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Nov., 2010
April, 2019

Integer k

Take $k = \frac{2}{\delta} - 3$.

$\Rightarrow \delta = \frac{2}{k+3}$

$$\frac{1}{1-\delta} = \frac{1}{1-\left(\frac{2}{k+3}\right)} = \frac{k+3}{k+1}$$

An optimal k-star of a metric graph is a $\frac{k+3}{k+1}$-approximation of an MRCT.

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k-star : $(S, T, \mathcal{L})$

internal nodes, tree topology, leaves.

n^k $\quad n^{k-1}$ $\quad$ constant

$$\boxed{O\left(\binom{n}{k} \times \binom{n-1}{k-1} \times k^{k-2}\right) \times O(n^3) = O(n^{2k+2}) \text{ time}}$$

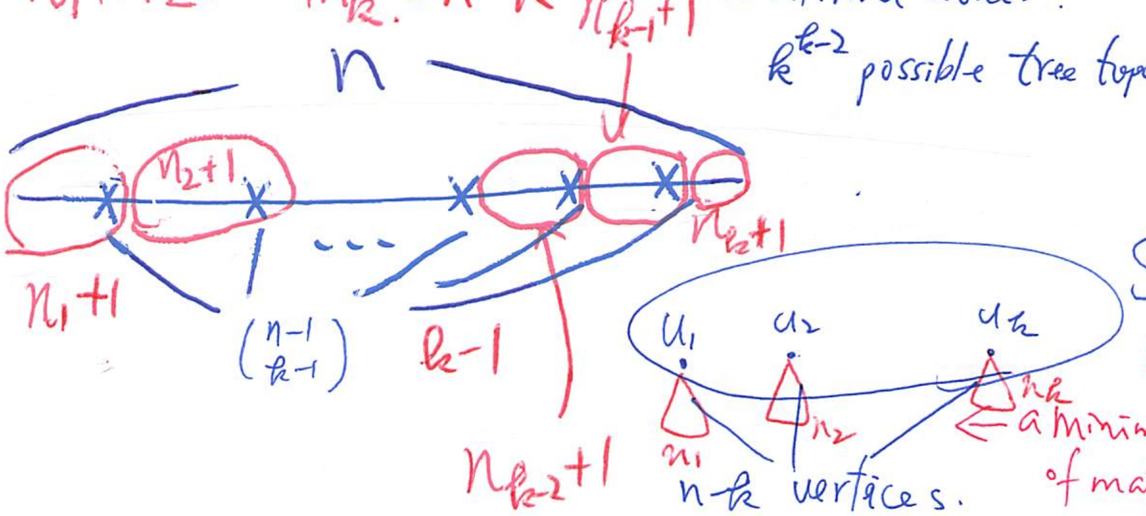
$v_1 \; v_2 \; \cdots\cdots \; v_n$

$\Downarrow \binom{n}{k}$

$n_1 + n_2 + \cdots + n_k = n - k$

k internal nodes.

k^{k-2} possible tree topologies.



$O(n^3)$-time (the assignment Problem)

$\Leftarrow$ a minimum-cost way of matching which obeys the degree constraints.

$k=7$

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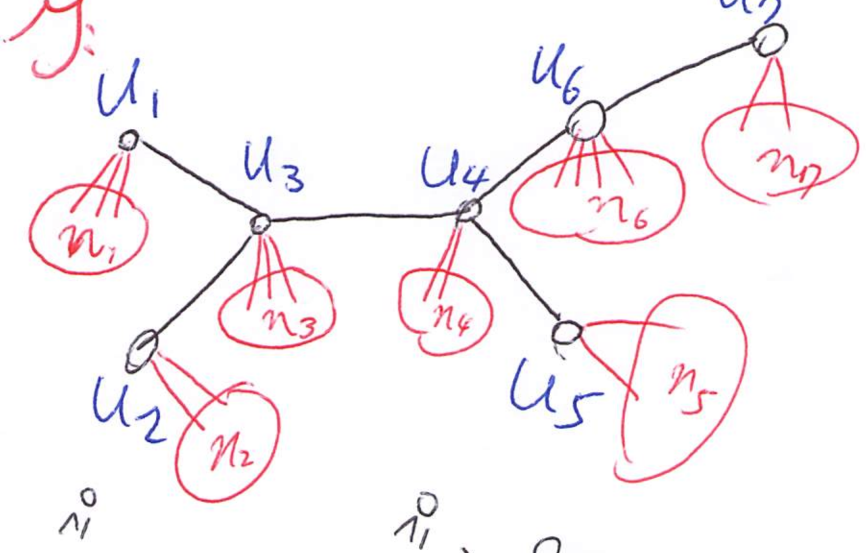
An optimal 7-star of a metric graph is a $\frac{7+3}{7+1}=\frac{5}{4}$-approximation of an MRCT.

* $\binom{n}{7}$ possible combinations of 7 internal nodes
* $7^{7-2}=7^5$ possible tree topologies for each 7-internal-node combination.
* For each tree topology, $\binom{n-1}{7-1}=\binom{n-1}{6}$ possible assignments of numbers of $n-7$ vertices to the 7 internal nodes.

e.g. T:



2 $(\mathring{n}_1, \mathring{n}_2, \ldots, \mathring{n}_7)$ & $n_1+n_2+\cdots+n_7=n-7$

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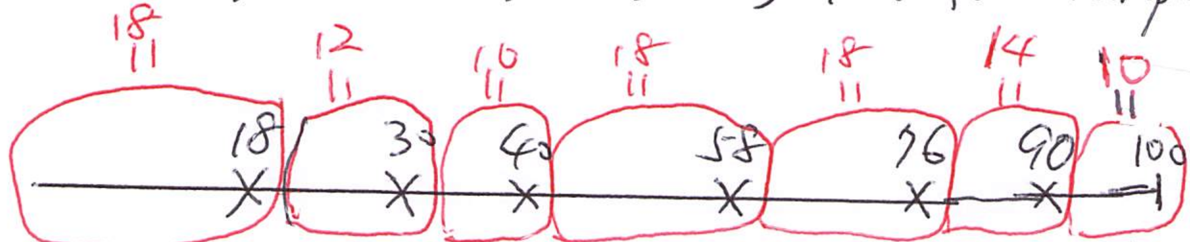
e.g. $n=100$

$k=7$

Choose $k-1=6$ numbers from $n-1=99$ number. $(1, 2, \cdots, 99)$

Take $(18, 30, 40, 58, 76, 90)$ for example.



$\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$

$n_1 = 18-1 = 17$ $\quad n_2 = 11$ $\quad n_3 = 9$ $\quad n_4 = 17$ $\quad n_5 = 17$ $\quad n_6 = 13$ $\quad n_7 = 9$

$$(n_1, n_2, n_3, n_4, n_5, n_6, n_7) = (17, 11, 9, 17, 17, 13, 9)$$

Note all these $n-k$ nodes are connected to T with a routing load of $2(n-1)$.

Thus, the best way of connecting these nodes to T is obtained by finding a minimum-cost way of matching up the nodes to $(u_1, u_2, \cdots, u_7)$ obeying the degree constraints $(n_1, n_2, \cdots, n_7)$.

This is an assignment problem which can be solved in $O(n^3)$ time.