defined as follows. Let $G = (V, E, w)$ be an undirected graph with nonnegative edge length function w . We are also given the requirements $\lambda(u, v)$ for each pair of vertices. For any spanning tree T of G , the communication cost between two vertices is defined to be the requirement multiplied by the path length of the two vertices on T , and the communication cost of T is the total communication cost summed over all pairs of vertices. Our goal is to construct a spanning tree with minimum communication cost. That is, we want to find a spanning tree T such that $\sum_{u, v \in V} \lambda(u, v) d_T(u, v)$ is minimized.

The requirements in the OCT problem are arbitrary nonnegative values. By restricting the requirements, several special cases of the problem were defined in the literature. We list the problems in the following, in which $r : V \rightarrow Z_0^+$ is a given vertex weight function and $S \subset V$ is a set of k vertices given as sources.

- $\lambda(u, v) = 1$ for each $u, v \in V$: This version is the MINIMUM ROUTING COST SPANNING TREE (MRCT) problem.
- $\lambda(u, v) = r(u)r(v)$ for each $u, v \in V$: This version is called the OPTIMAL PRODUCT-REQUIREMENT COMMUNICATION SPANNING TREE (PROCT) problem.
- $\lambda(u, v) = r(u) + r(v)$ for each $u, v \in V$: This version is called the OPTIMAL SUM-REQUIREMENT COMMUNICATION SPANNING TREE (SROCT) problem.
- $\lambda(u, v) = 0$ if $u \notin S$: This version is called the p -SOURCE OCT (p -OCT) problem. In other words, the goal is to find a spanning tree minimizing $\sum_{u \in S} \sum_{v \in V} \lambda(u, v) d_T(u, v)$.
- $\lambda(u, v) = 1$ if $u \in S$, and $\lambda(u, v) = 0$ otherwise: This version is called the p -Source MRCT (p -MRCT) problem. In other words, the goal is to find a spanning tree minimizing $\sum_{u \in S} \sum_{v \in V} d_T(u, v)$.

The relationship of the different versions of the OCT problems is illustrated in Figure 1. Table 1 summarizes the results.

Bibliographic Notes and Further Reading

The application of Yair Bartal's algorithm [1] to approximating the OCT problem was pointed out in [10]. A better bound was later given by Jittat Fakcharoenphol, Satish Rao, and Kunal Talwar [5].

The PROCT and SROCT problems were introduced in [8]. In that paper, Bang Ye Wu, Kun-Mao Chao, and Chuan Yi Tang gave a 1.577-approximation algorithm for the PROCT problem and a 2-approximation algorithm for the SROCT problem. The PTAS using the Scaling-and-Rounding technique for a PROCT problem was presented in [9] by the same authors. Scaling the input instances is a technique that has been used to balance the running time and the approximation ratio. For example, Oscar H. Ibarra and Chul E. Kim used the scaling technique to develop a *fully polynomial time approximation scheme* (FPTAS) for the

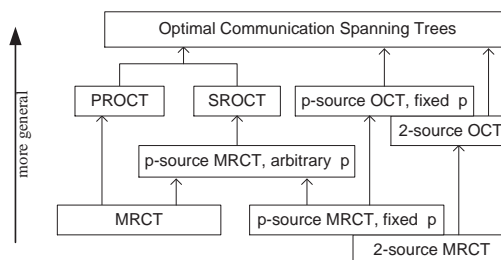


Figure 1: The relationship of the OCT problems

Table 1: The objectives and currently best ratios of the OCT problems.

Problem	Objective	Ratio
OCT	$\sum_{u,v} \lambda(u,v)d_T(u,v)$	$O(\log n)$
PROCT	$\sum_{u,v} r(u)r(v)d_T(u,v)$	PTAS
SROCT	$\sum_{u,v} (r(u) + r(v))d_T(u,v)$	2
MRCT	$\sum_{u,v} d_T(u,v)$	PTAS
p -MRCT	$\sum_{u \in S} \sum_{v \in V} d_T(u,v)$	2
2-MRCT	$\sum_v (d_T(s_1, v) + d_T(s_2, v))$	PTAS

knapsack problem [4], and some improvement was made by Eugene L. Lawler [6]. A nice explanation of the technique can also be found in [3](pp. 134–137).

The NP-hardness of the 2-MRCT was shown by Bang Ye Wu [7], in which the reduction is from the EXACT COVER BY 3-SETS (X3C) problem ([SP2] in [3]). The transformation is simpler and easier to extend to the weighted case, which is designed to show the NP-hardness of the p -MRCT problem for any fixed p . A similar reduction (for 2-MRCT) was also shown by Harold Connamacher and Andrzej Proskurowski [2]. They showed that the 2-MRCT problem is NP-hard. The PTAS for the 2-MRCT problem also appeared in [7]. In addition to the PTAS for the 2-MRCT problem, there is also a PTAS for the weighted 2-MRCT problem. But the PTAS works only for metric inputs and the counterpart on general graphs was left as an open problem.

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