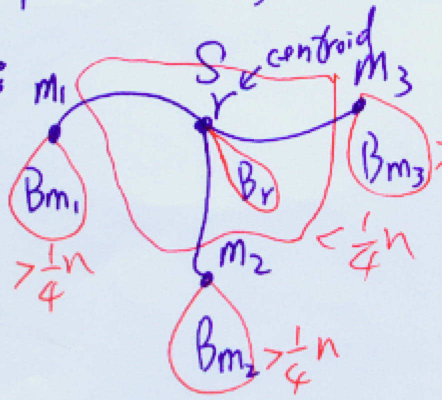
$\hat{T}$: an optimal MRCT; S: a minimal $\frac{1}{4}$-separator of $\hat{T}$ Kun-Mao Chao

Oct., 2010

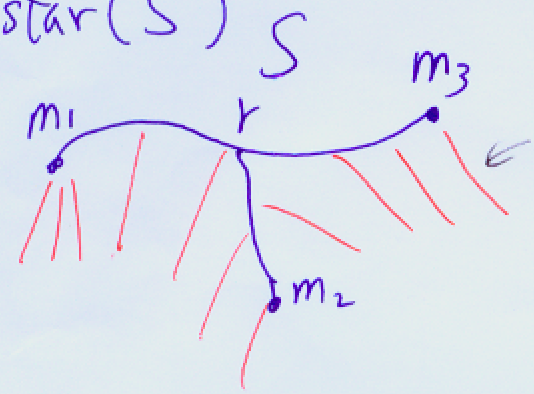
Oct. 2011

$\hat{T}$:



$$C(\hat{T}) \geq \frac{3}{2}n\sum_v d_{\hat{T}}(v,S) + \frac{3}{8}n^2 w(S)$$

$Y \in star(S)$



$\leftarrow d_Y(v,S) = d_G(v,S)$

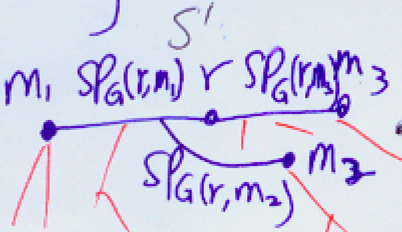
$$C(Y) \leq 2n\sum_v d_{Y \to G}(v,S) + \frac{1}{2}n^2 w(S)$$

$$\leq 2n\sum_v d_{\hat{T}}(v,S) + \frac{1}{2}n^2 w(S)$$

$$\frac{C(Y)}{C(\hat{T})} \leq \max\left\{\frac{2n}{\frac{3}{2}n}, \frac{\frac{1}{2}n^2}{\frac{3}{8}n^2}\right\} = \frac{4}{3}.$$

The thing is that we don't have "S"!

Y'



$\leftarrow d_{Y'}(v,S') = d_G(v,S')$ $\quad w(S') \leq w(S)$

$$C(Y') \leq 2n\sum_v d_G(v,S') + \frac{1}{2}n^2 w(S')$$

$(\# > \frac{3}{4}n)$ For B_{m_1}, B_{m_2}, B_{m_3}, and B_r, $d_G(v,S') \leq \min\{d_G(v,m_1), d_G(v,m_2), d_G(v,m_3), d_G(v,r)\} \leq d_{\hat{T}}(v,S)$

$(\# < \frac{1}{4}n)$ For others, $d_G(v,S') \leq d_{\hat{T}}(v,S) + \frac{1}{2}w(S)$

$$C(Y') \leq 2n\sum_v d_G(v,S') + \frac{1}{2}n^2 w(S)$$

$$\times \frac{1}{4}n \times \frac{1}{2}w(S)$$

$$\leq 2n\sum_v d_{\hat{T}}(v,S) + \frac{3}{4}n^2 w(S) \leq 2C(\hat{T})$$

$\uparrow$!

Kun-Mao Chao

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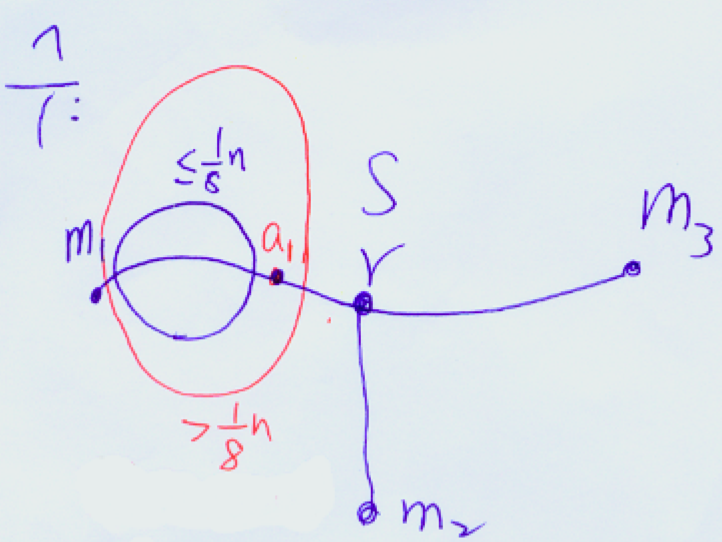
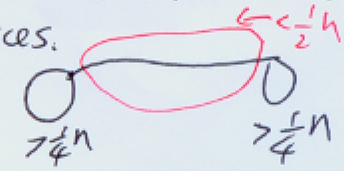
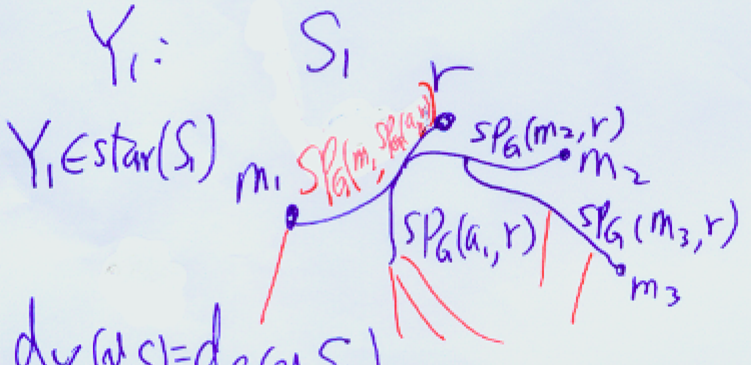


For degenerated cases, you might need a few more vertices.





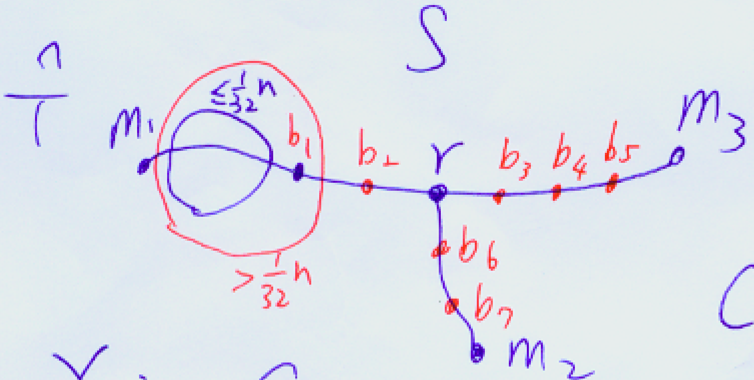
$d_{Y_1}(v, S_1) = d_G(v, S_1)$

$$C(Y_1) \le 2n \sum_v d_G(v, S_1) + \frac{1}{2}n^2 w(S_1)$$

$\times \frac{1}{8}n \times \frac{1}{2}w(S)$

$$\le 2n \sum_v d_{\hat{T}}(v, S) + \frac{5}{8}n^2 w(S)$$

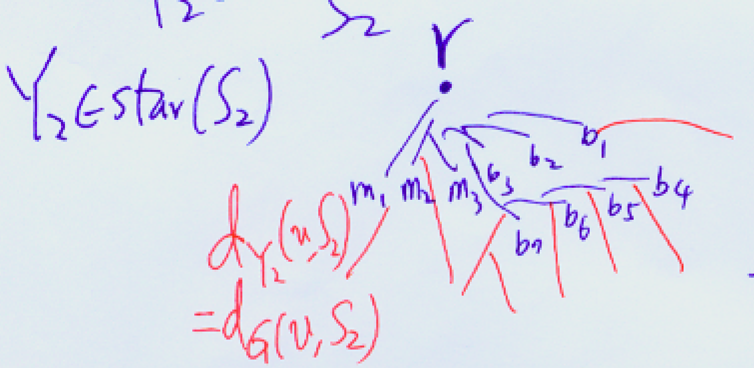
$$\frac{C(Y_1)}{C(\hat{T})} \le \max\left\{ \frac{2n}{\frac{3}{2}n}, \frac{\frac{5}{8}n^2}{\frac{3}{8}n^2} \right\} = \frac{5}{3}$$



$$C(Y_2) \le 2n \sum_v d_G(v, S_2) + \frac{1}{2}n^2 w(S_2)$$

$\times \frac{1}{32}n \times \frac{1}{2}w(S)$

$$\le 2n \sum_v d_{\hat{T}}(v, S) + \frac{17}{32}n^2 w(S)$$



$d_{Y_2}(v, S_2)$
$= d_G(v, S_2)$

$$\frac{C(Y_2)}{C(\hat{T})} \le \max\left\{ \frac{2n}{\frac{3}{2}n}, \frac{\frac{17}{32}n^2}{\frac{3}{8}n^2} \right\} = \frac{17}{12}$$

$$\frac{\frac{1}{2}n^2 + \Delta n^2}{\frac{3}{8}n^2} = \frac{4}{3} + \boxed{\frac{8}{3}\Delta}$$

$\Delta = \frac{1}{32} \Rightarrow \frac{4}{3} + \frac{8}{3} \times \frac{1}{32} = \frac{17}{12}$

$\Delta = \frac{1}{8} \Rightarrow \frac{4}{3} + \frac{8}{3} \times \frac{1}{8} = \frac{5}{3}$

$\Delta = \frac{1}{10000}$
$\Rightarrow \frac{4}{3} + \frac{8}{3} \times \frac{1}{10000} \approx \frac{4}{3}$