The Complexity of the Network Design Problem

D.S. Jihnson, J.K. Lenstra, A.H.G. Rinnooy Kan

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Outline

- Introduction
- P and NP
- Network Design Problem (NDP)
- KNAPSACK and NDP
- SNDP is NP-complete
- Proof of SNDP
- Conclusion

Introduction

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Introduction

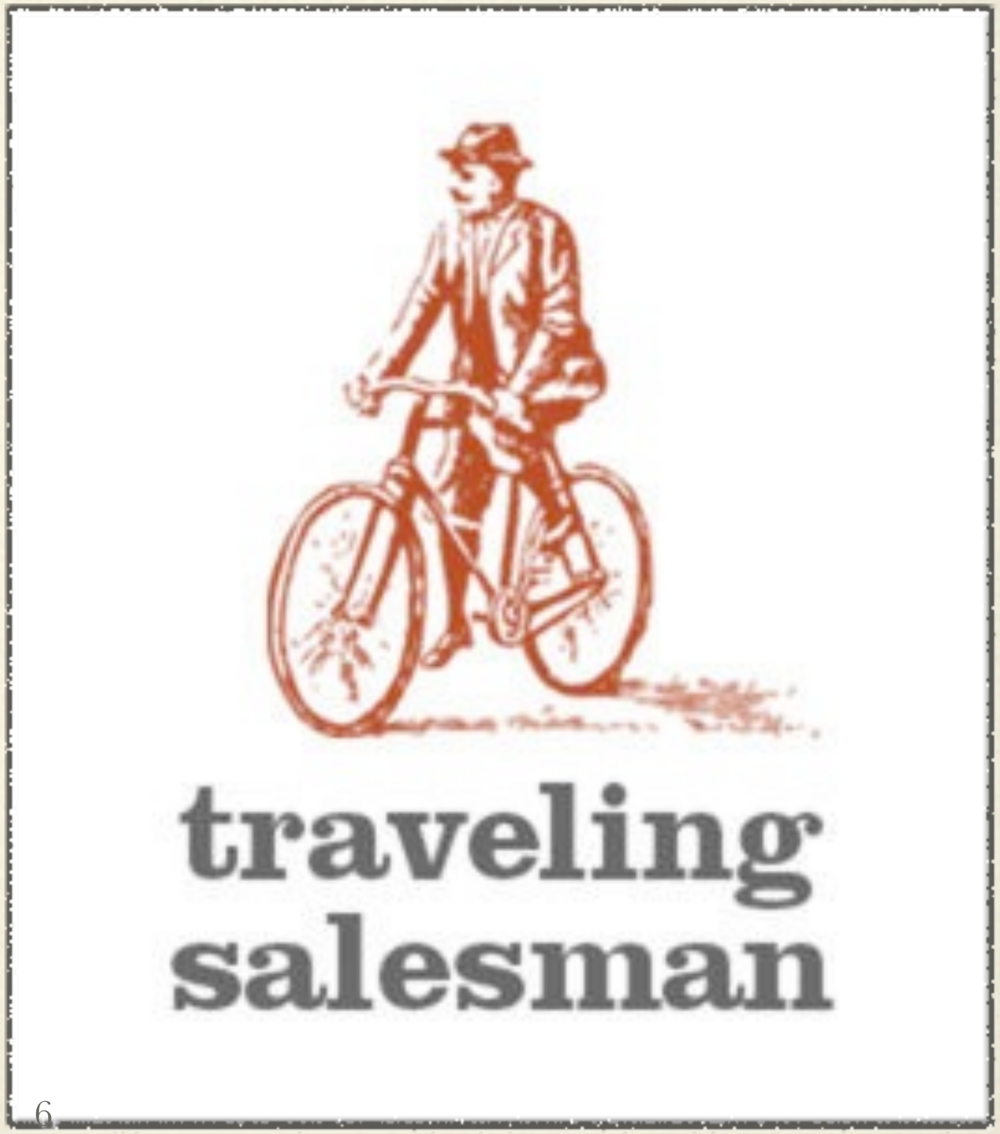
- Why we use computer?
- Solve computational problems!
- pure theoretical CS
- applied Mathematics

Introduction

- For some specific purposes:
- Optimization- define energy/cost/profit...... functions
- Maximize/minimize them

Introduction

- Opt. Examples:
- Shortest path
- Traveling salesman
- Texture synthesis
- Photo cuts



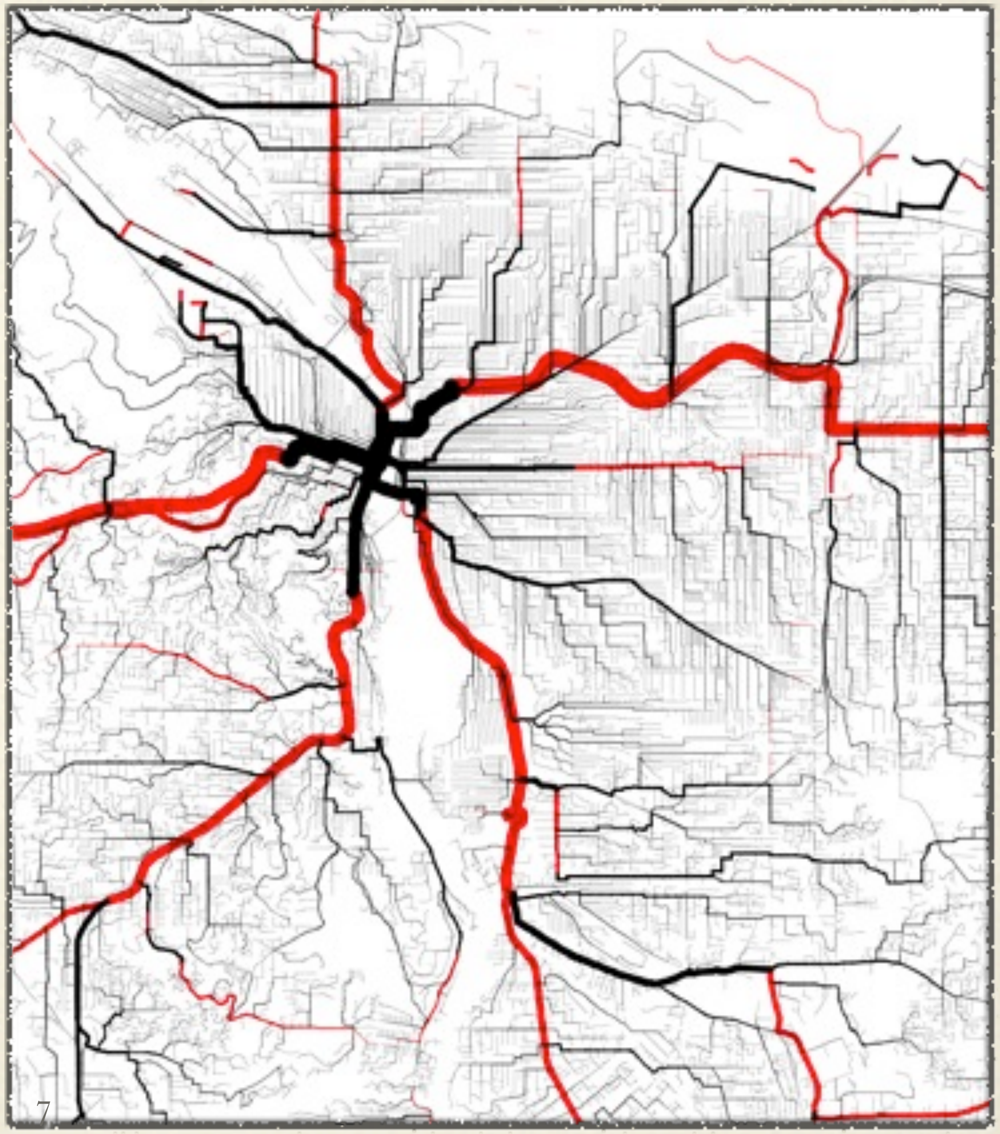
Introduction

- Opt. Examples:
- Shortest path
- Traveling salesman
- Texture synthesis
- Photo cuts

We just met this
man in midterm!

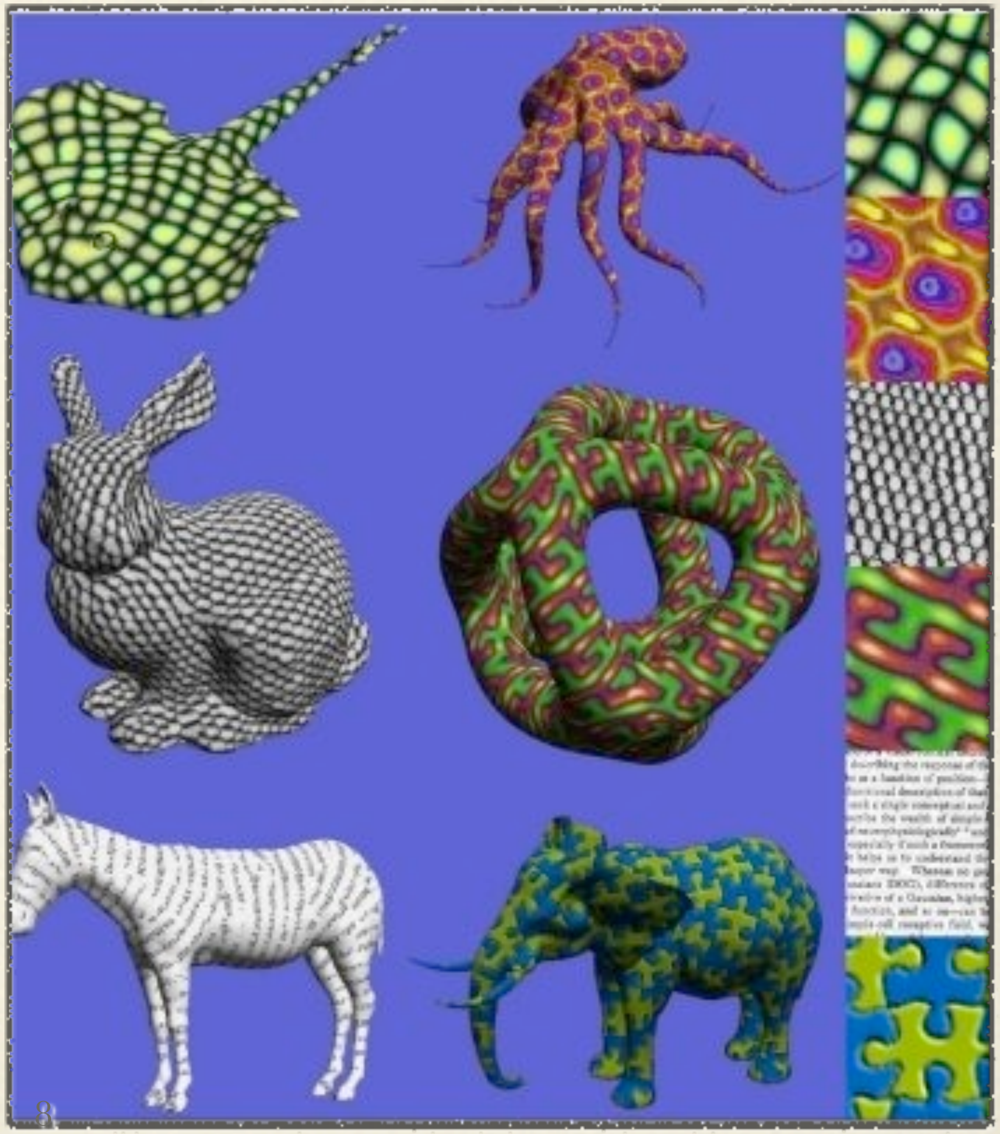
Introduction

- Shortest path:
- weighted graph
- directed/undirected
- Bellman–Ford/ Dijkstra



Introduction

- Texture synthesis:
- define energy functions
- overlap and immerse texture patches
- computed by energy function minimization

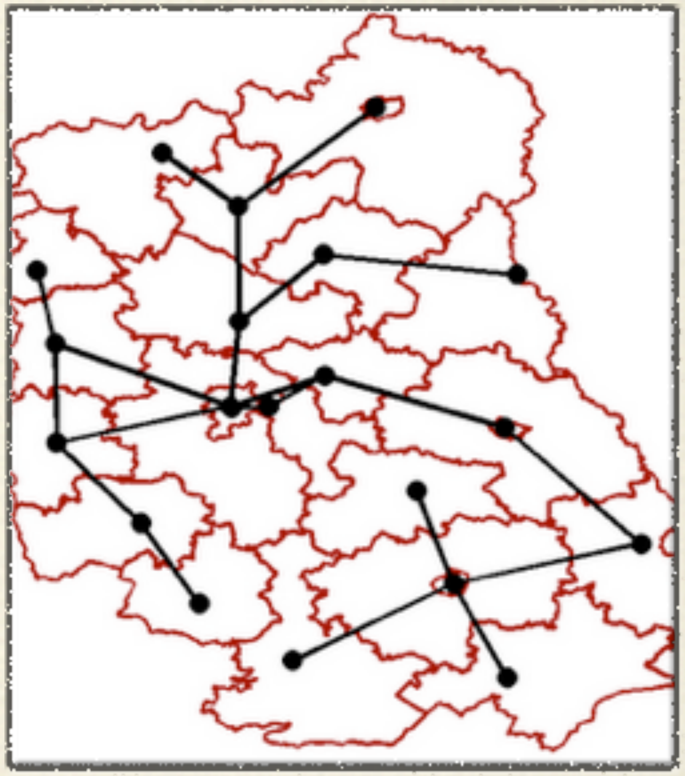


Introduction

- Photo cuts:
- define energy function
- combine photos for panorama purpose
- computed by energy function minimization

Introduction

- Network Design Problem
- given a undirected graph
- find subgraph connects all vertices
- minimize sum of shortest path weights
- subject to budget constraint



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Introduction

- This paper try to:
- show NP-completeness of NDP
- even for special case:
  all edge weights are equal and
  budget restricts to spanning tree

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Introduction

- The result implies:
- construct similar algorithms for other combinatorial problems
  - traveling salesman problem
  - multi-commodity network flow problem

- none are solvable in poly-time

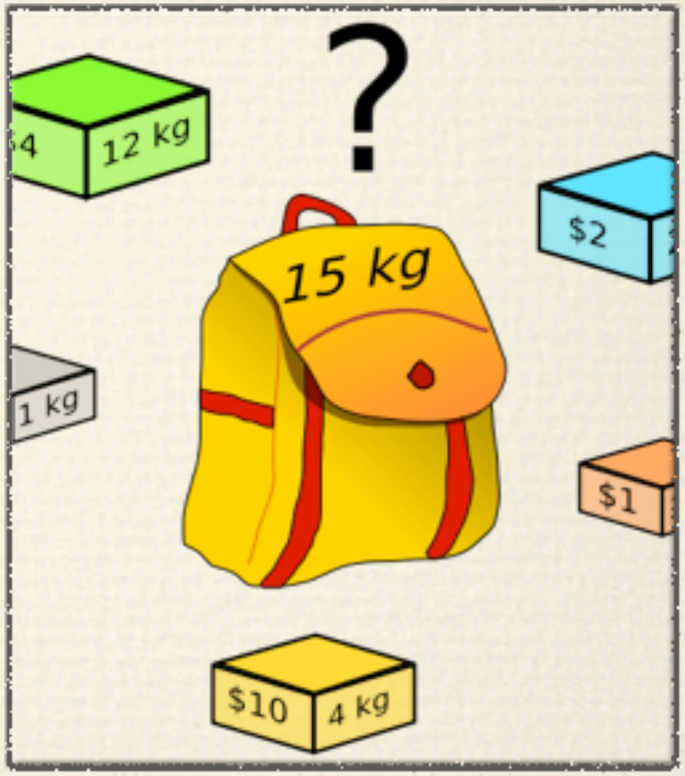
Introduction

- The result implies:
- construct similar algorithms for other combinatorial problems
  - traveling salesman problem
  - multi-commodity network flow problem
- none are solvable in poly-time

Discuss more precisely later

Introduction

- In the rest of this presentation, we are going to:
- Demonstrate what's P and NP.
- Give formal definition for Network Design Problem(NDP).
- Discuss relation between NDP and KNAPSACK.
- Prove NDP and SNDP is NPC.



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P and NP

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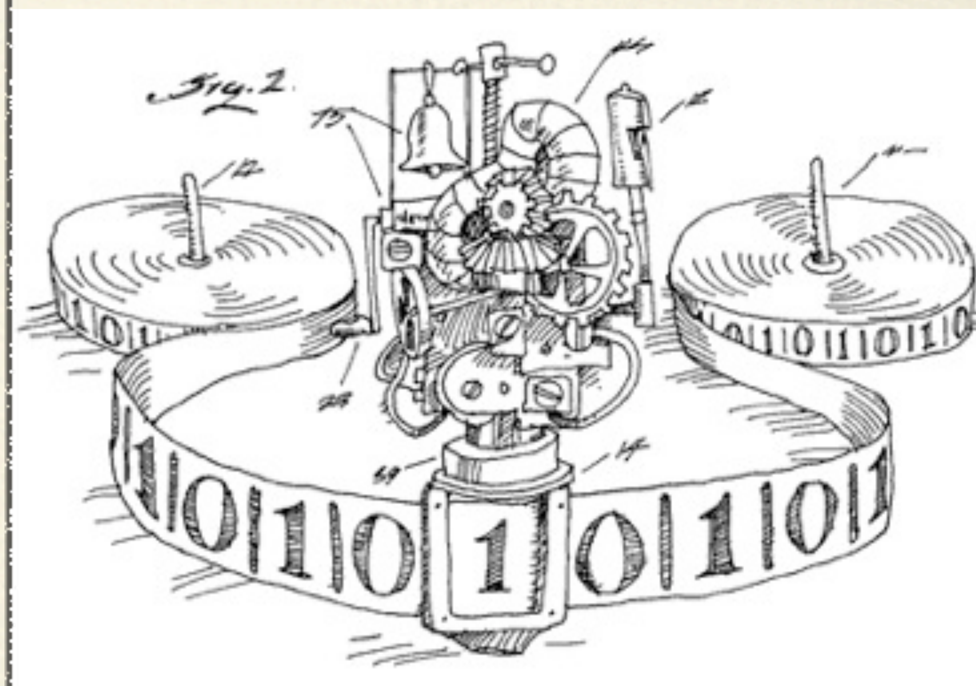
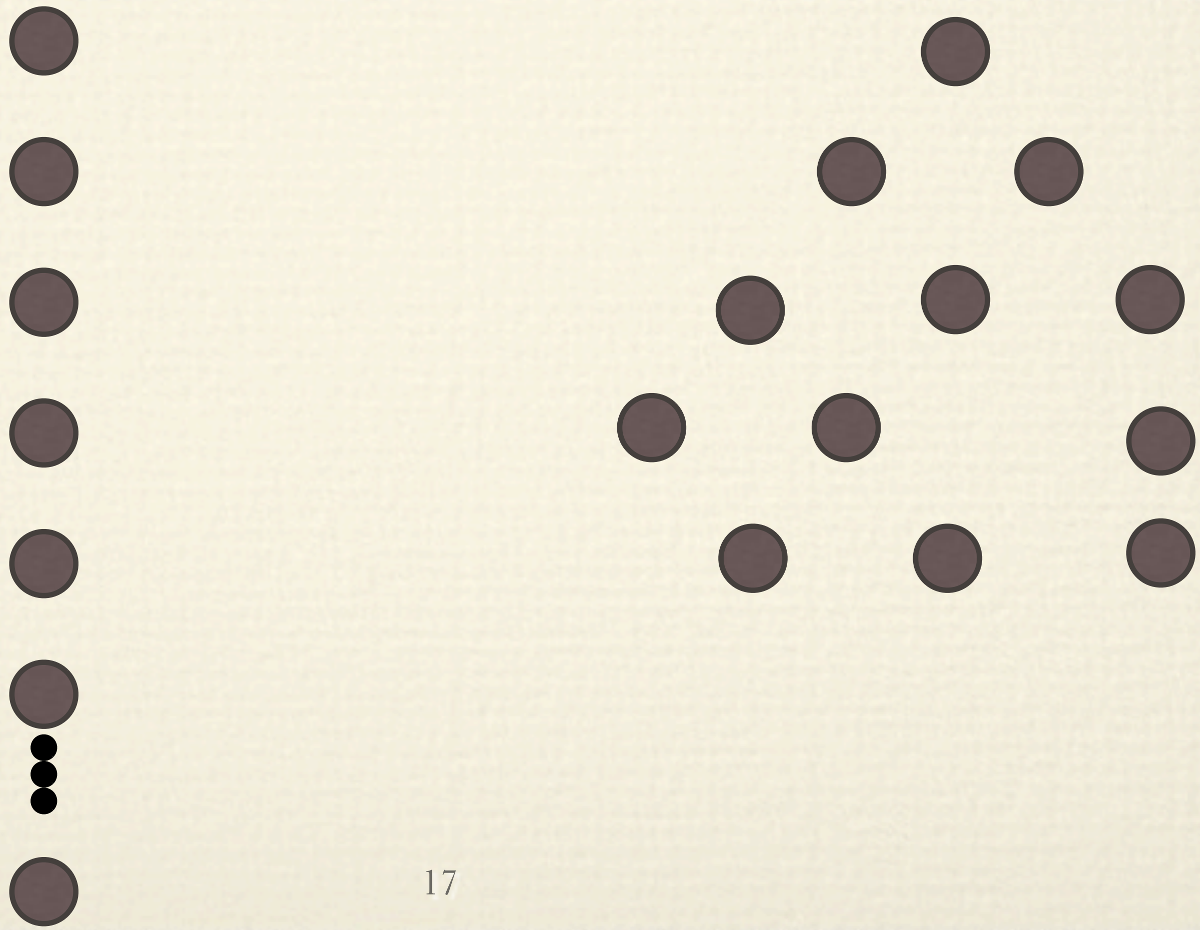
What is Turing Machine anyway ?

- A simple machine conceptually exists for computation.
- Almost every computation model can be transformed to Turing Machine.



Deterministic and Non-deterministic

- Take a closer look on the main difference between these two types.
  (Transition Functions and Transition Relations)



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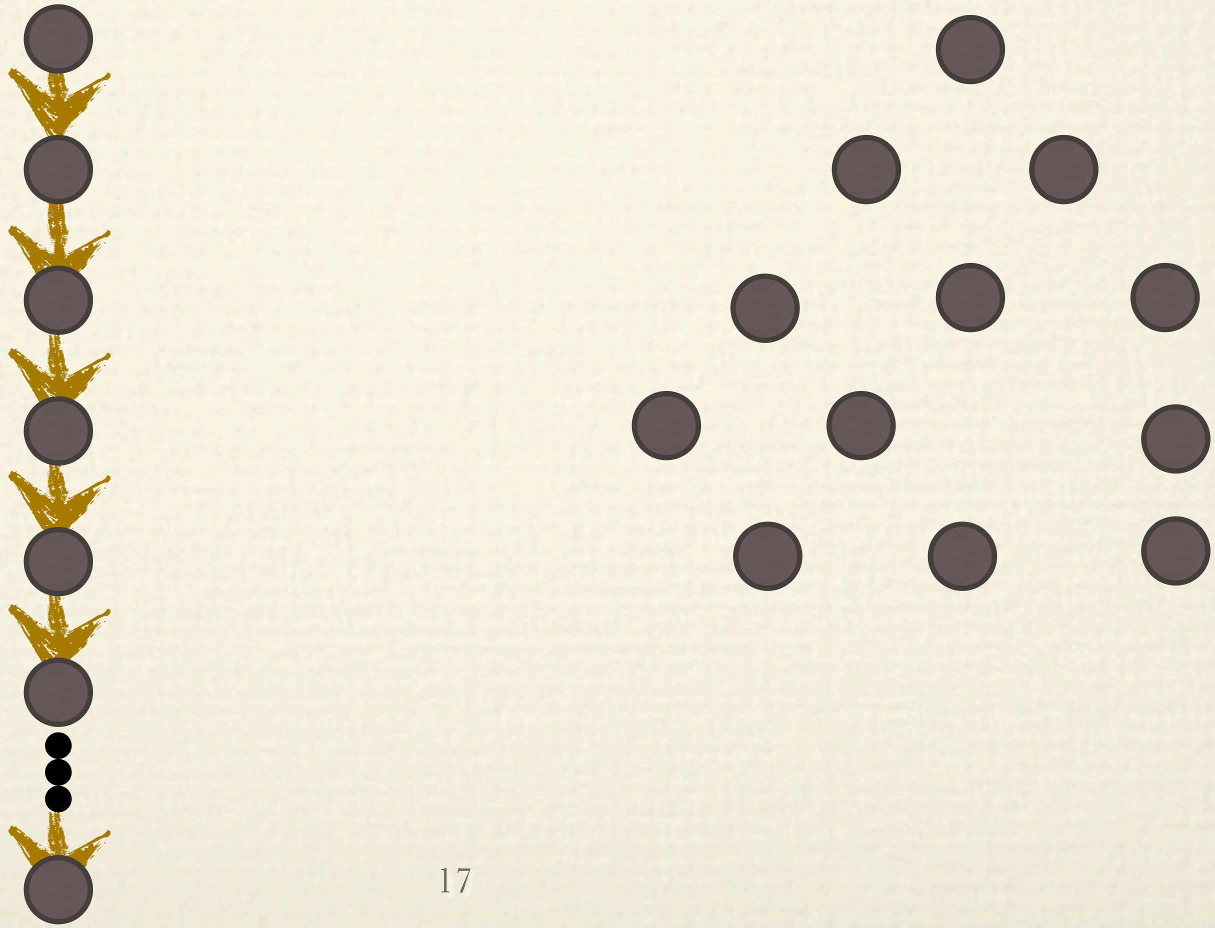

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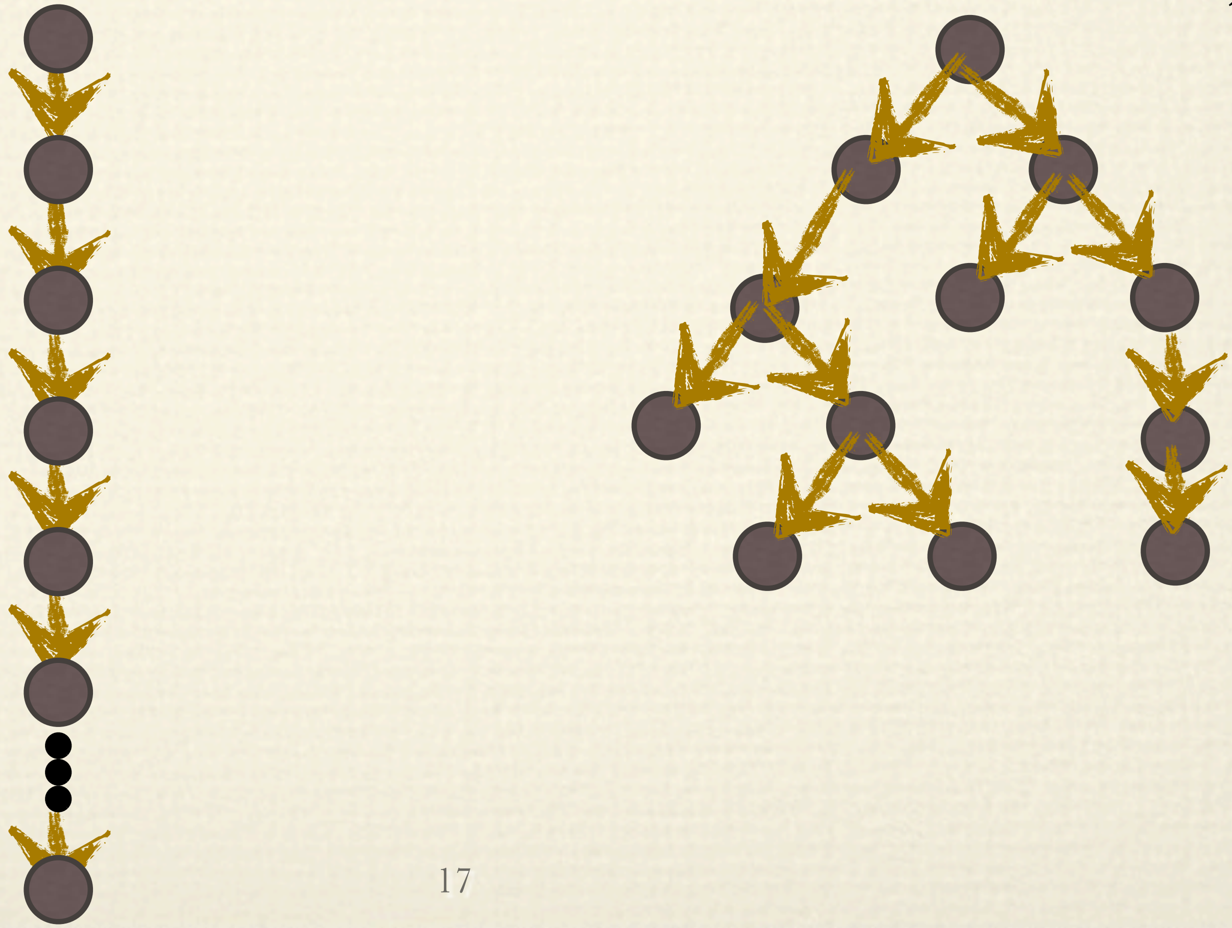


P vs NP

- A common misunderstanding
- Polynomial Time v.s. Non-Polynomial Time

Polynomial Time v.s. Non-deterministic Polynomial Time

$P = \bigcup_{k>0} TIME(n^k)$ $NP = \bigcup_{k>0} NTIME(n^k)$

P vs NP

- A common misunderstanding
- ~~Polynomial Time v.s. Non-Polynomial Time~~

Polynomial Time v.s. Non-deterministic Polynomial Time

$$P = \bigcup_{k>0} TIME(n^k) \quad NP = \bigcup_{k>0} NTIME(n^k)$$

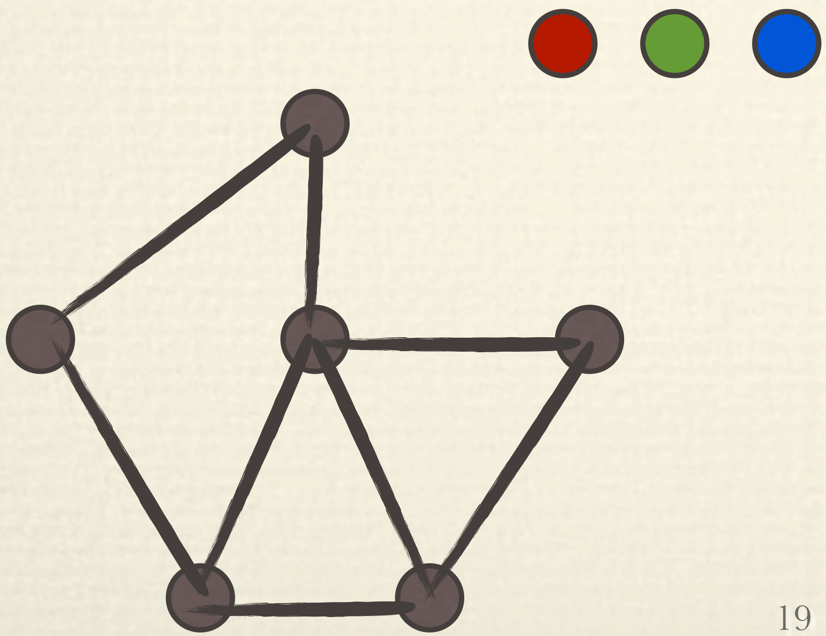
Reduction

- What's reduction for?

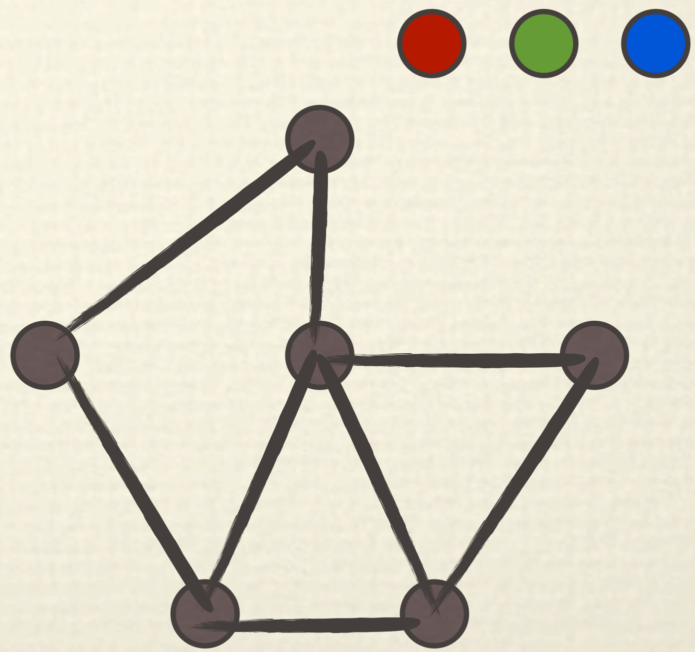
  we say "A reduce to B" if there exists a transformation R which $x \in A \Leftrightarrow R(x) \in B$, this prove problem B is "at least hard as " problem A

  Example: 3-coloring reduce to 4-coloring

3-coloring



4-coloring



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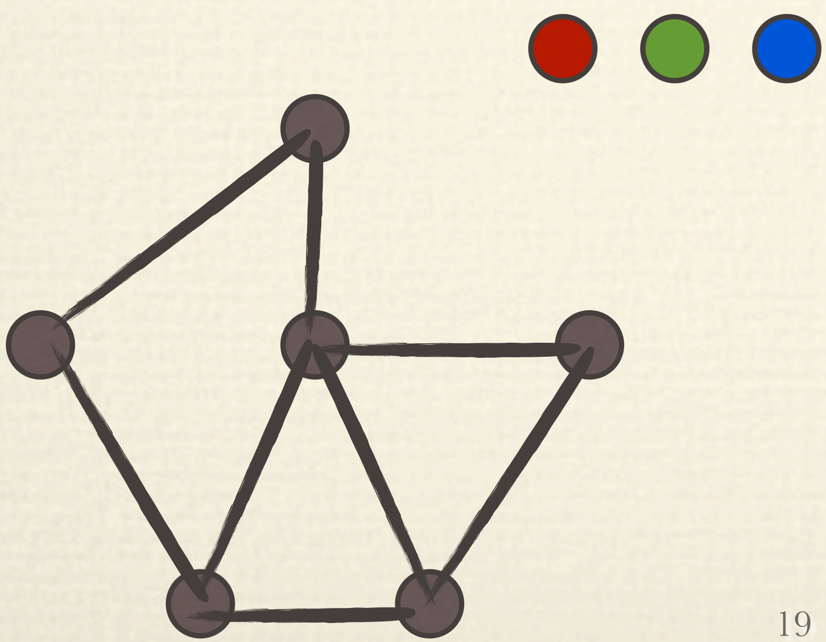

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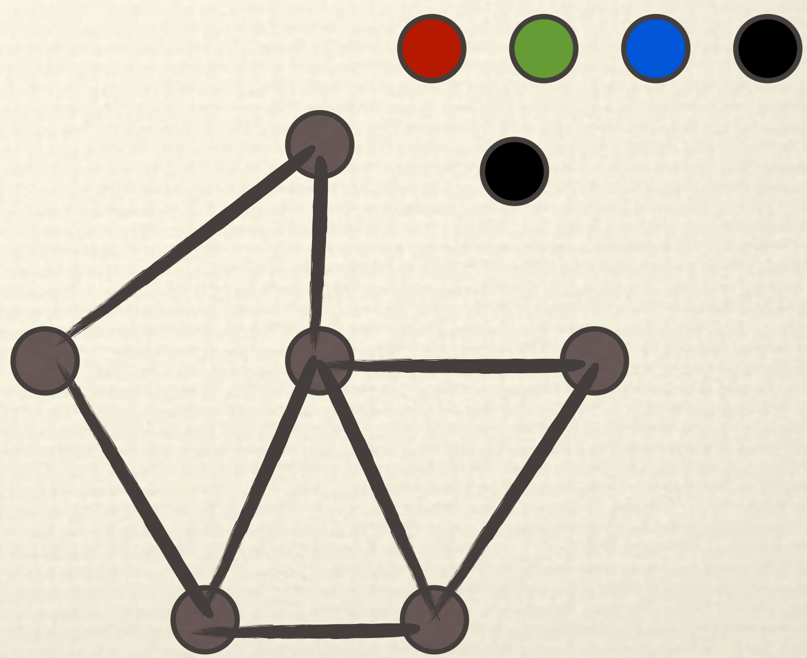

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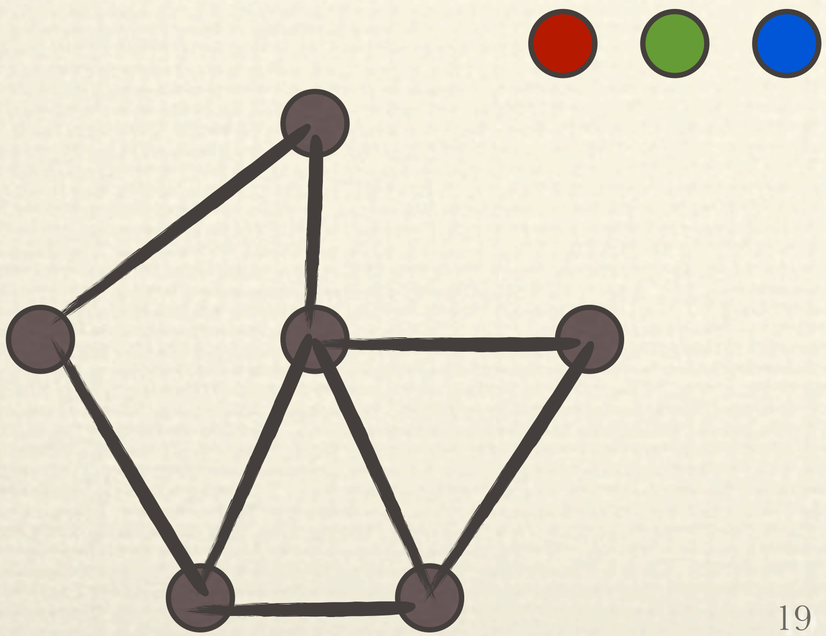

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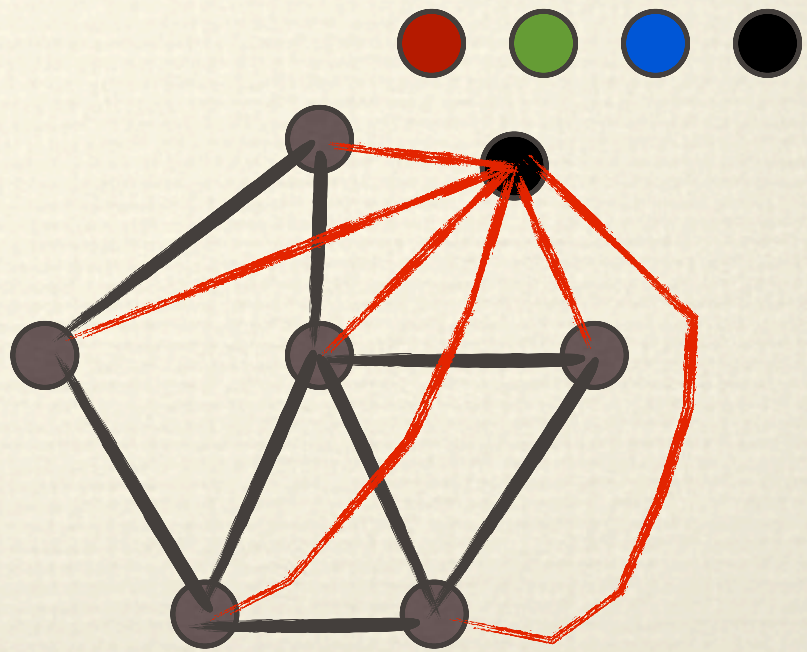
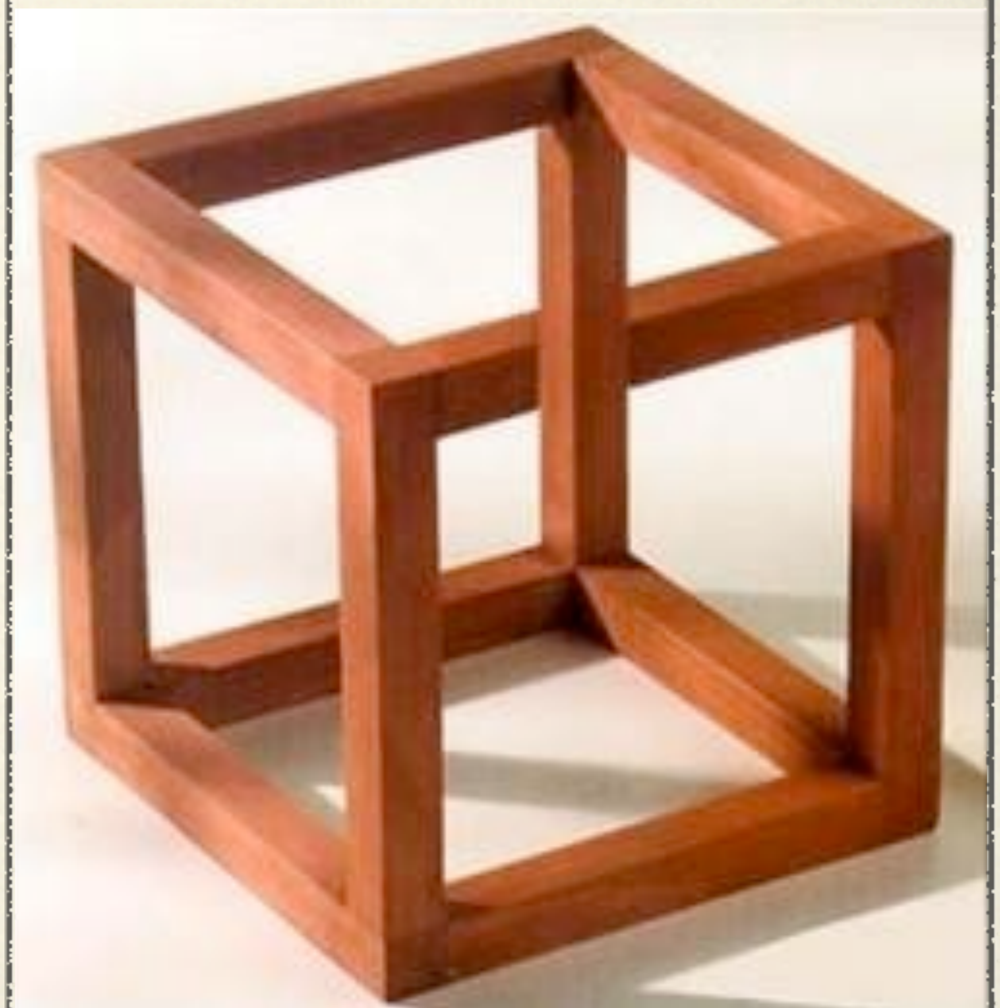


Reduction - Paradox

- Reduction must be polynomial to prevent paradox



P-hard, NP-hard, NP-complete

- We call a problem X is NP-hard if all the problems in the NP can be reduced to X.
- In addition, if X belongs to NP, X is NP-complete.
- This also applied to the P-hard and P-complete.

- Examples of P and NP

- Examples of P and NP

Examples

- NPC examples
  - Hamiltonian path
  - Vertex cover
  - Integer linear programming
  - 3-satisfiability
- P examples
  - Circuit Value Problem (CVP)
  - Linear programming

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Network Design Problem (NDP)

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NDP

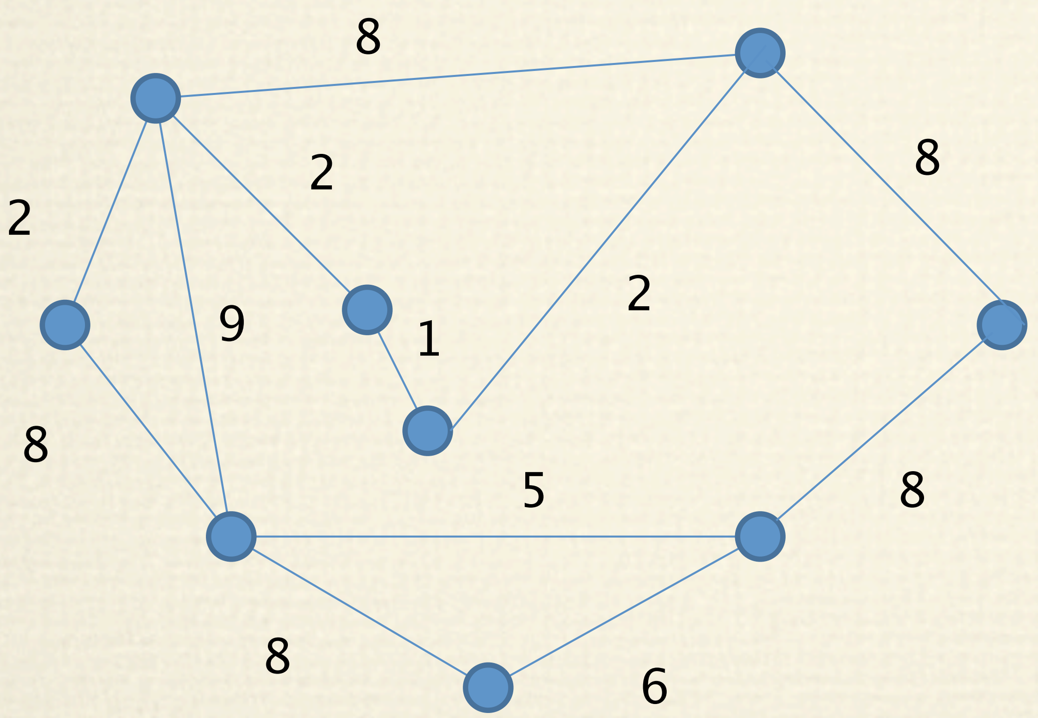
- Goal : NDP is NP-complete
- Step 1 : NDP is in NP
- Step 2 : Reduce a NP-complete
    - problem to NDP

NDP(cont.)

- NETWORK DESIGN PROBLEM(NDP):
- Given an undirected graph G=(V,E), a weight function L:E->N, a budget B and a criterion threshold C(B,C ∈N), does there exist a subgraph G'=(V,E') of G with weight $\sum_{\{i,j\}\in E'} L(\{i,j\}) <= B$ and criterion value F(G')<=C
- where F(G') denotes the sum of the weights of the shortest paths in G' between all vertex pairs?

NDP(cont.)

- Weighted Graph G

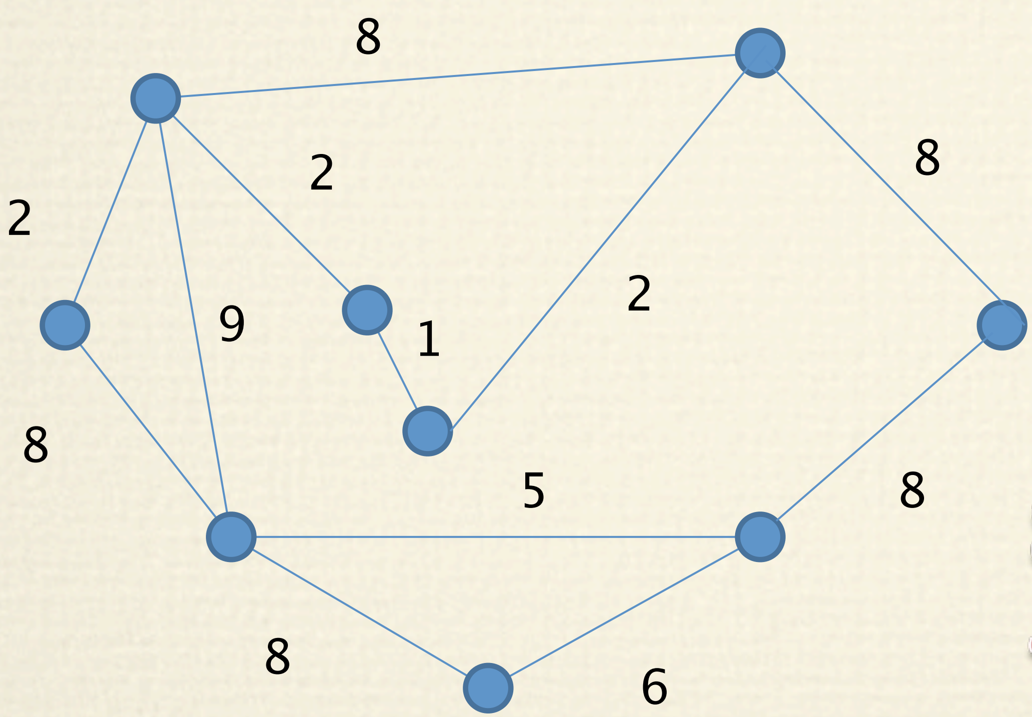


Budget B=50

Criterion threshold C=500

NDP(cont.)

- Weighted Graph G



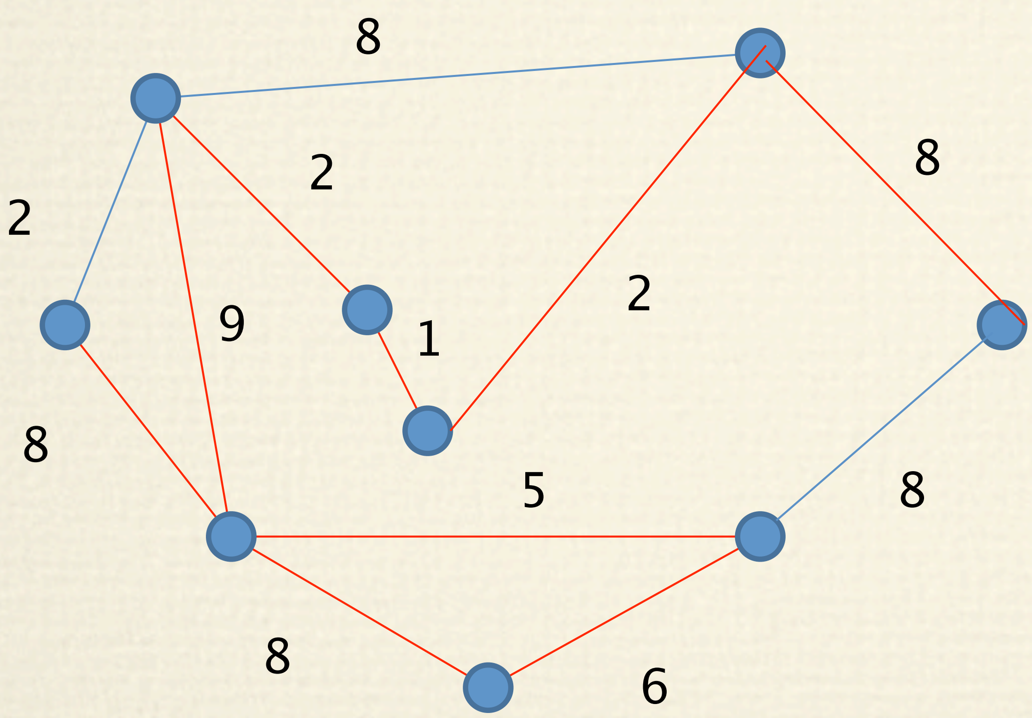
Subgraph exists?

Total weight<=B

Criterion value<=C

NDP(cont.)

- Now guess and verify!!



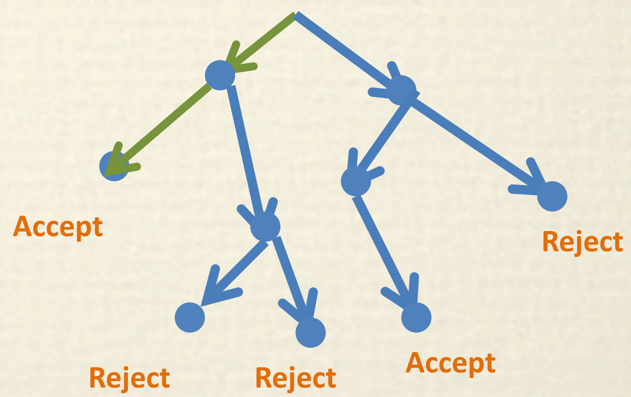
B=50,C=500

Total weight=49

Criterion value=491

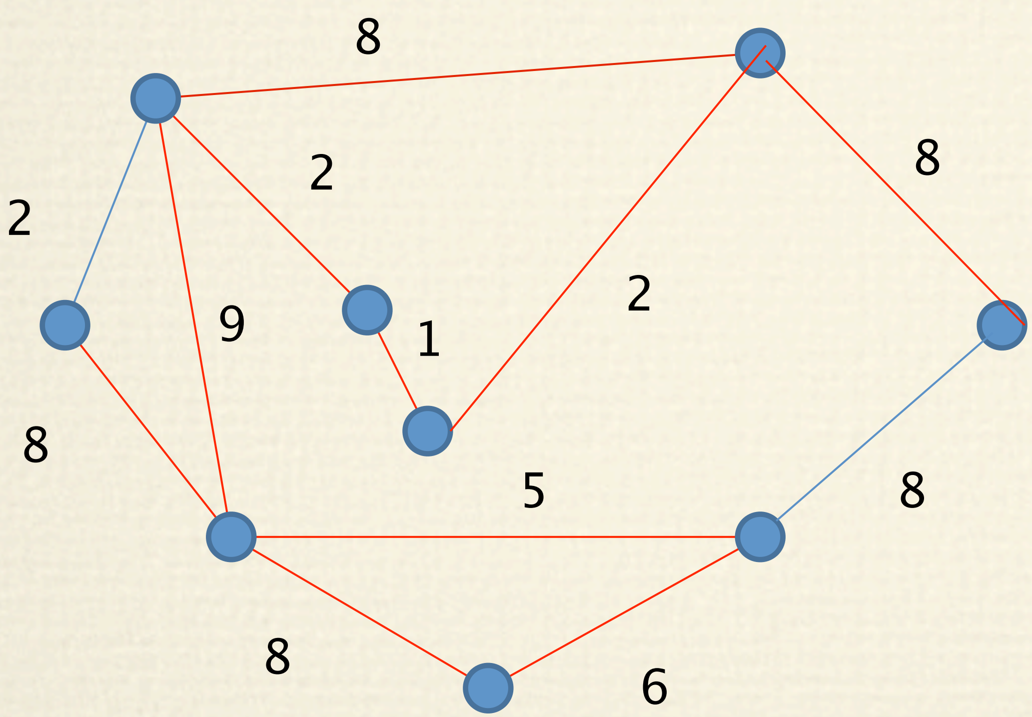
49<=50,491<=500

Yes!!!!!!!



NDP(cont.)

- Now guess and verify!!



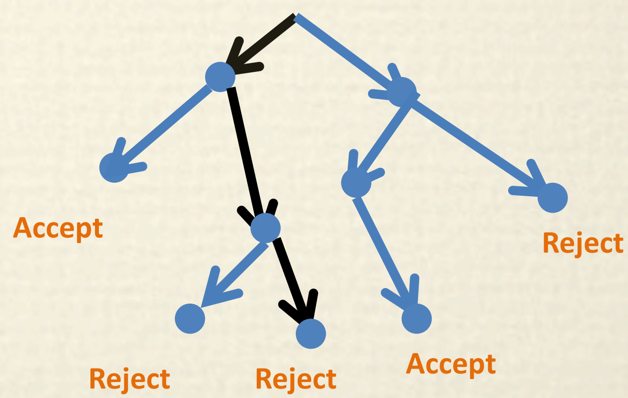
B=50,C=500

Total weight=57

Criterion value=463

57>50,463<=500

NDP is NO??????



NDP(cont.)

- NDP is in NP
- NP-complete?
- Reduce Knapsack problem to NDP

Knapsack problem

- n items(T={1,2,3,4,5,6,….,t})
- Item x has value v_x and weight w_x
- Given V and W
- Does there exist a subset S $\subset$ T such that
- $\sum_{i \in S} w_i <= W$ and $\sum_{i \in S} v_i >= V$?

Example

0.5kg,
200NT

80kg,6000NT

5kg,100NT

0.6kg,
22NT

1200kg,
700000NT

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Knapsack problem (another def.)

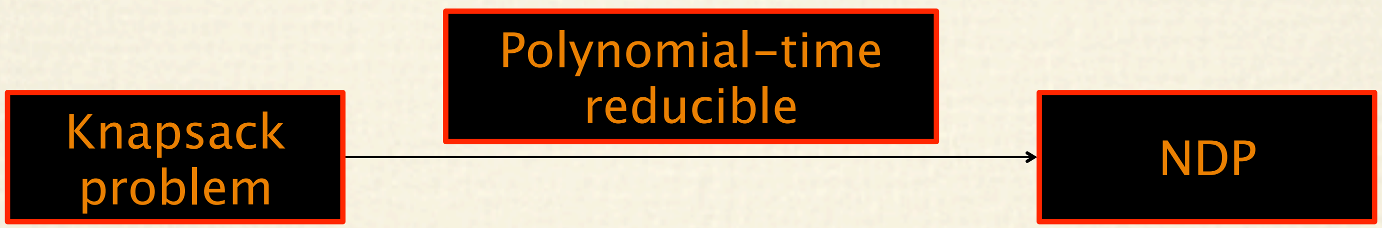
- W = V
- $v_x = w_x$ for x=1,2,…..t

Knapsack problem (another def.)

- n items(T={1,2,3,4,5,6,….,t})
- Item i has value a_i
- Given b
- Does there exist a subset S ⊂ T such that
- $\sum_{i \in S} a_i = b$?

Knapsack problem

- Knapsack problem is NP-complete



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KNAPSACK & NDP

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A KNAPSACK Example

t = 4

$$\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases}$$

b = 7

- There is a solution => $a_1 + a_3 = 2 + 5 = 7$

KNAPSACK => NDP

- KNAPSACK

$t = 4$	$\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases}$	$b = 7$

Let $A = \sum_{i=1}^{t} a_i = 16$

- NDP

Reduce

$V = \{0\} \cup \{ i , i' \mid i \in [1, t]\}$

$E = \{ (0 , i) , (0 , i') , (i , i') \mid i \in [1, t]\}$

$L((0 , i)) = L((0 , i')) = L((i , i')) = a_i$

$B = 2A + b$

$C = 4tA - b$

KNAPSACK => NDP

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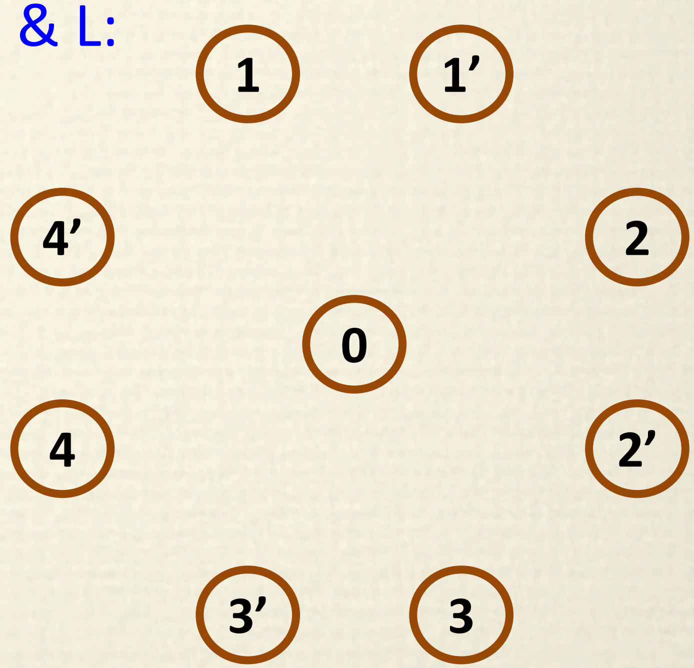
G & L:



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KNAPSACK => NDP

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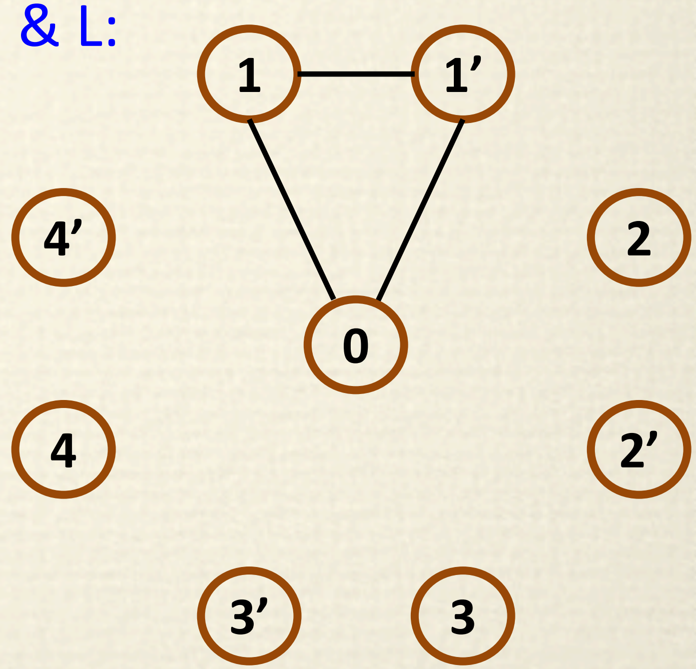

KNAPSACK => NDP

- KNAPSACK

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Let A = $\sum_{i=1}^{t} a_i$ = 16

- NDP

V = {0} ∪ { i , i' | i ∈ [1, t]}
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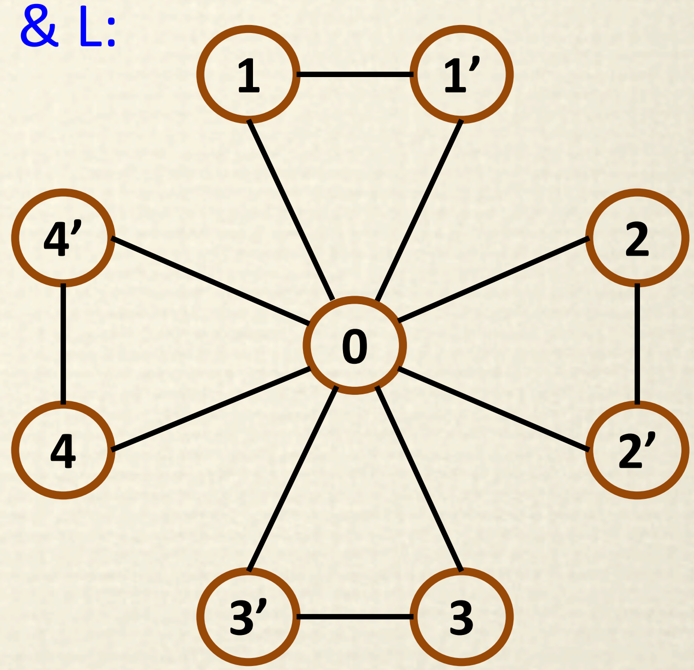

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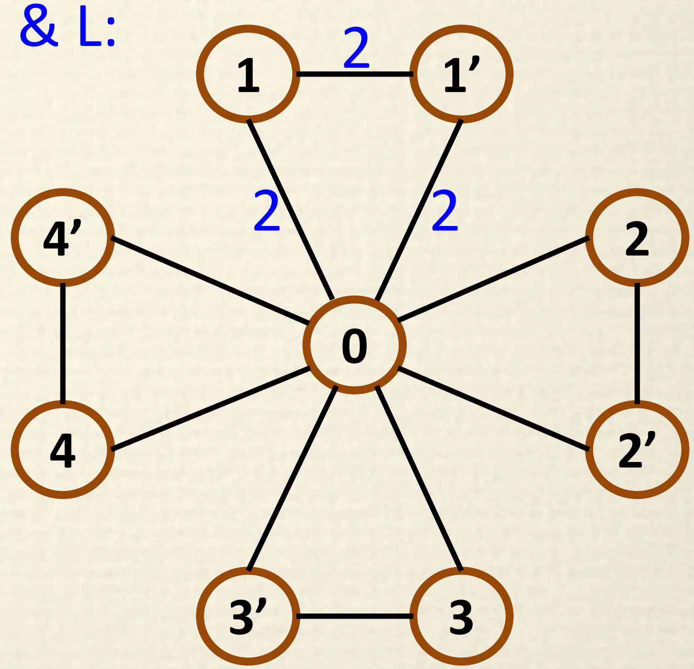

KNAPSACK => NDP

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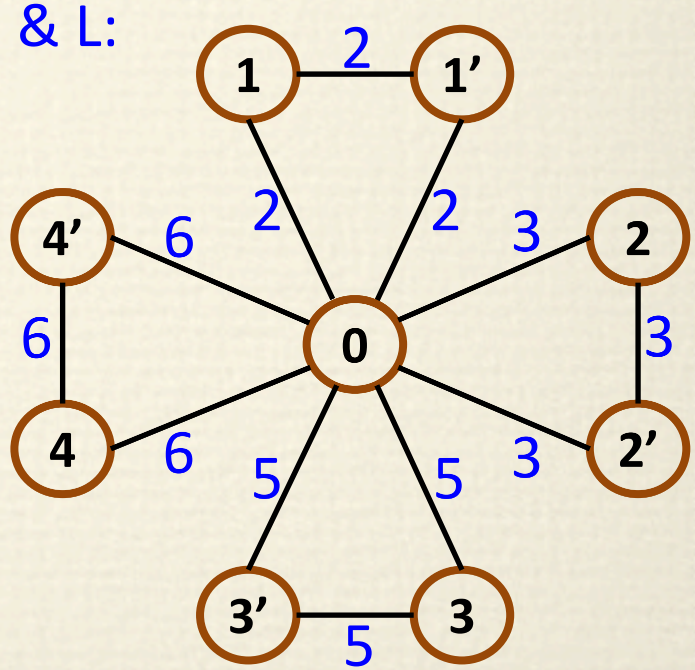

KNAPSACK => NDP

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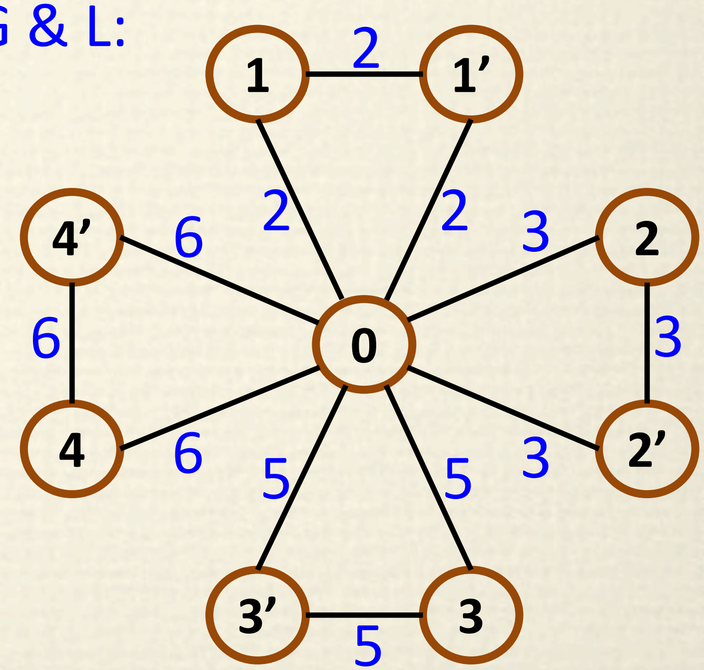
$L((0 , i)) = L((0 , i')) = L((i , i')) = a_i$

$B = 2A + b = 39$ (budget)

$C = 4tA - b = 249$ (criterion threshold)

G & L:



$t = 4$ $\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases}$ $b = 7$

Solve NDP

$A = \sum_{i=1}^{t} a_i = 16$
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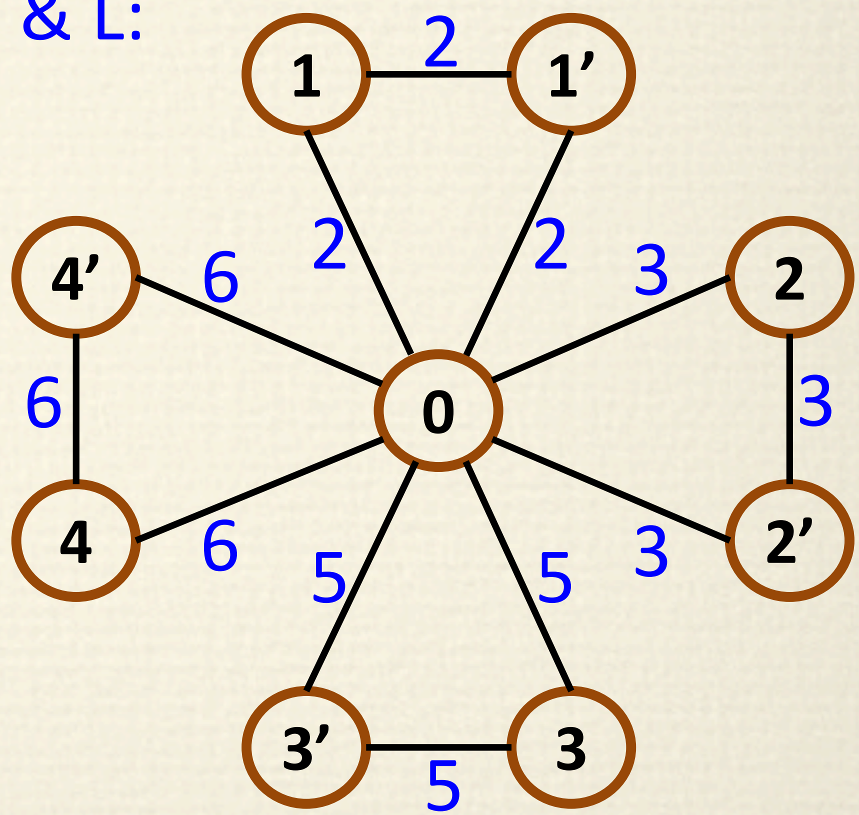

$$t = 4 \quad \begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases} \quad b = 7$$

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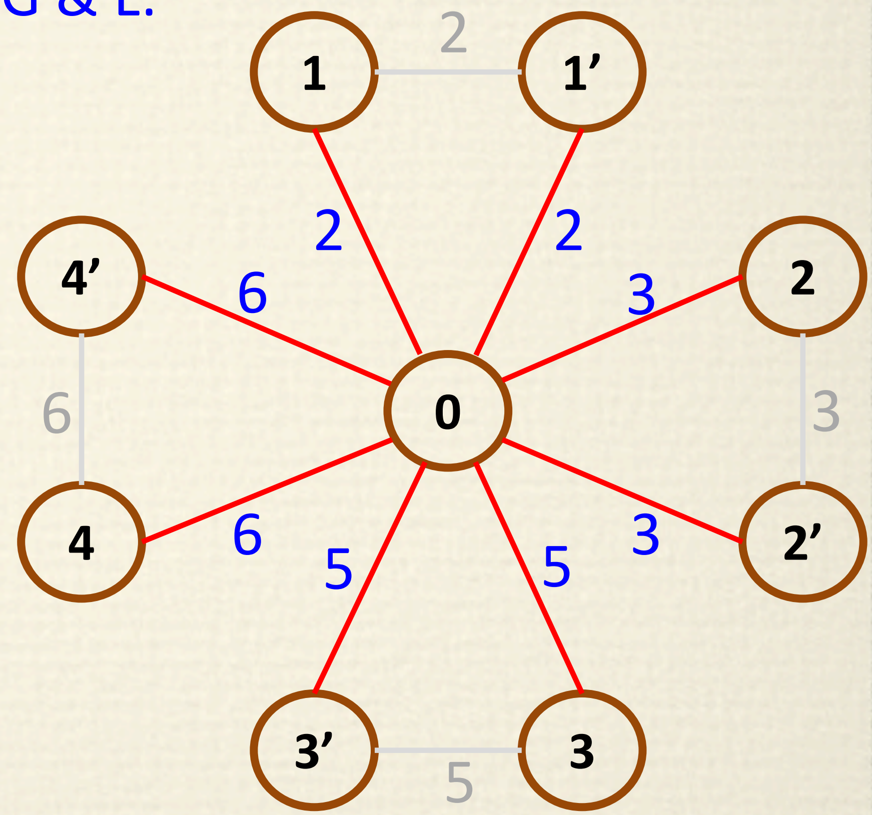

- Any feasible solution can be assumed to contain the star graph.

Solve NDP

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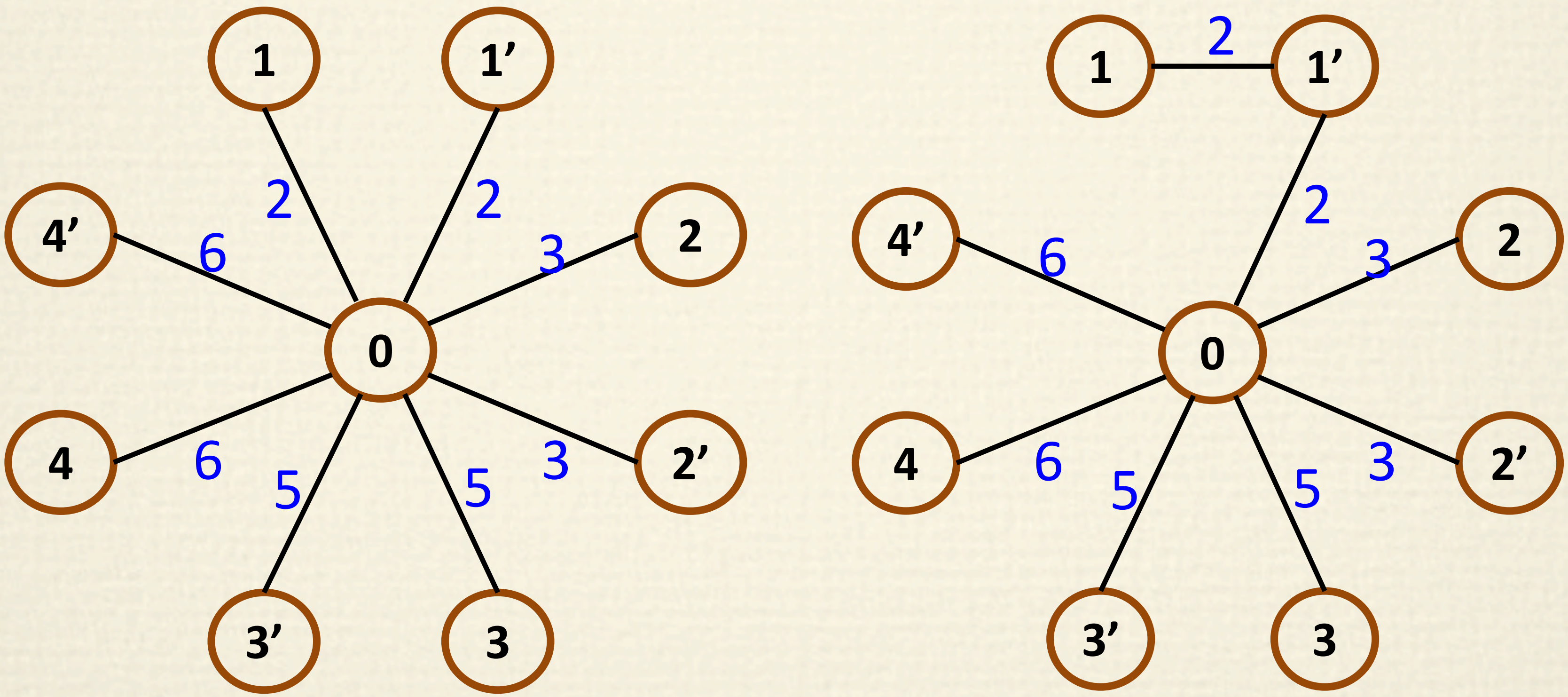

Both consume same budget, but the criterion value of right one is larger!

$t = 4$ $\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases}$ $b = 7$

Solve NDP

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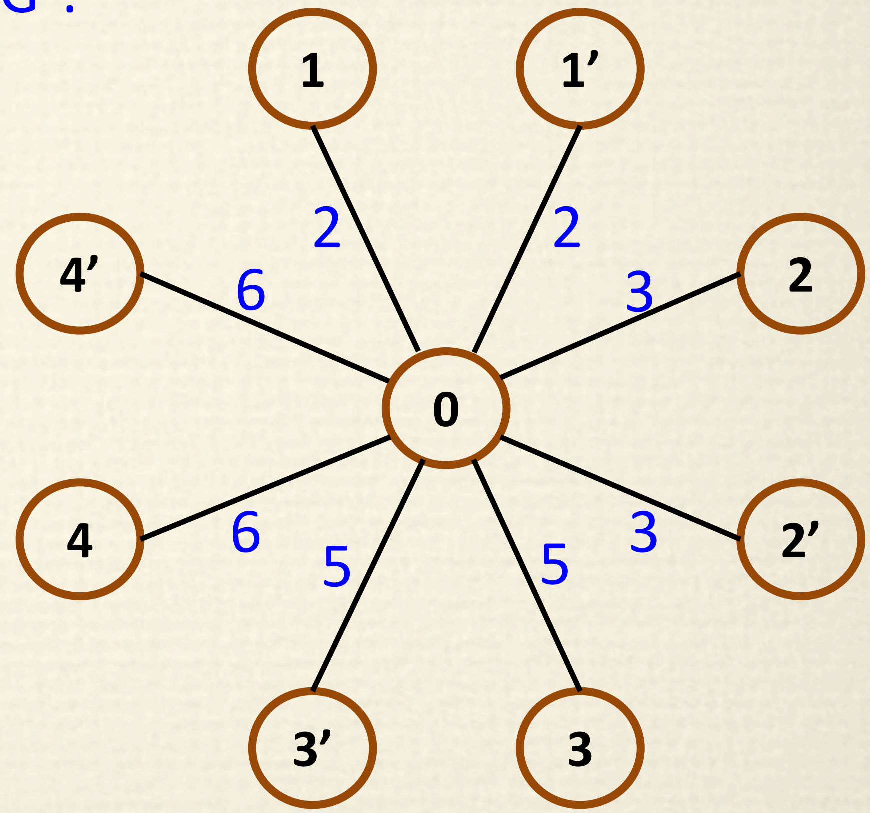
G^*:



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Solve NDP

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$B = 2A + b = 39$ (budget)

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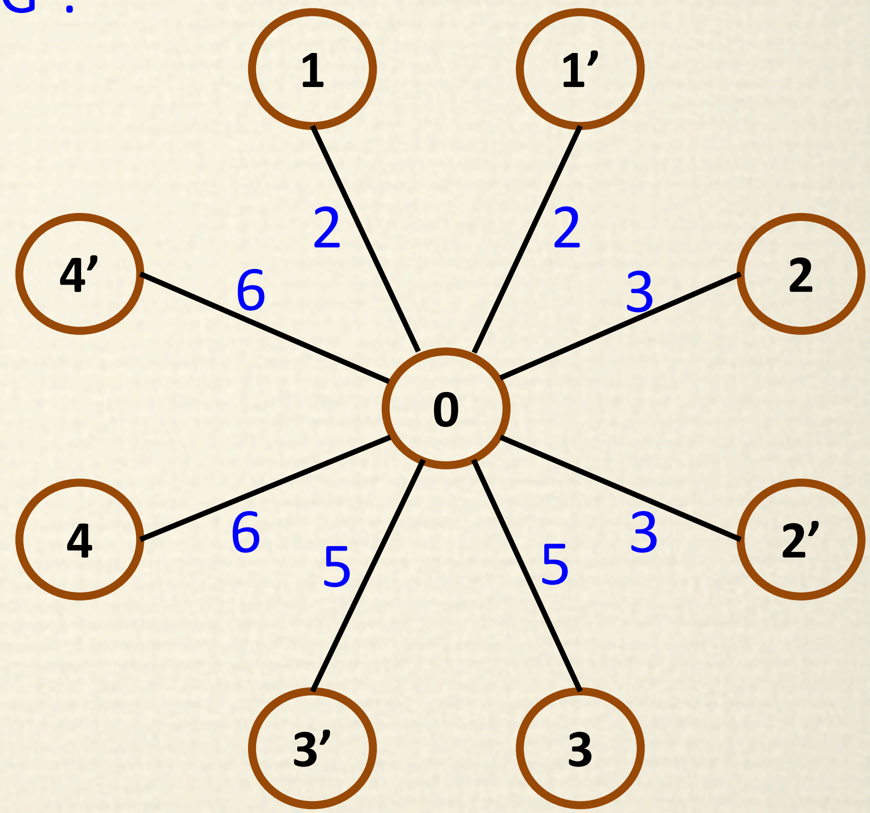
Sum of weight: $2A = 32$

Criterion value: $4tA = 256$

G^*:



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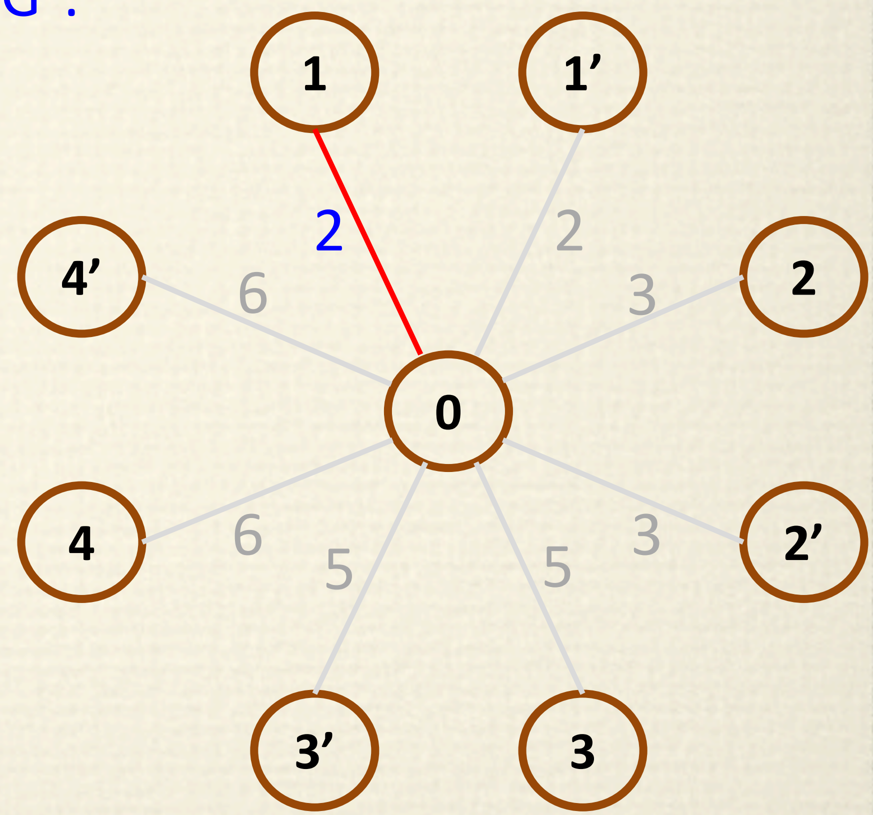
$B = 2A + b = 39$ (budget)
$C = 4tA - b = 249$ (criterion threshold)

Sum of weight: $2A = 32$
Criterion value: $4tA = 256$

routing load
$2 \times 1 \times 2t$

G^*:



$t = 4$ $\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases}$ $b = 7$

Solve NDP

$A = \sum_{i=1}^{t} a_i = 16$

$B = 2A + b = 39$ (budget)

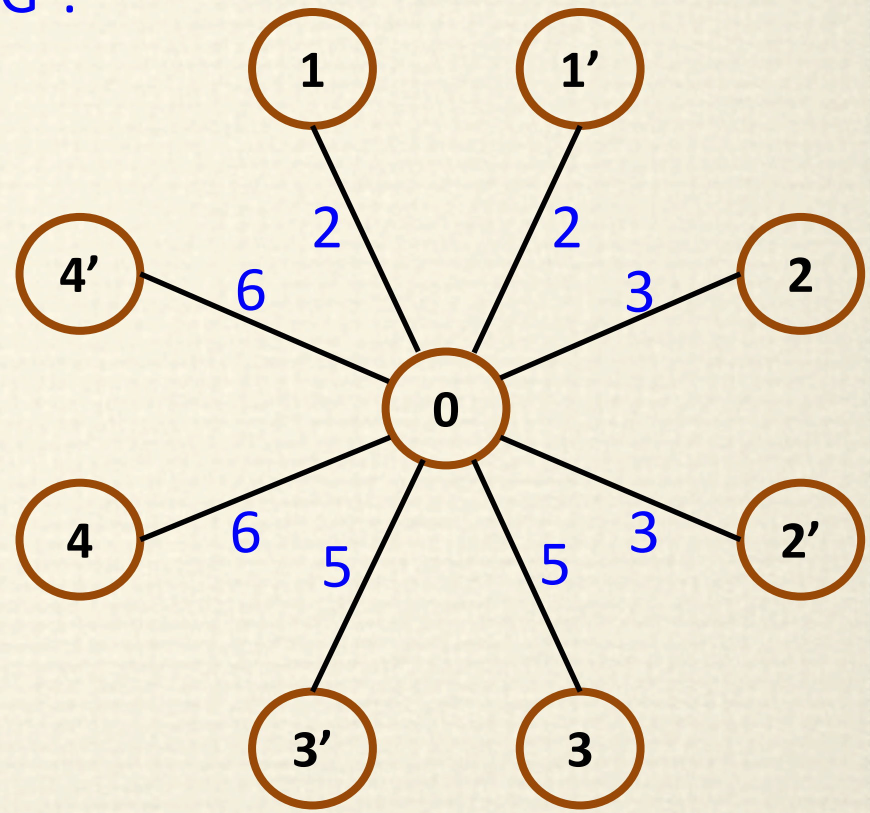
$C = 4tA - b = 249$ (criterion threshold)

Sum of weight: $2A = 32$

Criterion value: $4tA = 256$

$2A \times (\ 2 \times 1 \times 2t\ )$

G^*:



Solve NDP

$t = 4 \quad \begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases} \quad b = 7$

$A = \sum_{i=1}^{t} a_i = 16$

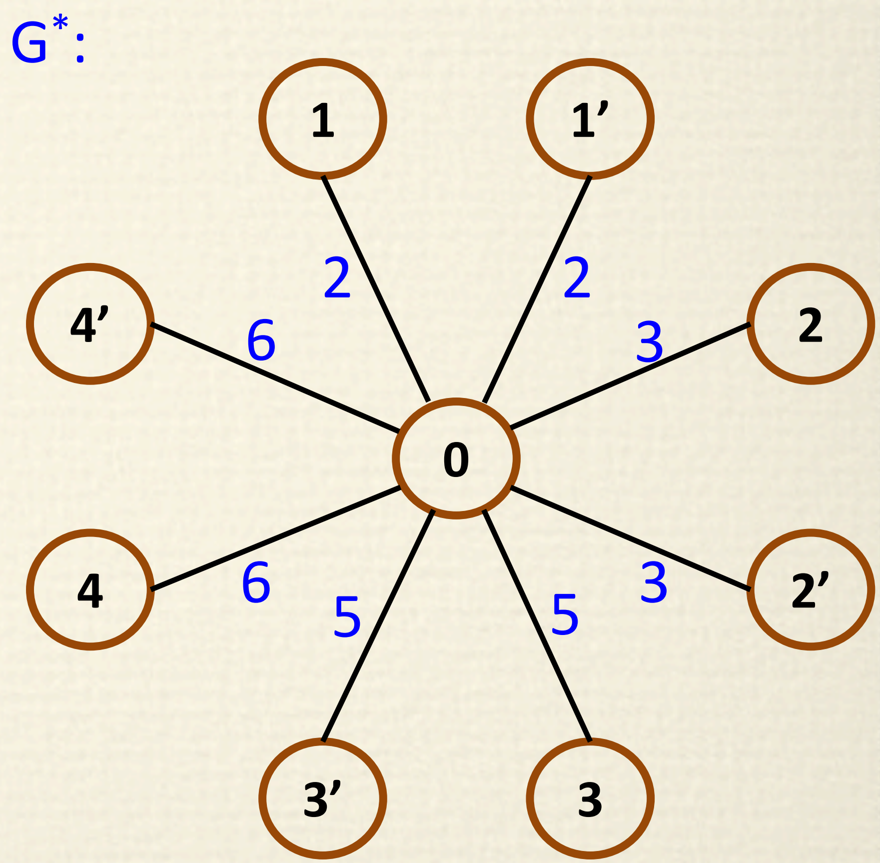
$B = 2A + b = 39$ (budget)

$C = 4tA - b = 249$ (criterion threshold)

Sum of weight: $2A = 32$

Criterion value: $4tA = 256$

$$\frac{2A \times (\ 2 \times 1 \times 2t\ )}{2}$$



Solve NDP

$$t = 4 \quad \begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases} \quad b = 7$$

$A = \sum_{i=1}^{t} a_i = 16$

$B = 2A + b = 39$ (budget)

$C = 4tA - b = 249$ (criterion threshold)

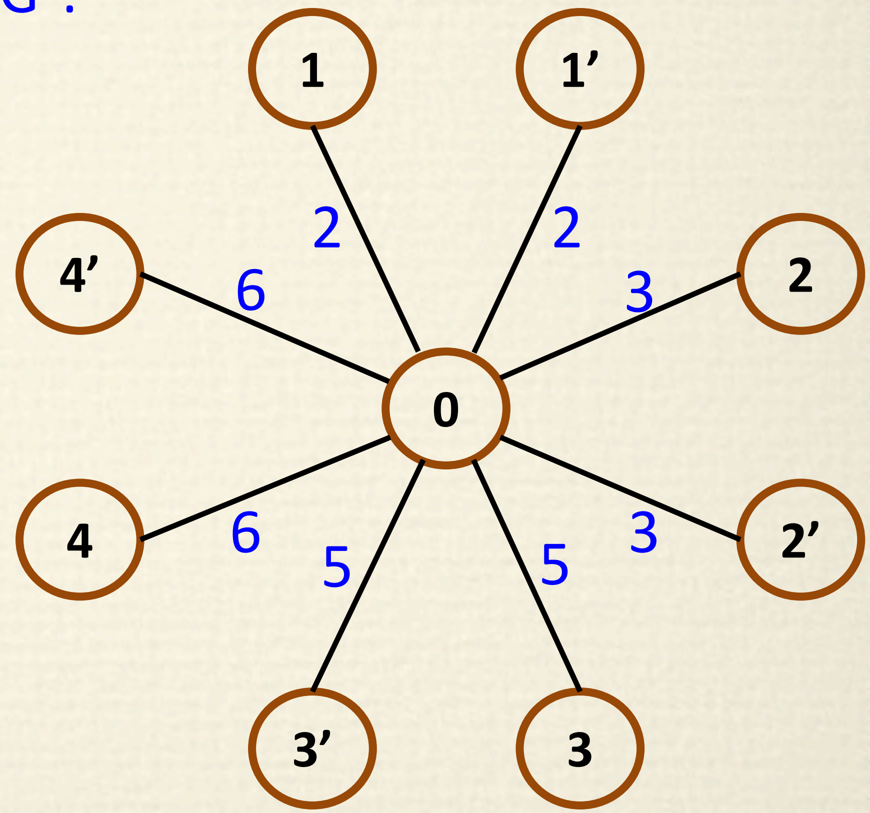
Sum of weight: $2A = 32$

Budget: $2A + b = 39$

Criterion value: $4tA = 256$

Criterion threshold: $4tA - b = 249$ ✗

G^*:



$t = 4$ $\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases}$ $b = 7$

Solve NDP

$A = \sum_{i=1}^{t} a_i = 16$

$B = 2A + b = 39$ (budget)

$C = 4tA - b = 249$ (criterion threshold)

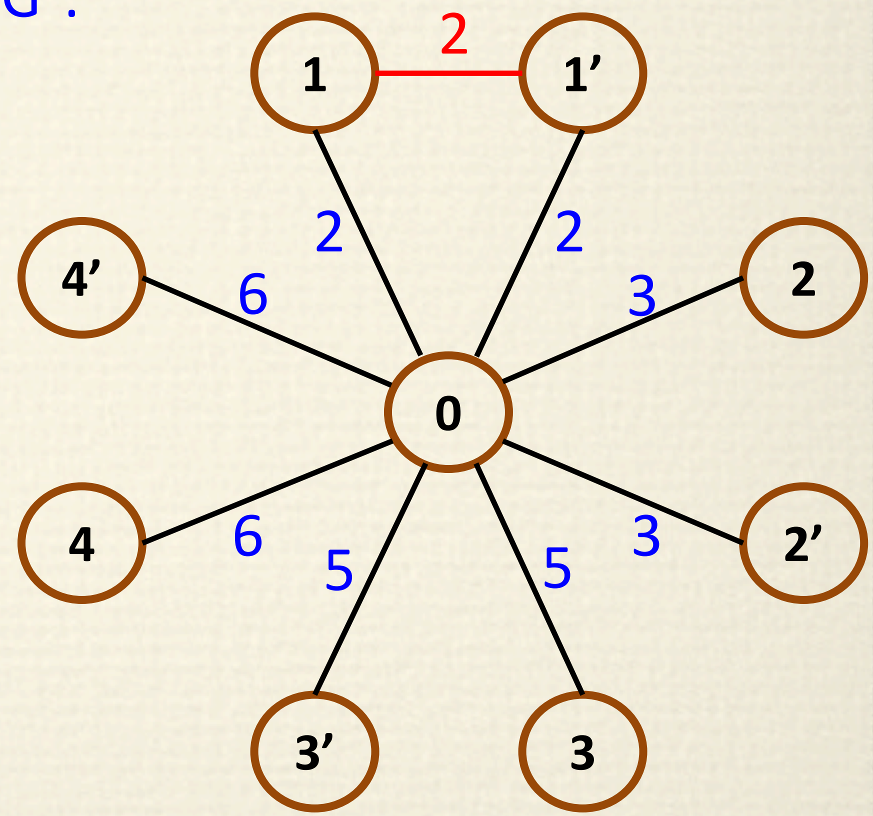
Sum of weight: 2A + 2 = ~~32~~ = 34

Budget: 2A + b = 39

Criterion value: 4tA - 2 = ~~256~~ = 254

Criterion threshold: 4tA - b = 249

G*:



t = 4 $\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases}$ b = 7

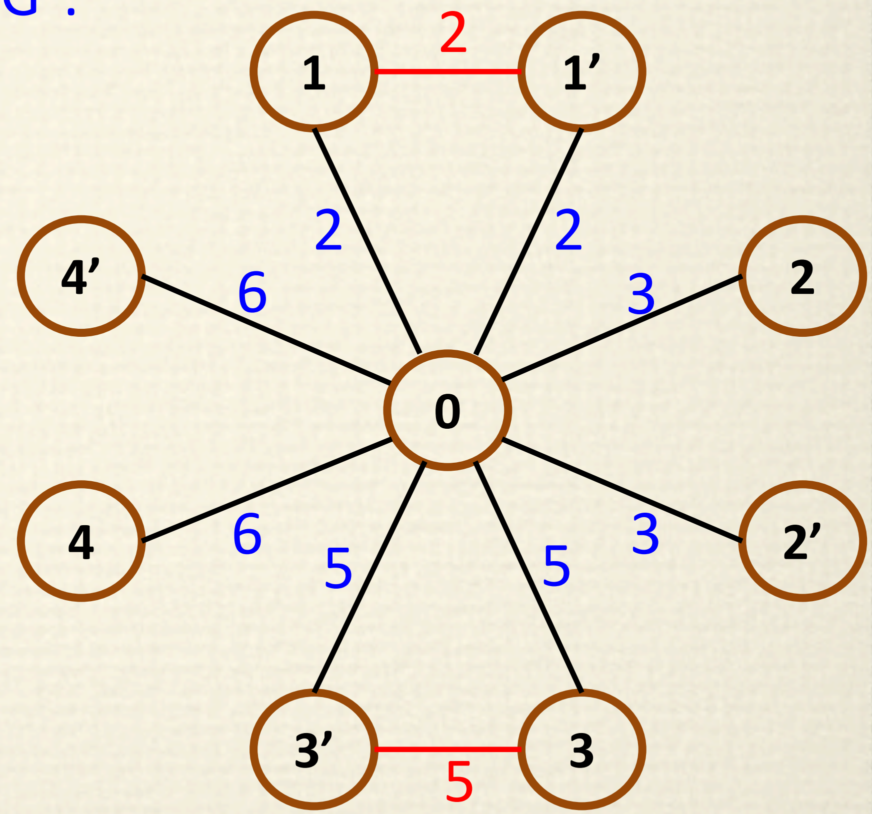
Solve NDP

$A = \sum_{i=1}^{t} a_i = 16$
$B = 2A + b = 39$ (budget)
$C = 4tA - b = 249$ (criterion threshold)

Sum of weight: 2A + 7 = ~~32~~ = 39
Budget: 2A + b = 39

Criterion value: 4tA - 7 = ~~256~~ = 249
Criterion threshold: 4tA - b = 249

G^*:



Solve NDP

t = 4 $\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_3 = 5 \\ a_4 = 6 \end{cases}$ b = 7

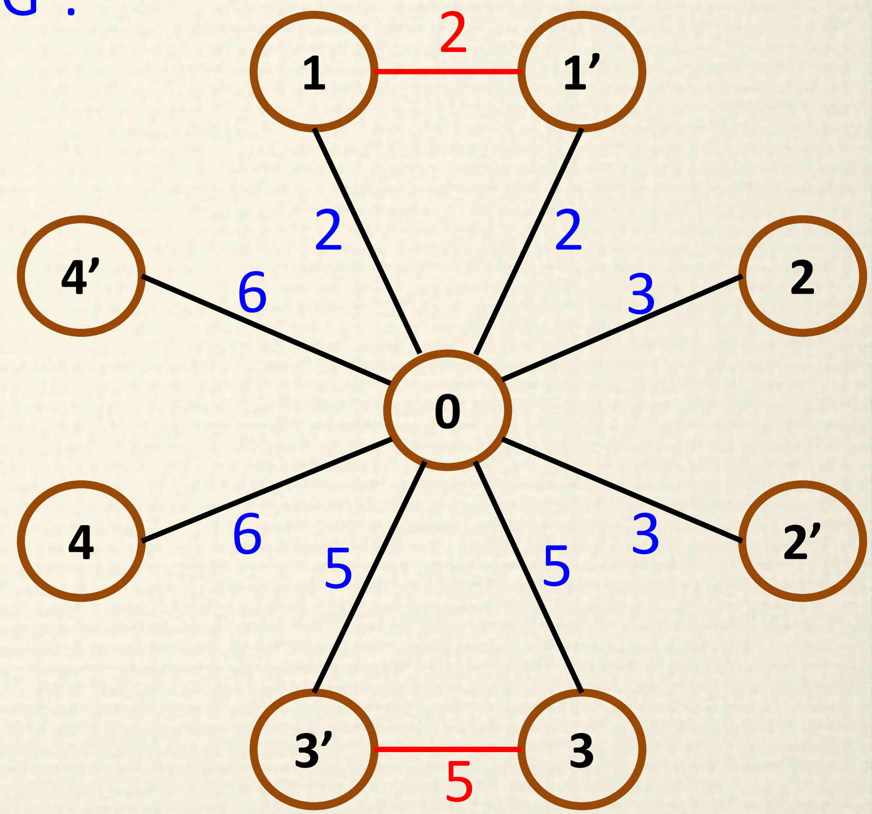
$a_1 + a_3 = b$

$A = \sum_{i=1}^{t} a_i = 16$
$B = 2A + b = 39$ (budget)
$C = 4tA - b = 249$ (criterion threshold)

Sum of weight: 2A + 7 = ~~32~~ = 39
Budget: 2A + b = 39

Criterion value: 4tA - 7 = ~~256~~ = 249
Criterion threshold: 4tA - b = 249

G^*:



NDP is NP-complete

- **KNAPSACK is NP-complete and reducible to NDP.**
  - For any instance of KNAPSACK, an instance of NDP can be constructed in polynomial-bounded time.
- **KNAPSACK has solution ⇔ NDP has solution**
  - Solving the instance of NDP solves the instance of KNAPSACK as well.
- **NDP belongs to NP.**
  - Any feasible subgraph can be recognized in polynomial time.
- **NDP is NP-complete**

Outline

- Introduction
- P and NP
- Network Design Problem (NDP)
- KNAPSACK and NDP
- SNDP is NP-complete
- Proof of SNDP
- Conclusion

SNDP is NP-complete

R00922157 李庚謙
R00922156 陳子筠

We will show that EXACT 3-COVER is reducible to SNDP.

Problem Formulation of SNDP

Simple Network Design Problem (SNDP):

- NDP with $L(\{i,j\}) = 1$ for all $\{i, j\} \in E$ and $B = |V| - 1$

- unit edge weight
- spanning tree
- SNDP $\in$ NP

A Known NPC Problem: Exact 3-cover

- Given a collection $S = \{\sigma_1, \sigma_2, \ldots, \sigma_s\}$ of 3-element subsets of a set $T = \{\tau_1, \tau_2, \ldots, \tau_{3t}\}$, does there exist a sub-collection $S' \subset S$ of pairwise disjoint sets such that $\sum_{\sigma \in S'} \sigma = T$?

- Example:
  - $T = \{1,2,3,4,5,6\}$, $S=\{\{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,2,6\}\}$
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A Known NPC Problem: Exact 3-cover

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- Example:
  - $T = \{1,2,3,4,5,6\}$, $S=\{\{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,2,6\}\}$

    No!
  - $T = \{1,2,3,4,5,6\}$, $S=\{\{1,2,3\}, \{3,4,5\}, \{1,2,5\}, \{1,2,6\}\}$

A Known NPC Problem: Exact 3-cover

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    No!
  - $T = \{1,2,3,4,5,6\}$, $S = \{\{1,2,3\}, \{3,4,5\}, \{1,2,5\}, \{1,2,6\}\}$

    ***S'*={{3,4,5},{1,2,6}}** Yes!

Problem Formulation of SNDP

Simple Network Design Problem (SNDP):

- NDP with $L(\{i, j\}) = 1$ for all $\{i, j\} \in E$ and $B = |V| - 1$

- special case of NDP
- unit edge weight
- spanning tree
- SNDP $\in$ NP

Reduction

- Given any instance of EXACT 3-COVER, we define an instance of SNDP as follows:

$$V = R \cup S \cup T$$

$$R = \{\rho_0, \rho_1, \ldots, \rho_r\}$$

$$r = C_{SS} + C_{ST} + C_{TT}$$

$$E = \{\{\rho_i, \rho_0\} : i = 1, \ldots, r\}$$
$$\cup \{\{\rho_0, \sigma\} : \sigma \in S\}$$
$$\cup \{\{\sigma, \tau\} : \tau \in \sigma \in S\}$$

$$C = C_{RR} + C_{RS} + C_{RT} + C_{SS} + C_{ST} + C_{TT}$$

Reduction

- Given any instance of EXACT 3-COVER, we define an instance of SNDP as follows:

$$V = R \cup S \cup T$$

$$R = \{\rho_0, \rho_1, \ldots, \rho_r\}$$

$$r = C_{SS} + C_{ST} + C_{TT}$$

$$E = \{\{\rho_i, \rho_0\} : i = 1, \ldots, r\}$$
$$\cup \{\{\rho_0, \sigma\} : \sigma \in S\}$$
$$\cup \{\{\sigma, \tau\} : \tau \in \sigma \in S\}$$

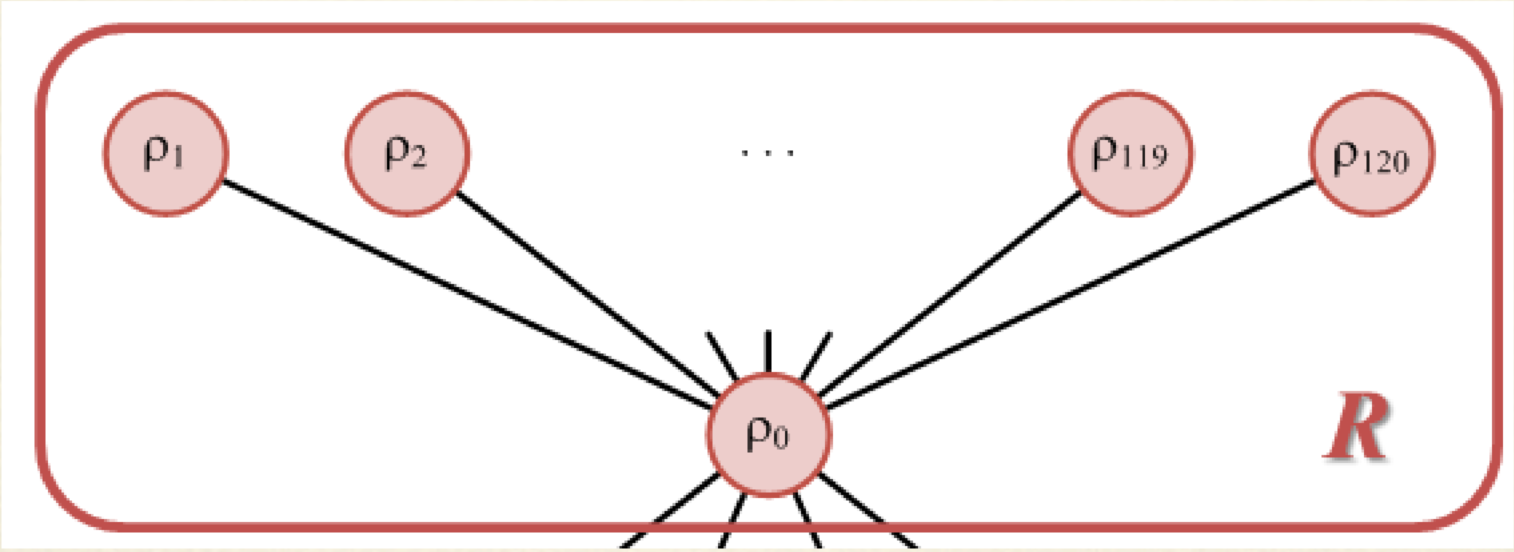
$$C = C_{RR} + C_{RS} + C_{RT} + \boxed{C_{SS} + C_{ST} + C_{TT}} = r$$

Meaning of parameters

- V: vertex
- R: vertex of S.T construct
- r: C_{SS}+C_{ST}+C_{TT}
- E: edge of R.S.T
- C: Total routing cost

Criterion Threshold

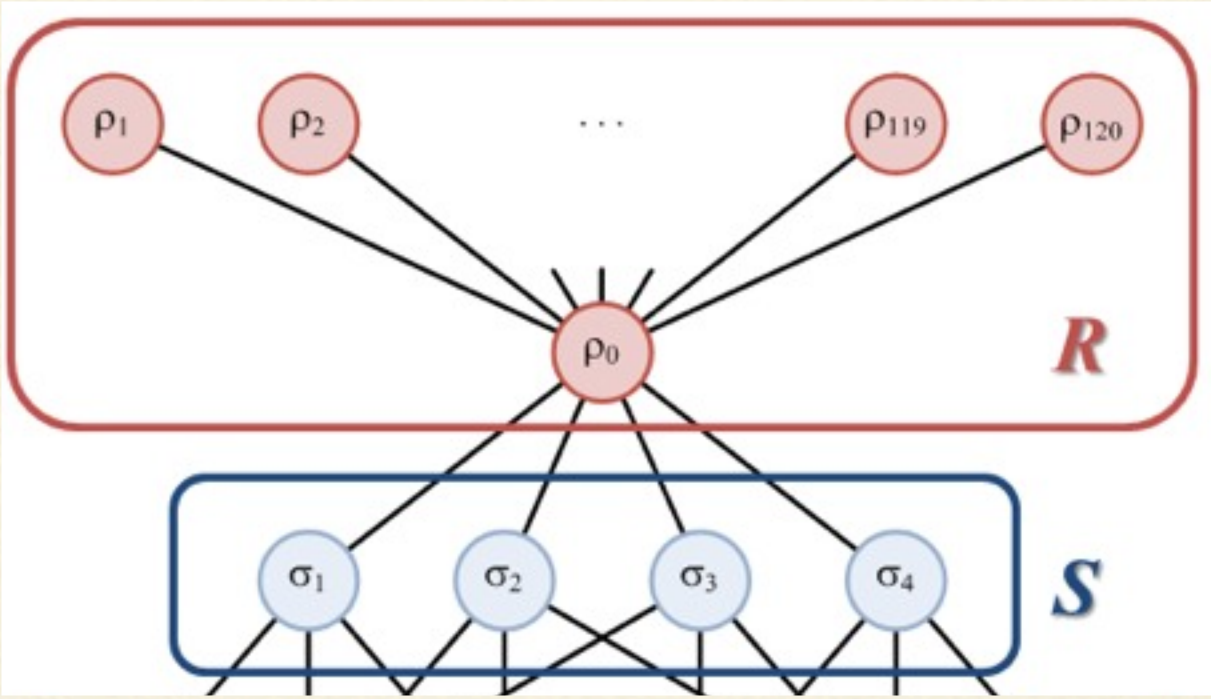
- $C_{RR}=r^2$



- $C_{RR}=\{[2*(r-1)+1]*r+r\}/2=r^2$

Criterion Threshold

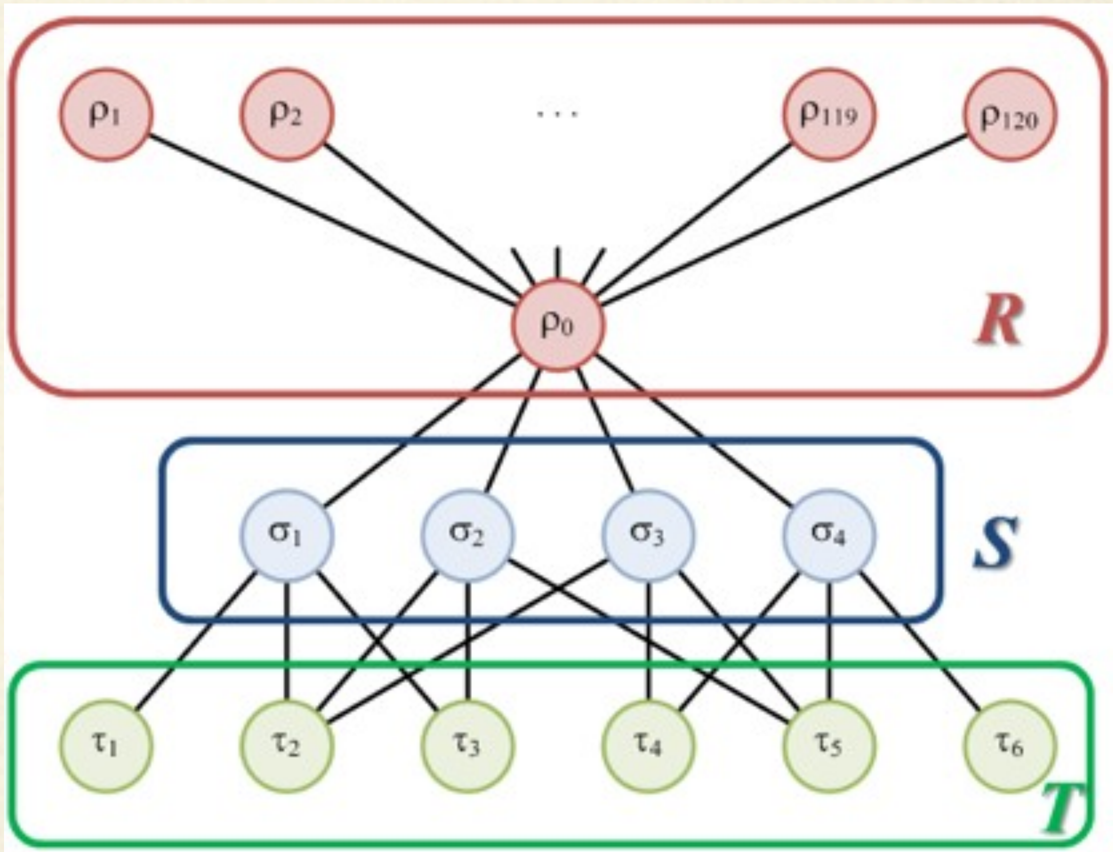
- C_{RS}=2rs+s



- C_{RS}=(2*s)*r+s=2rs+s

Criterion Threshold

- C_{RT}=9rt+6t



- C_{RT}=(3*3t)*r+2*3t=9rt+6t

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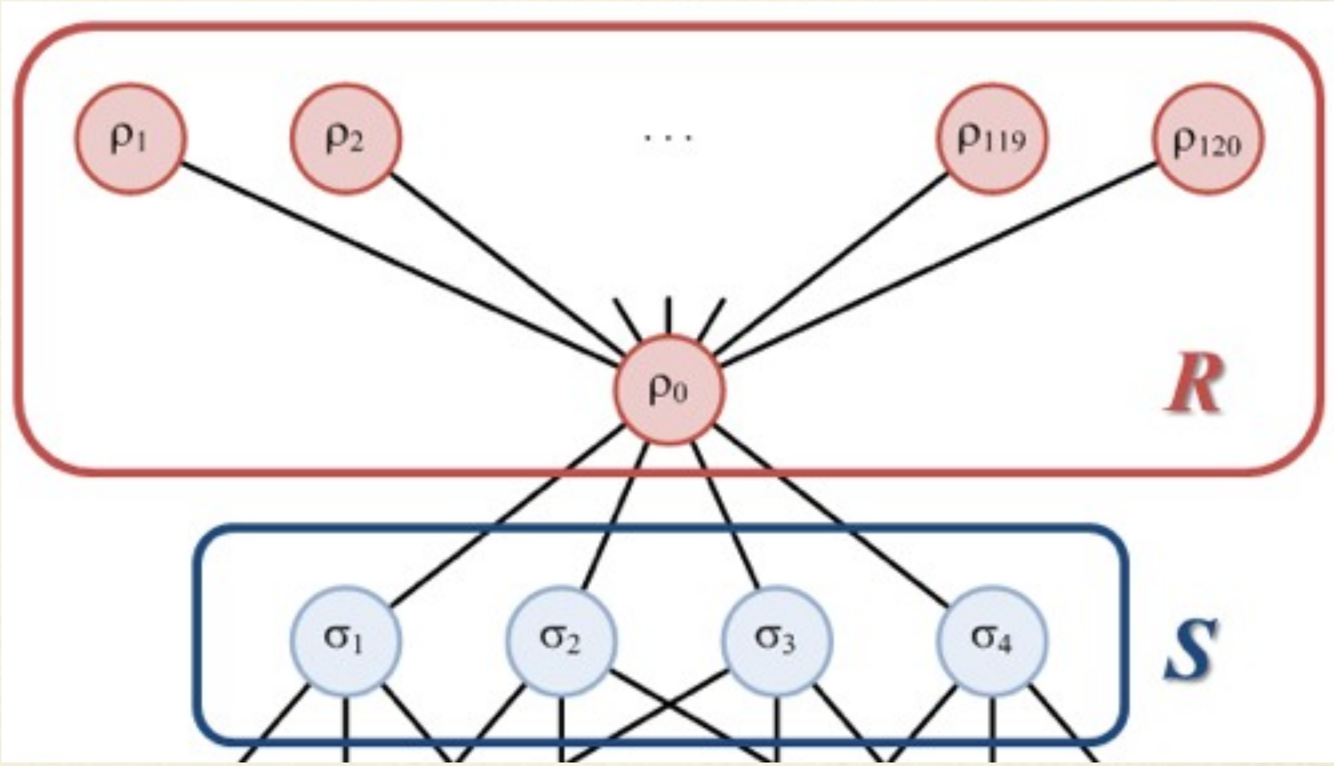
Criterion Threshold

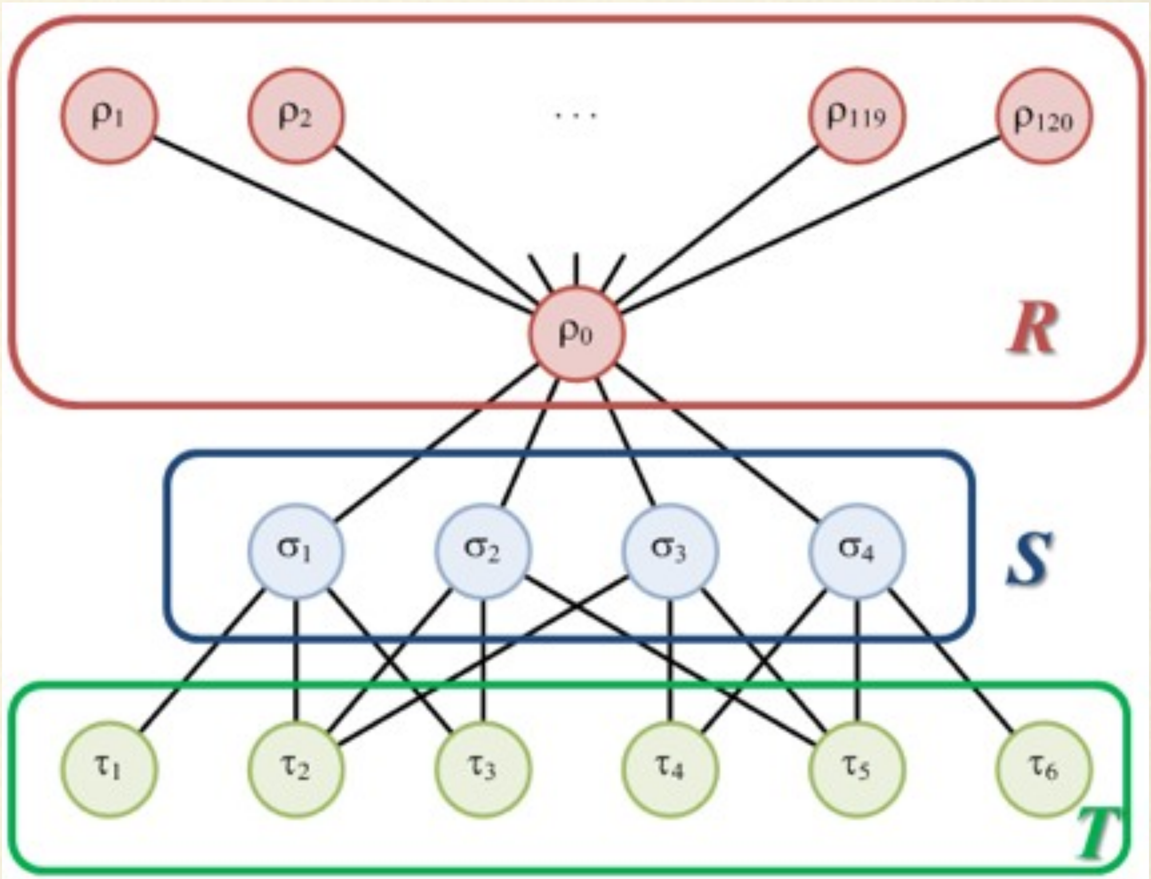
- $C_{SS}=s^2$-s



- C_{SS}={[2*(s-1)]*s}/2=s^2-s

Criterion Threshold

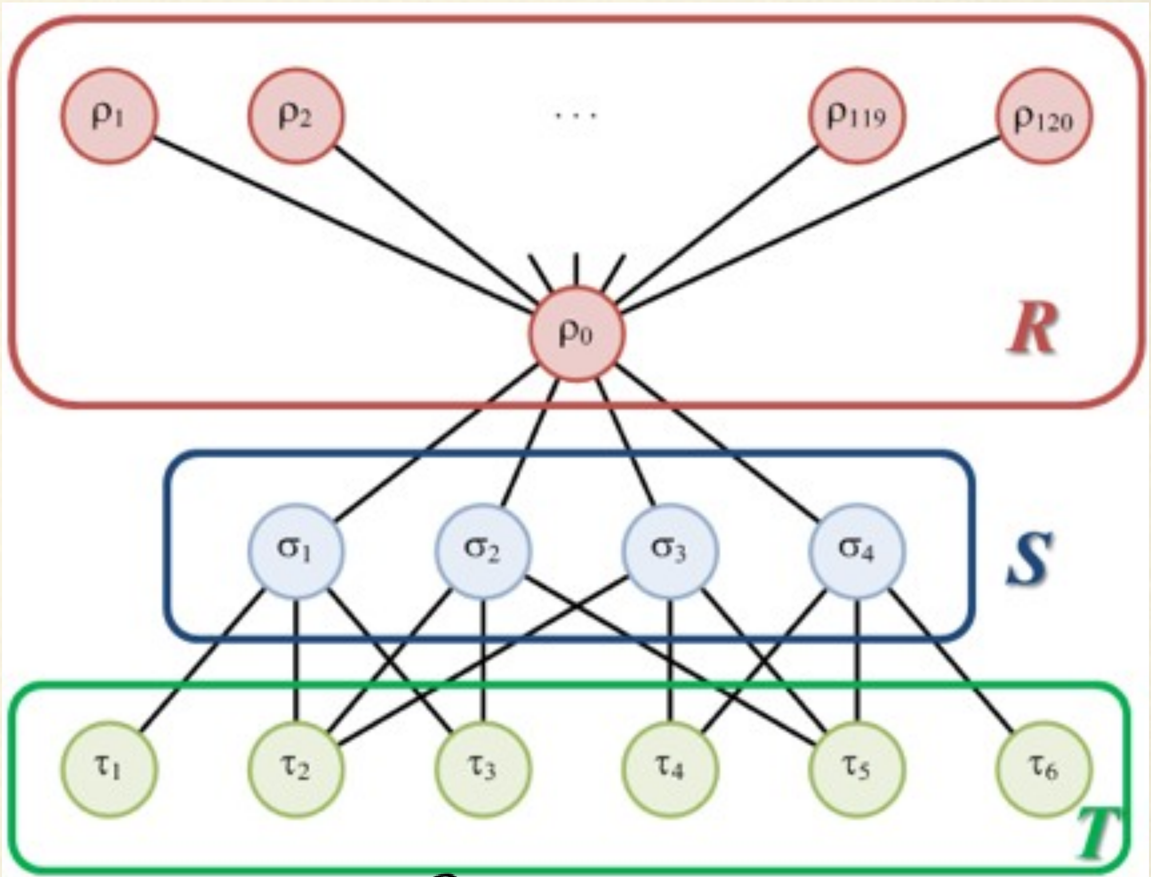
- C_{ST}=9st-6t



- C_{ST}=[1+3*(s-1)]*3t= 9st-6t

Criterion Threshold

- C_{TT}=18t^2-12t



- C_{TT}={[2*2+4*(3t-3)]*3t}/2=18t^2-12t

Criterion Threshold

- C_{RR}=r^2
- C_{RS}=2rs+s
- C_{RT}=9rt+6t
- C_{SS}=s^2-s
- C_{ST}=9st-6t
- C_{TT}=18t^2-12t

Illustration of Reduction

Illustration of Reduction

$S = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$

Illustration of Reduction

$S = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\} \qquad s = 4$

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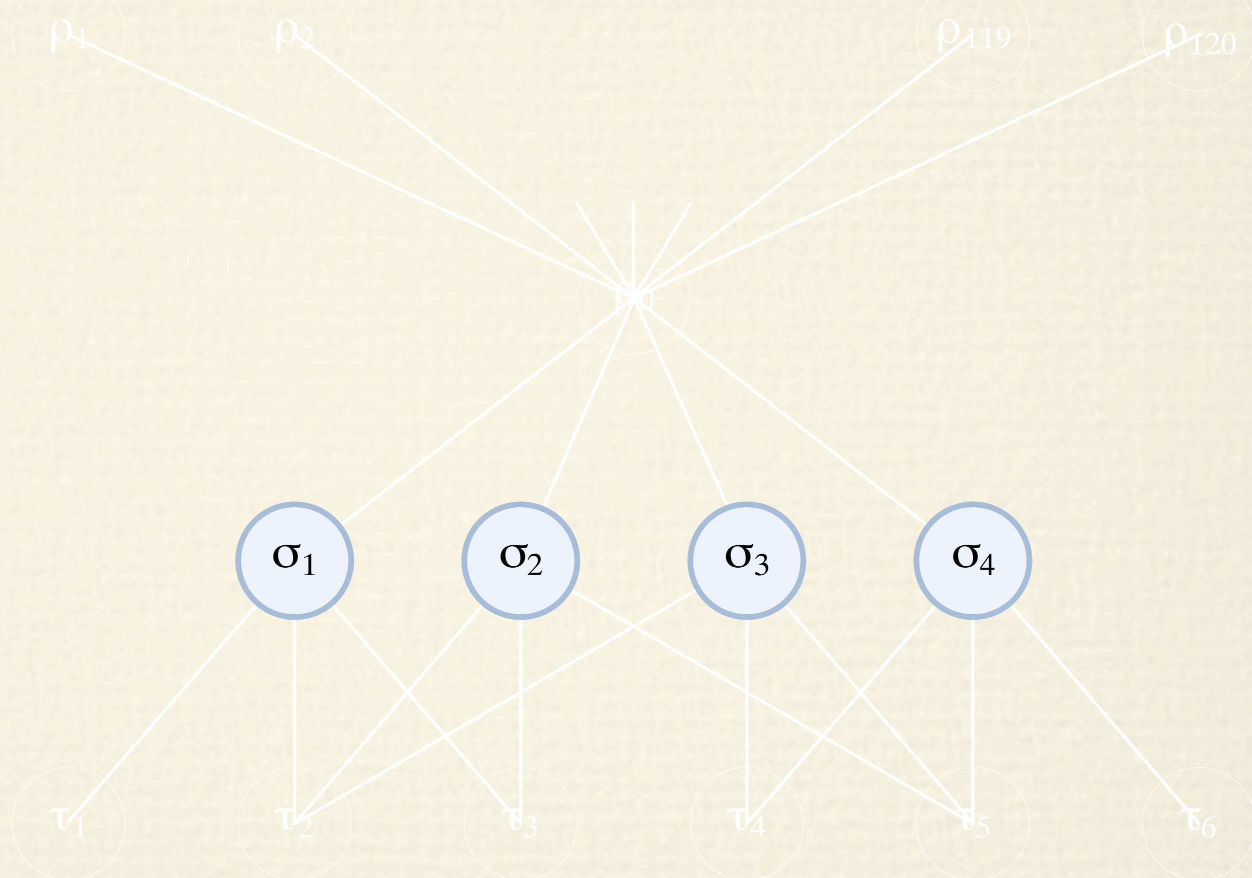

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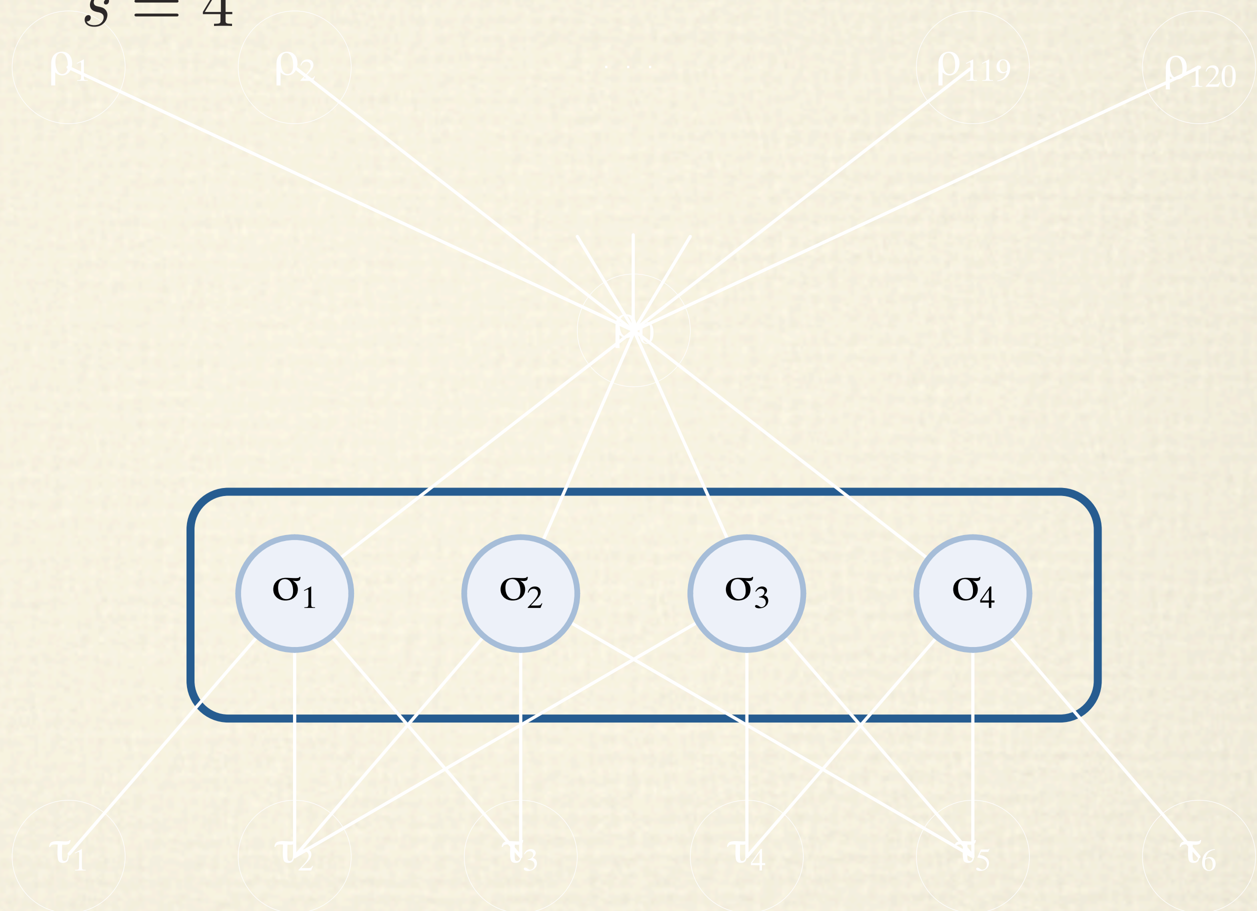

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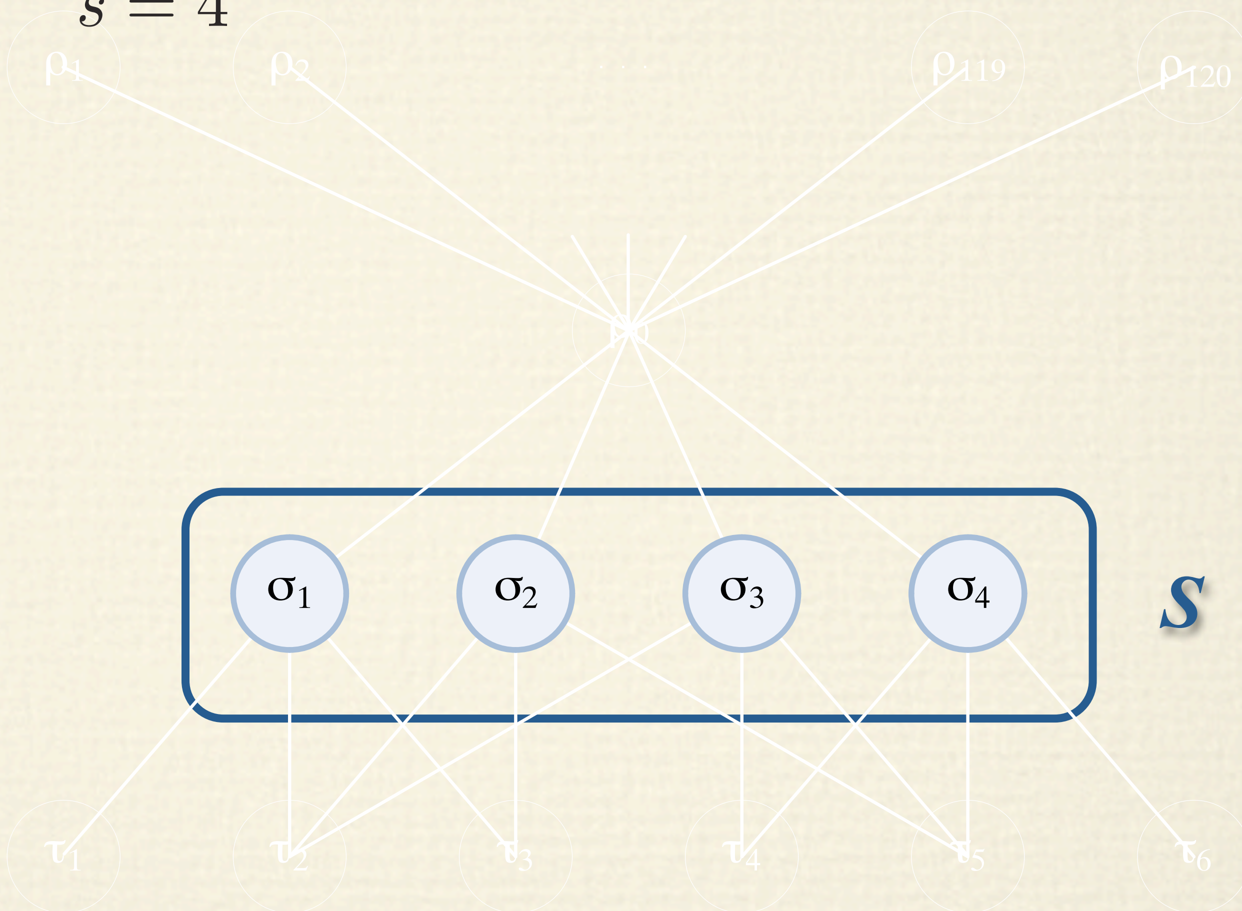
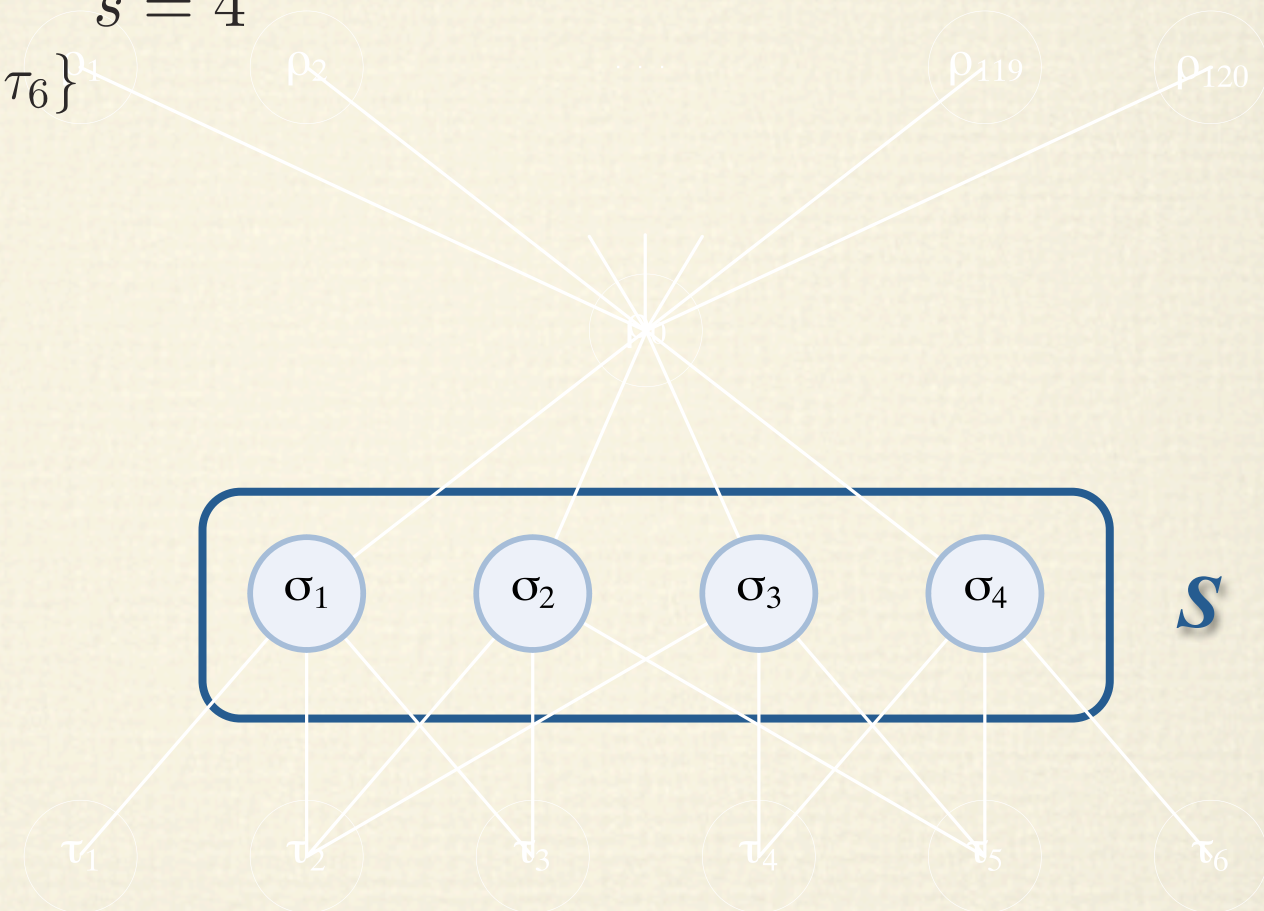


Illustration of Reduction

$S = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ $\quad s = 4$

$T = \{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6\}$



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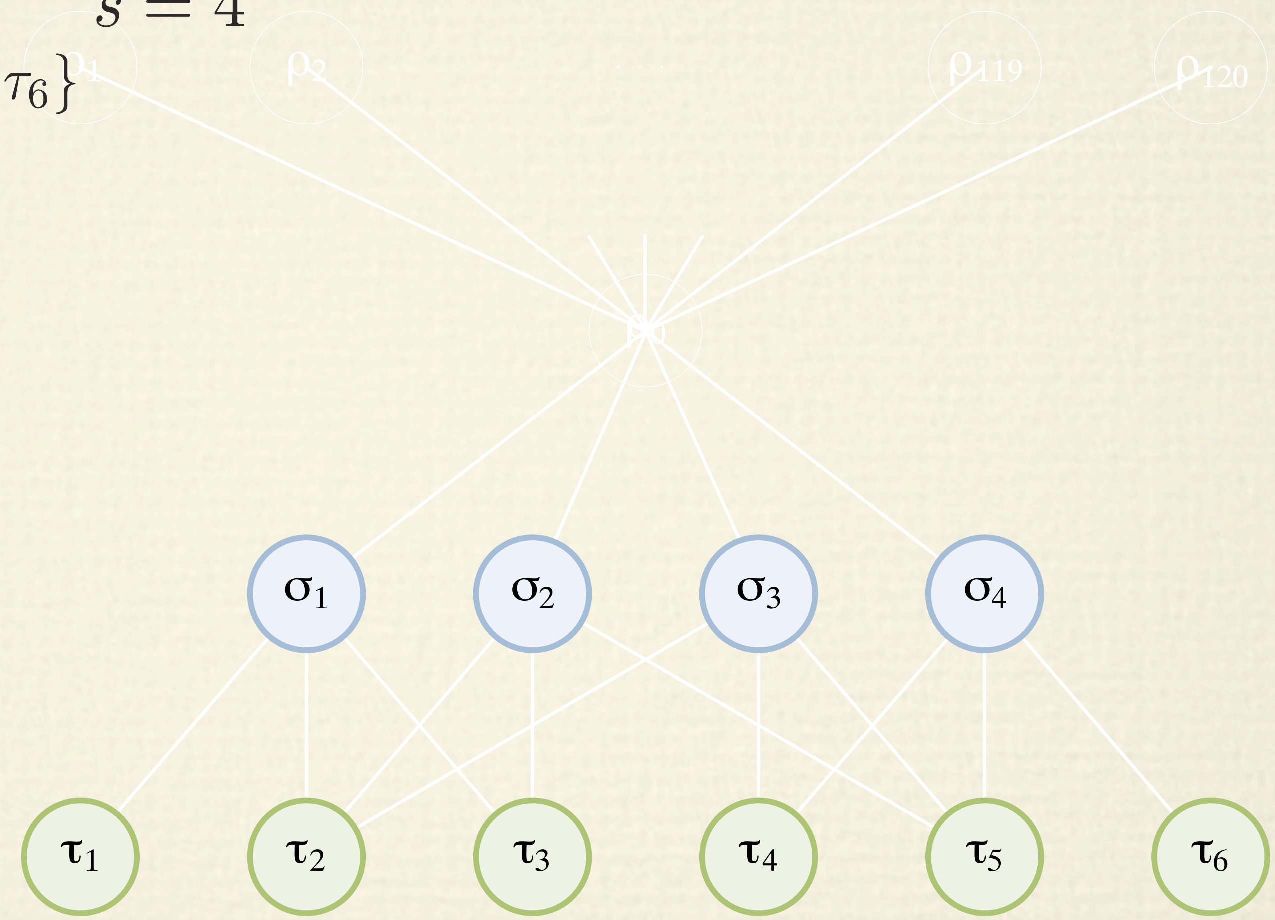
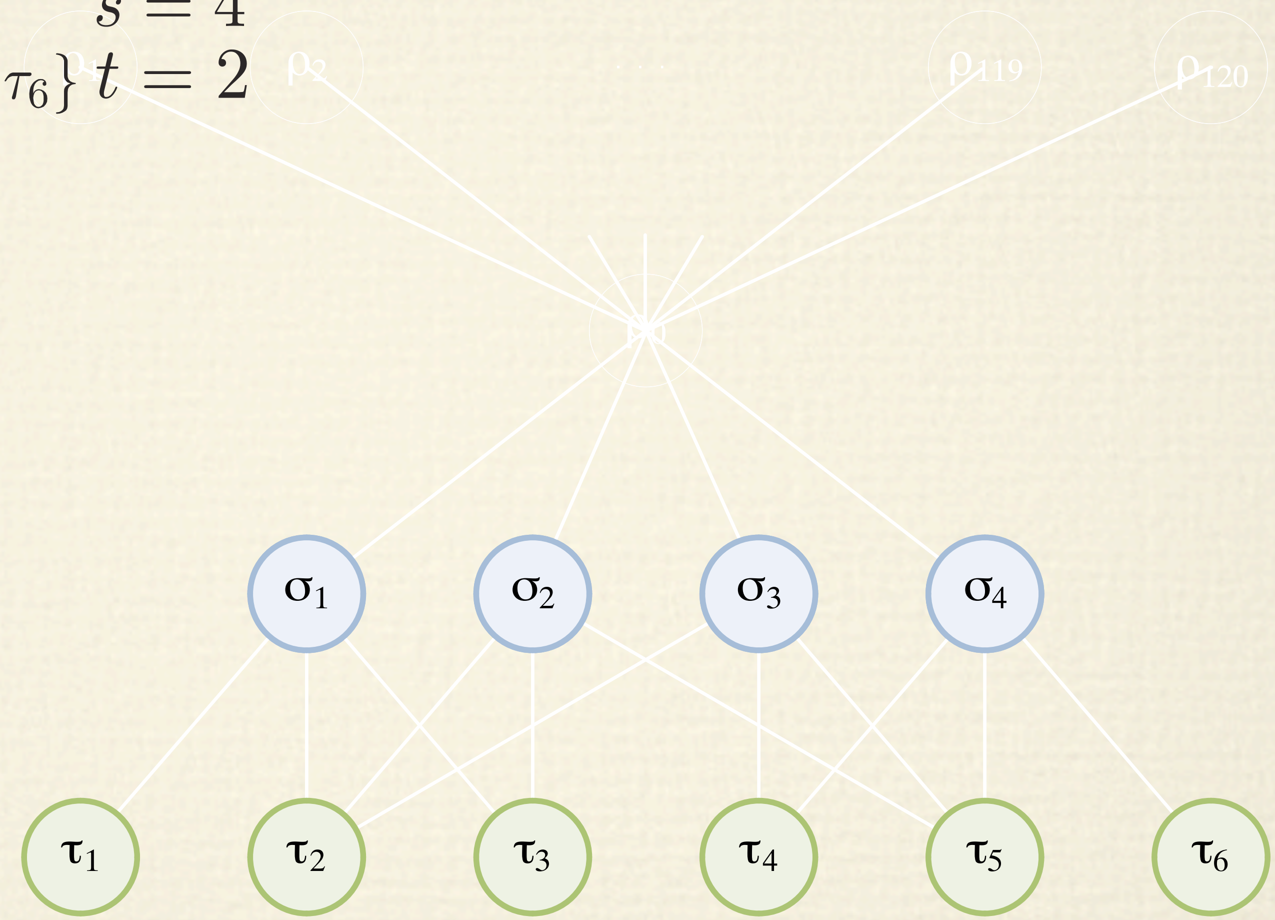


Illustration of Reduction

$S = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\} \qquad s = 4$

$T = \{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6\} \quad t = 2$



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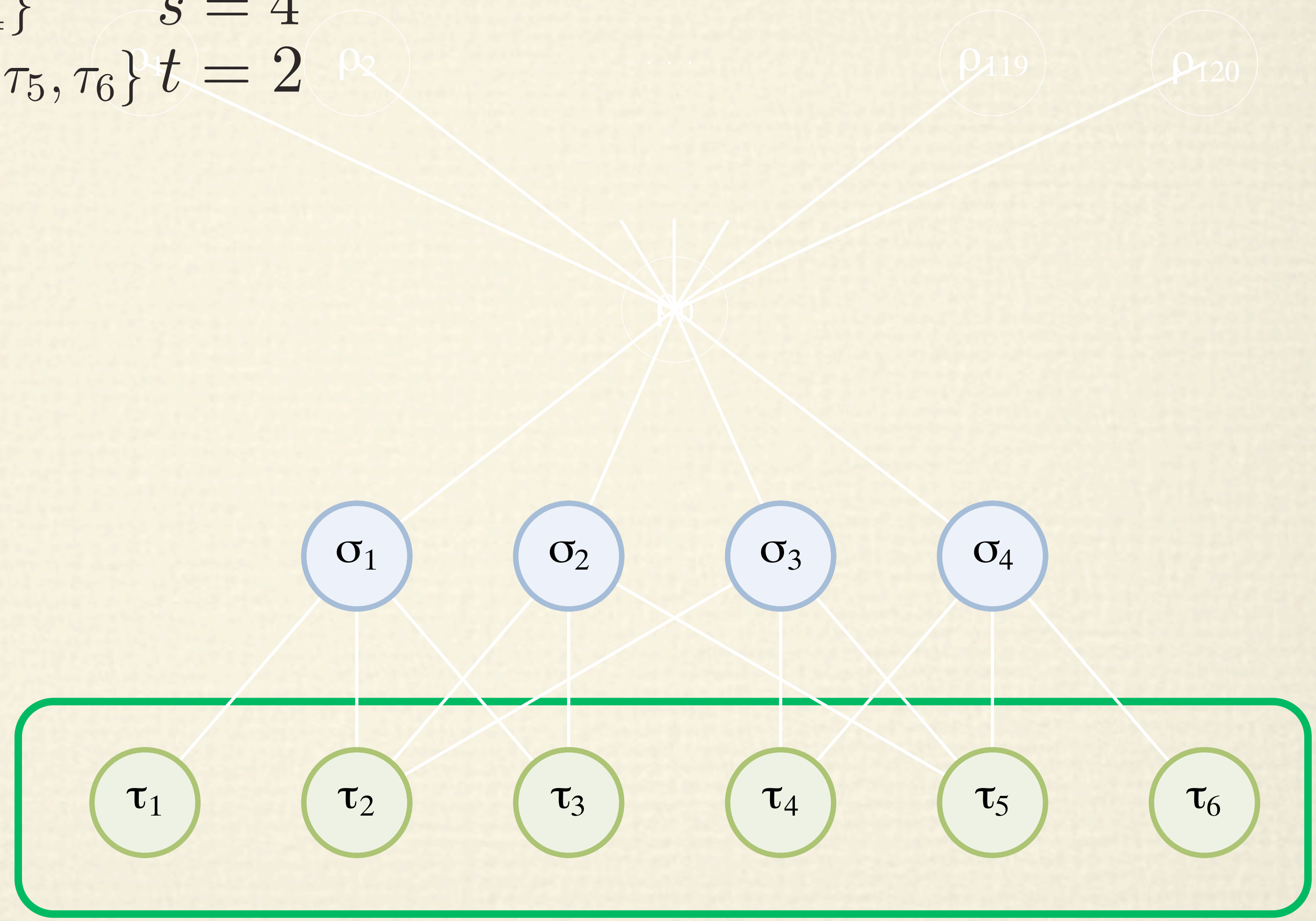

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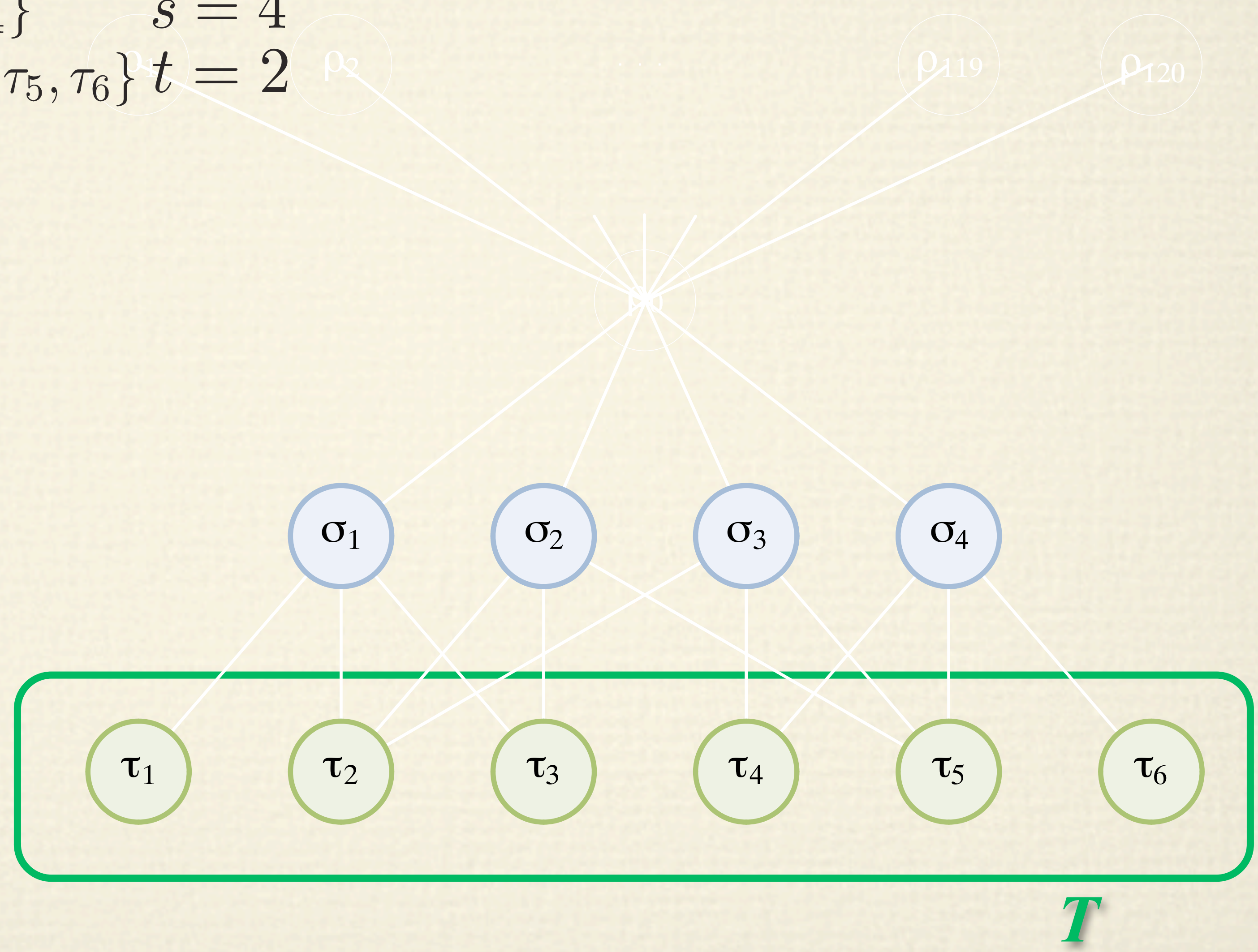

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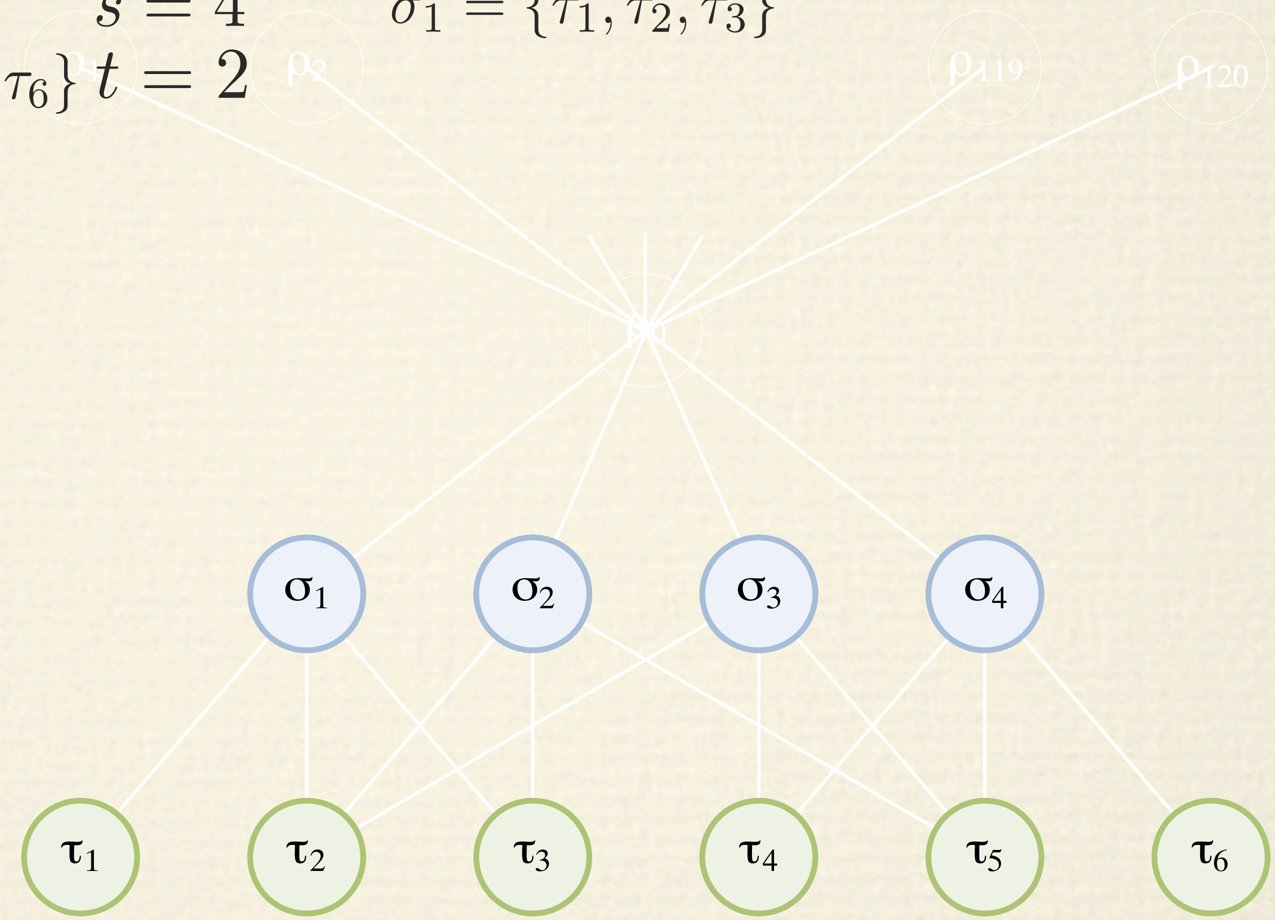

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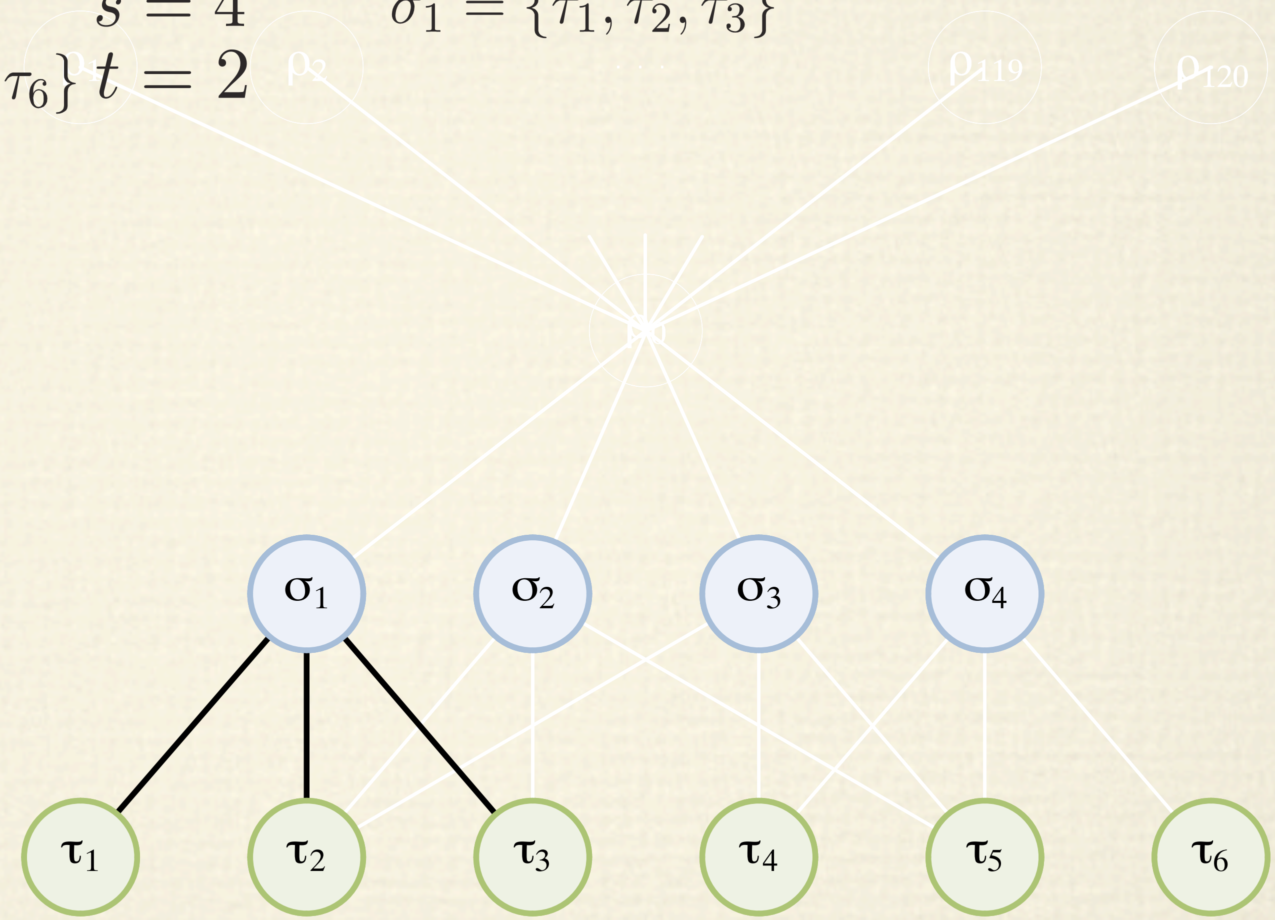

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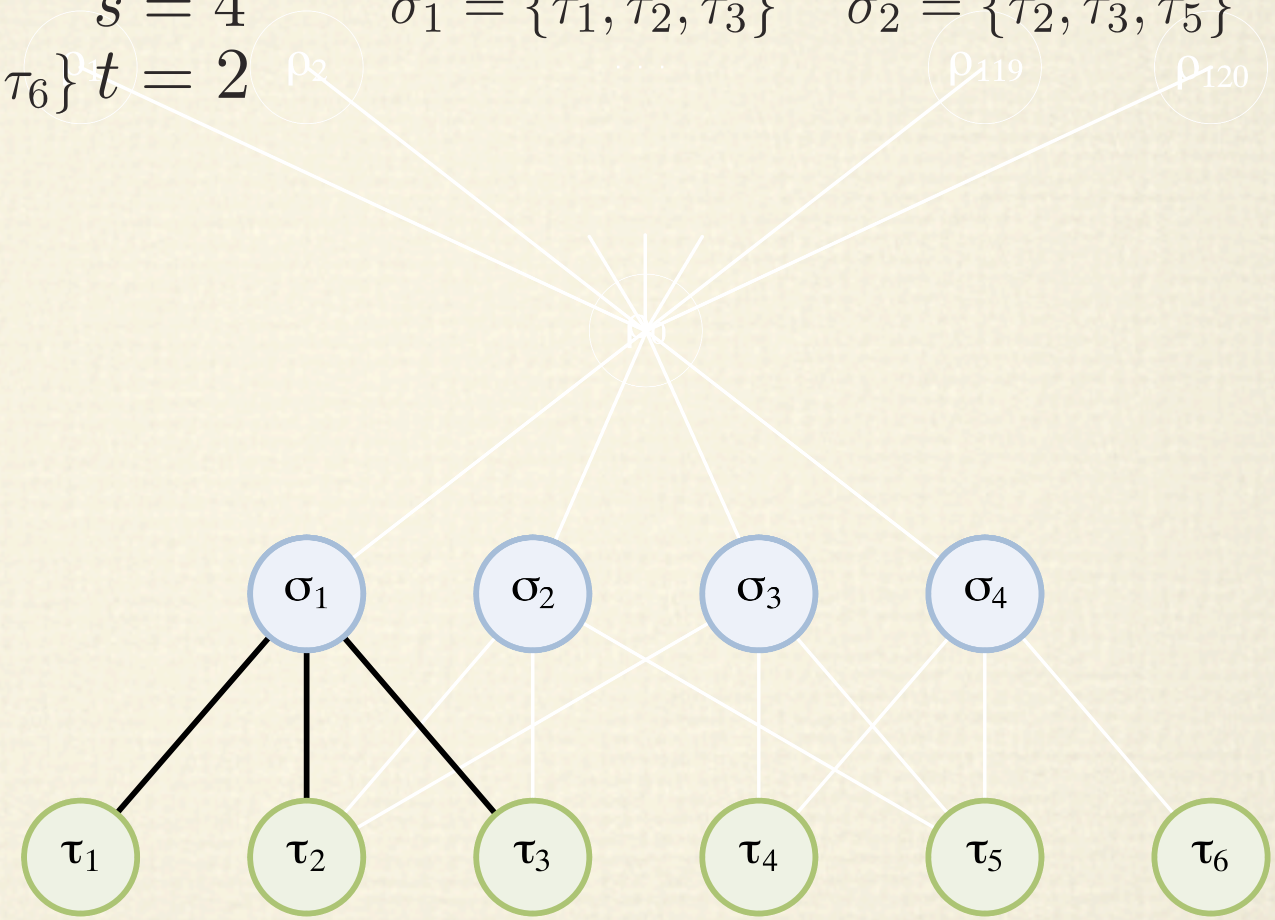

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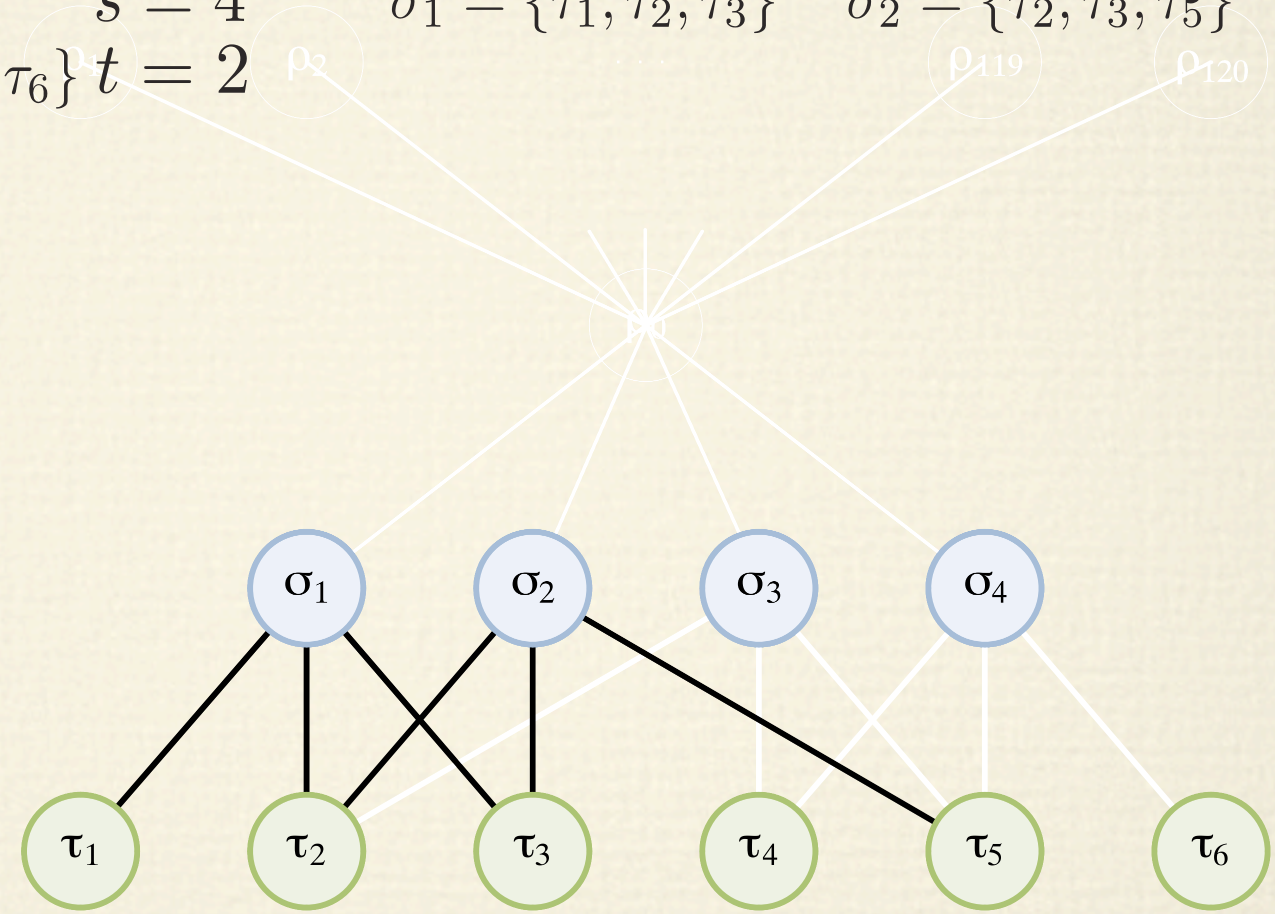

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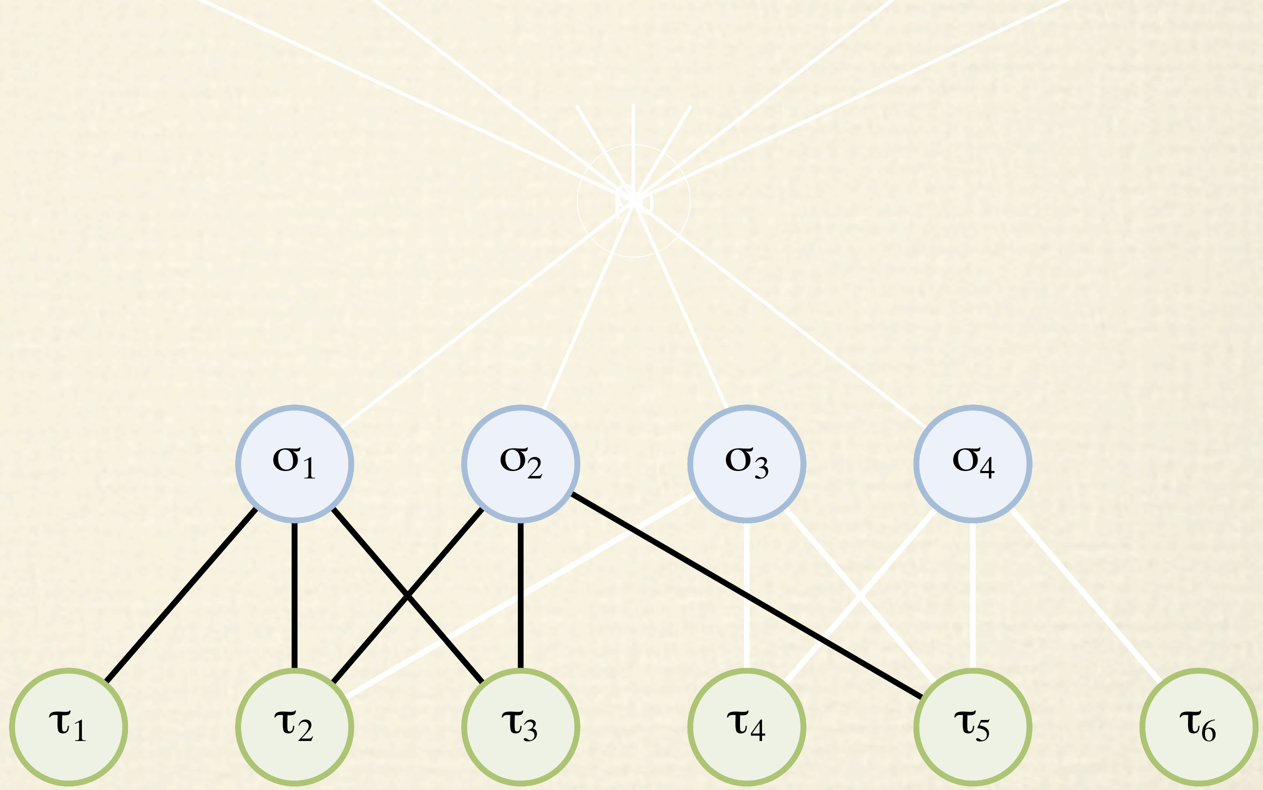

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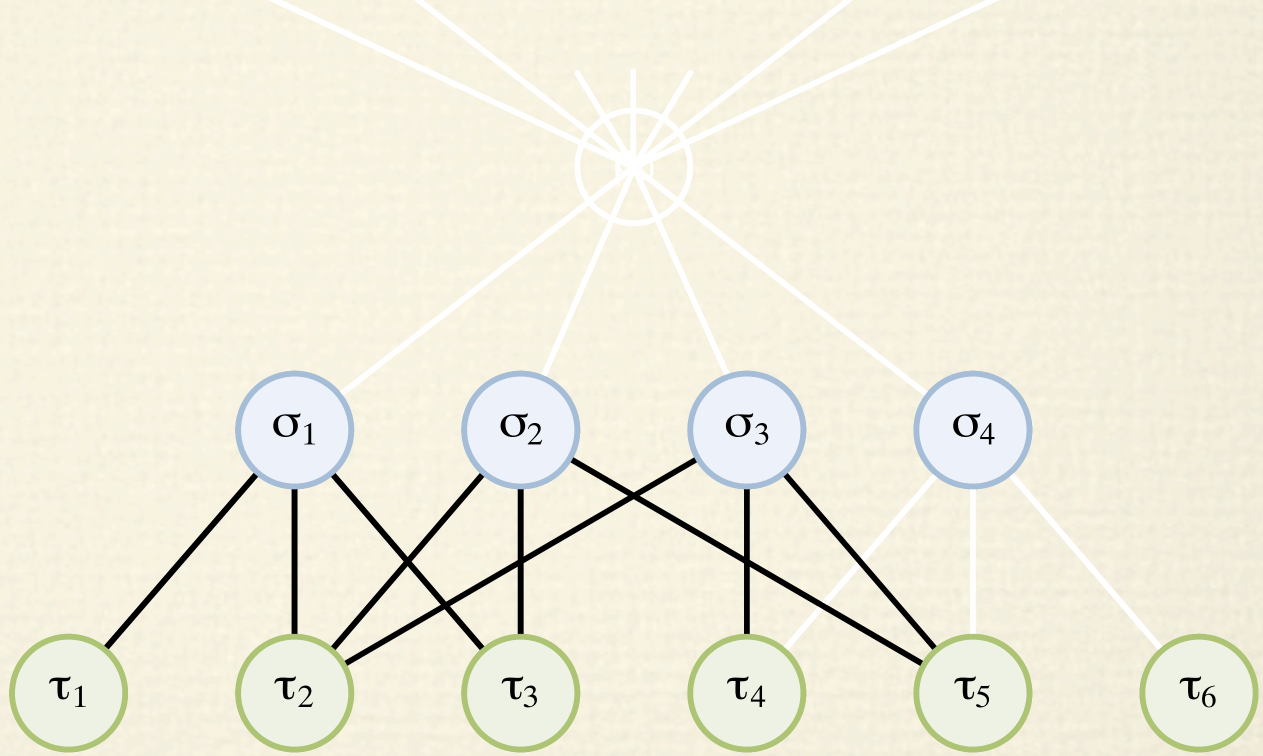

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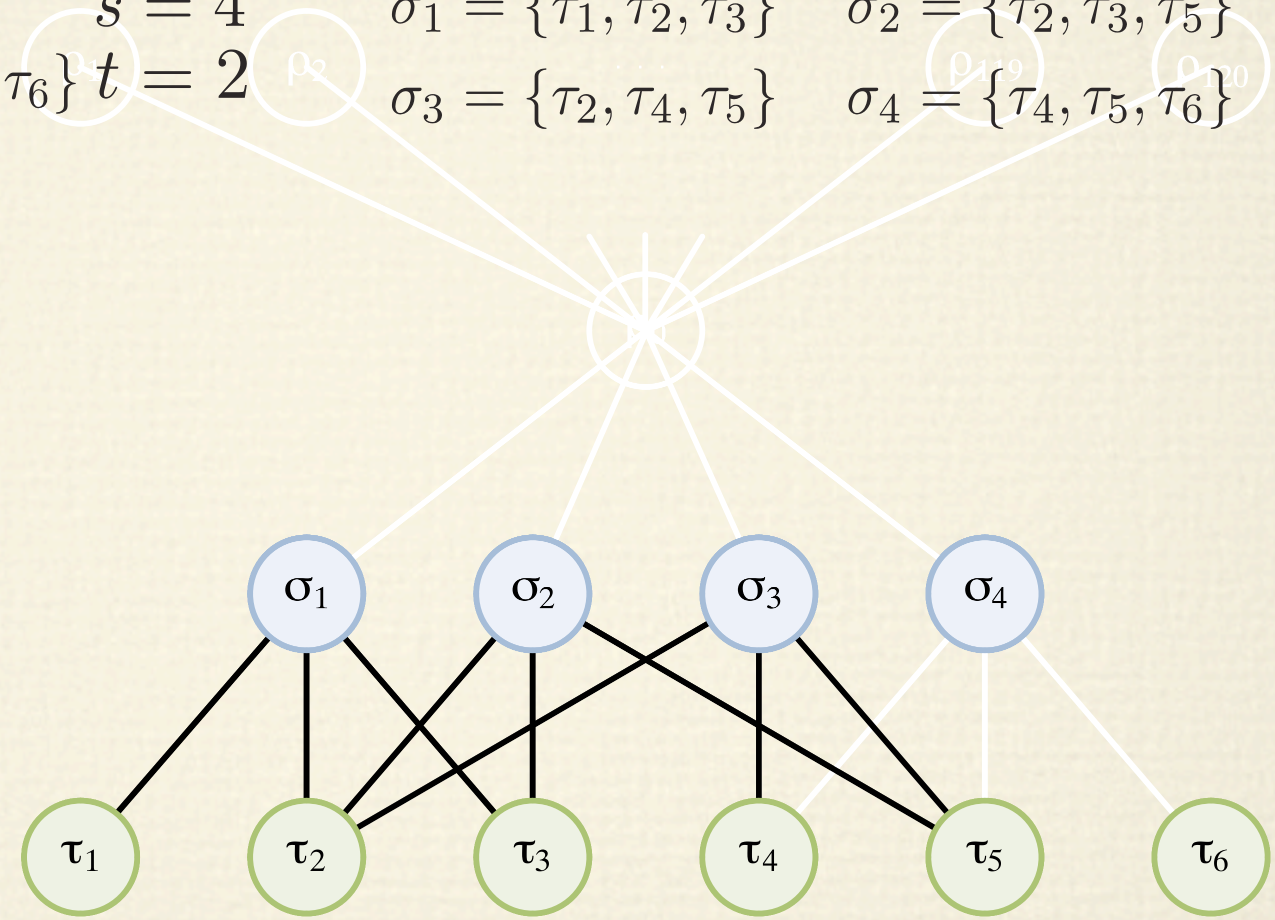
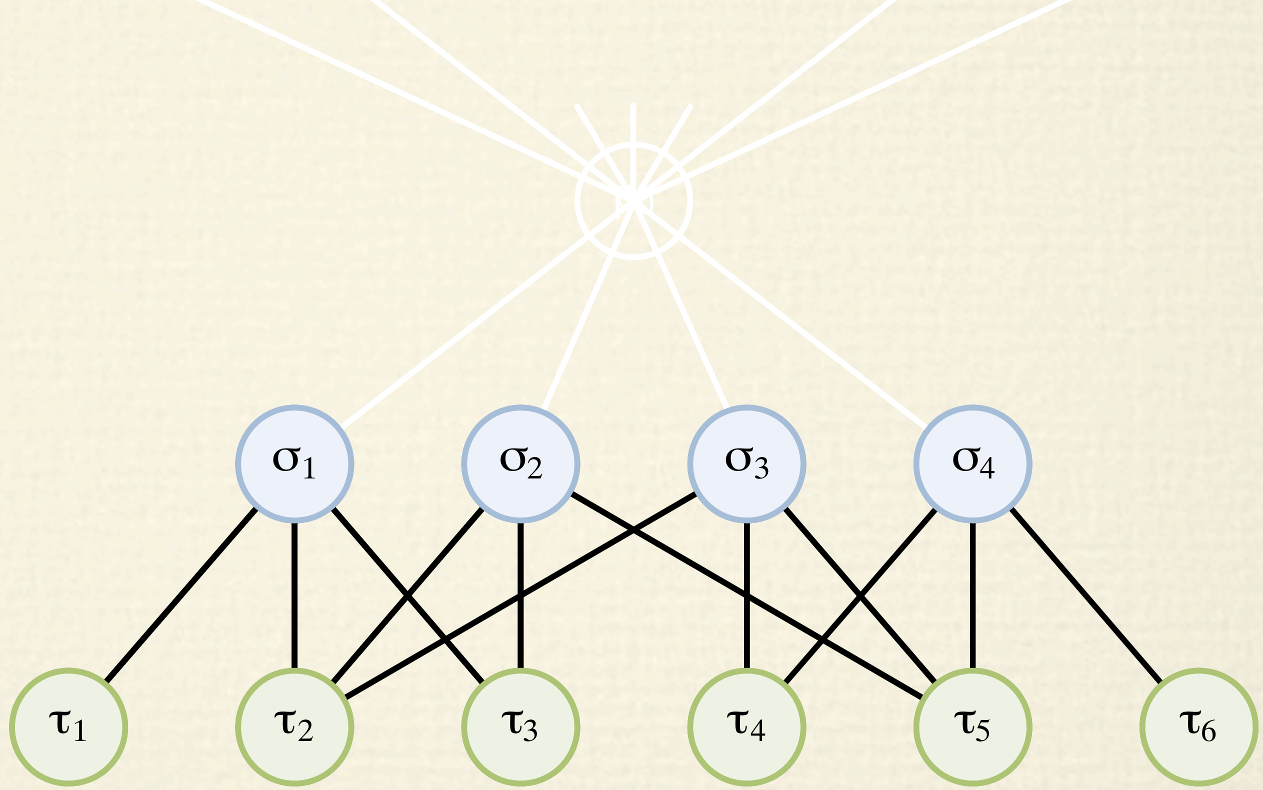


Illustration of Reduction

$S = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ $\quad s = 4$ $\quad \sigma_1 = \{\tau_1, \tau_2, \tau_3\}$ $\quad \sigma_2 = \{\tau_2, \tau_3, \tau_5\}$

$T = \{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6\}$ $\quad t = 2$ $\quad \sigma_3 = \{\tau_2, \tau_4, \tau_5\}$ $\quad \sigma_4 = \{\tau_4, \tau_5, \tau_6\}$



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$C_{SS} = s^2 - s = 16 - 4 = 12$

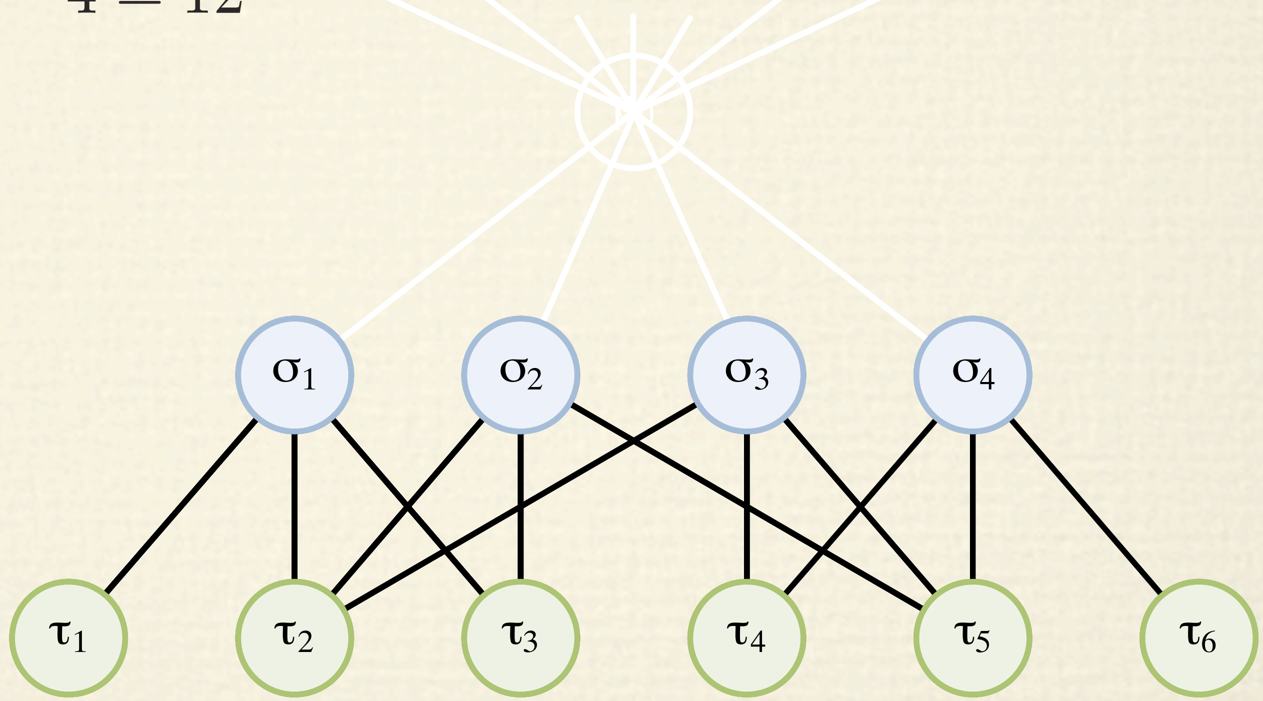

Illustration of Reduction

$S = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ $s = 4$

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$C_{SS} = s^2 - s = 16 - 4 = 12$

$$C_{ST} = 9st - 6t$$
$$= 9 \times 4 \times 2 - 6 \times 2 = 60$$

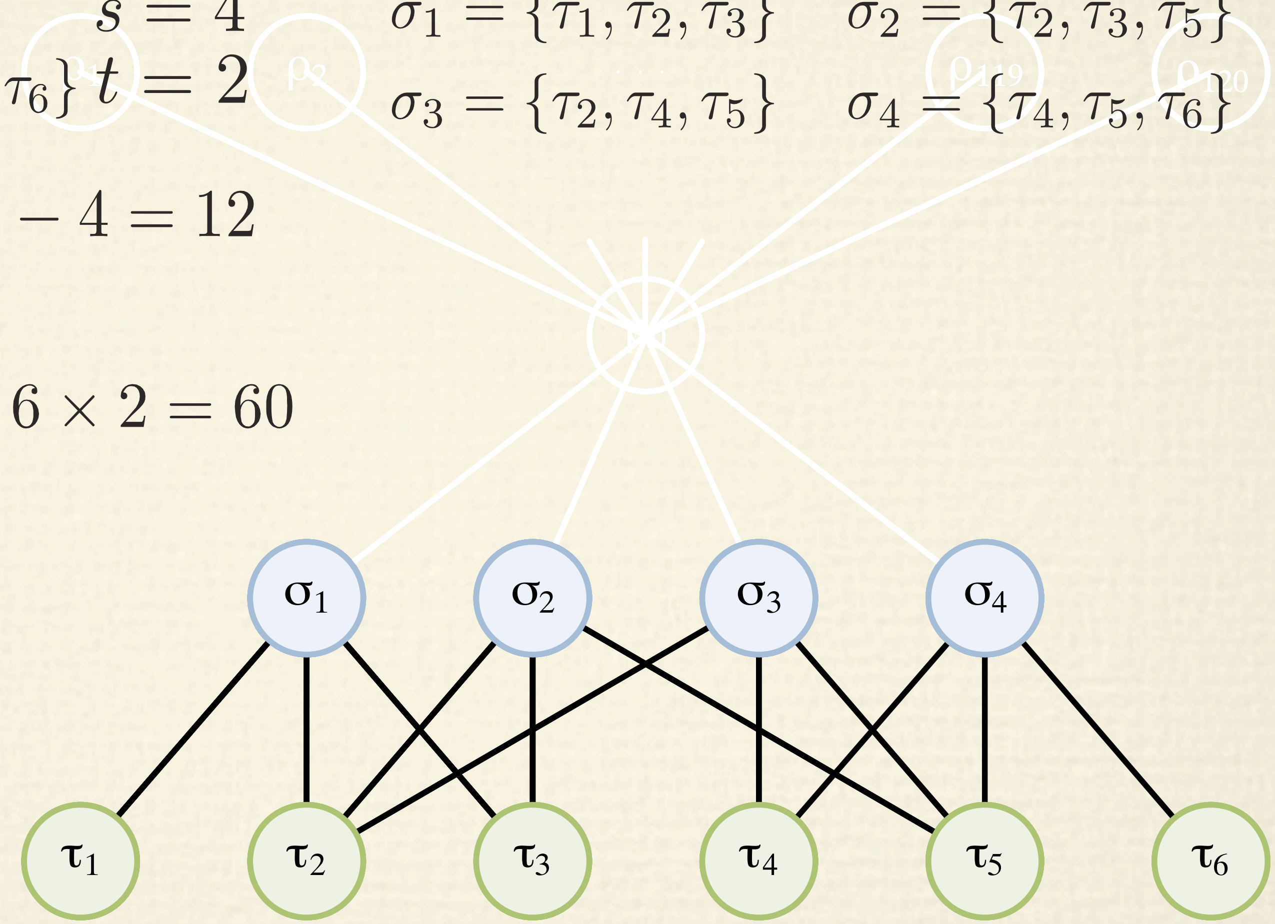

Illustration of Reduction

$S = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ $\quad s = 4$

$T = \{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6\}$ $\quad t = 2$

$\sigma_1 = \{\tau_1, \tau_2, \tau_3\}$ $\quad \sigma_2 = \{\tau_2, \tau_3, \tau_5\}$

$\sigma_3 = \{\tau_2, \tau_4, \tau_5\}$ $\quad \sigma_4 = \{\tau_4, \tau_5, \tau_6\}$

$$C_{SS} = s^2 - s = 16 - 4 = 12$$

$$C_{ST} = 9st - 6t$$
$$= 9 \times 4 \times 2 - 6 \times 2 = 60$$

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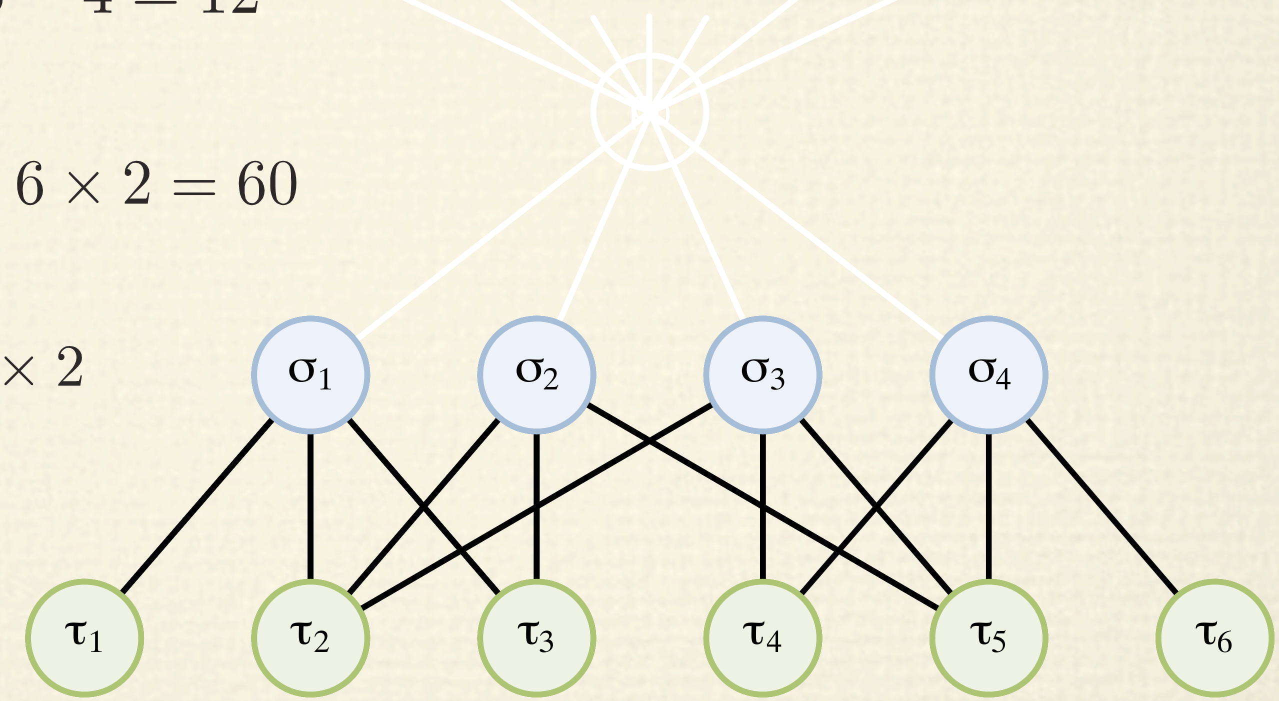

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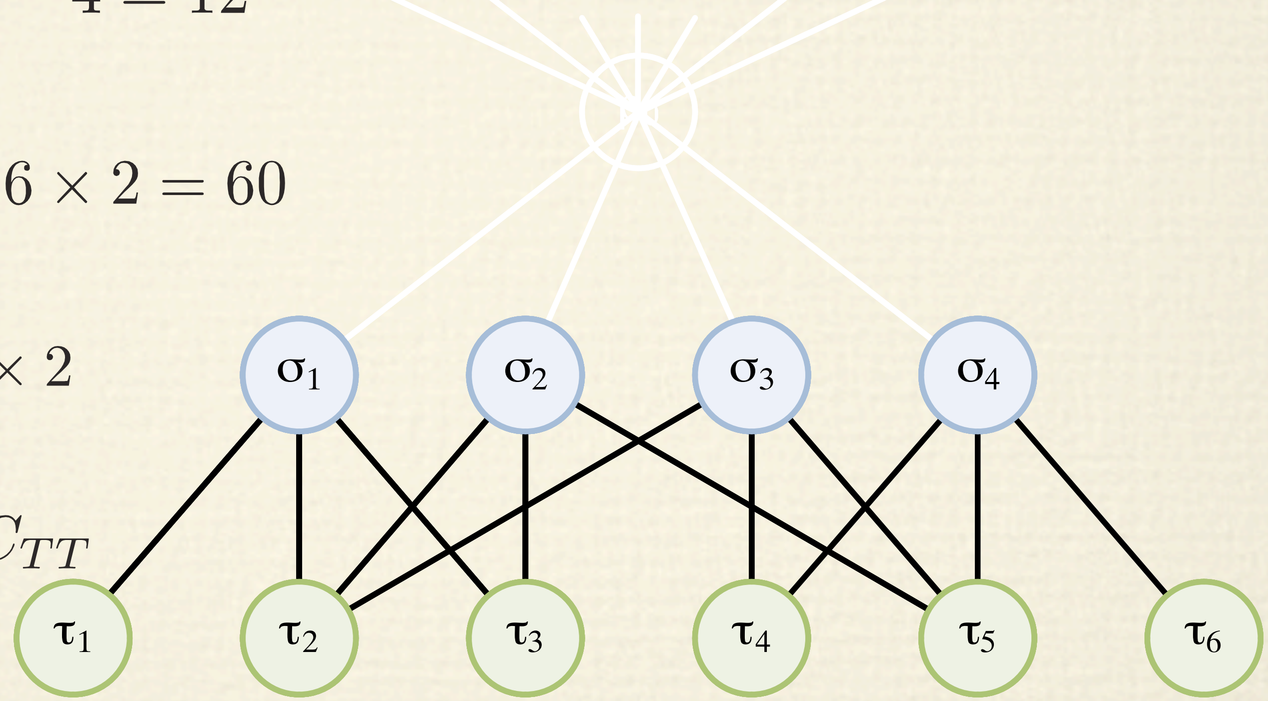
$$r = C_{SS} + C_{ST} + C_{TT}$$
$$= 12 + 60 + 48$$
$$= 120$$



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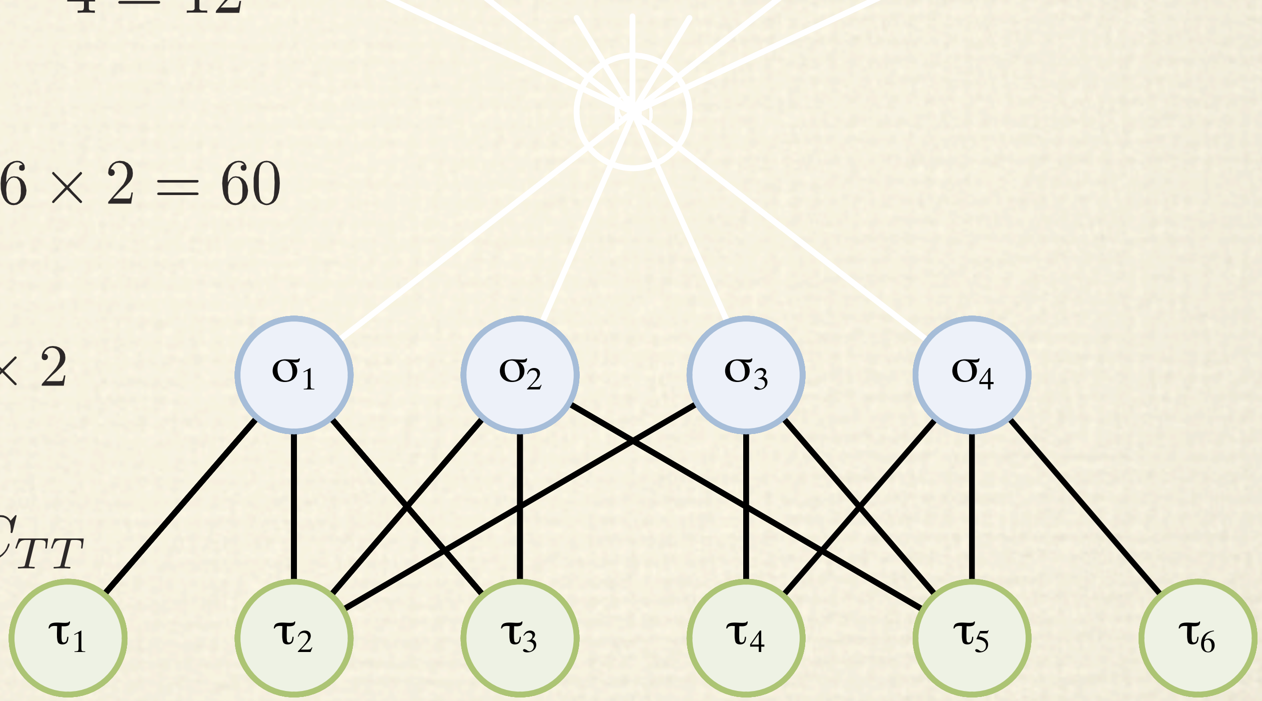

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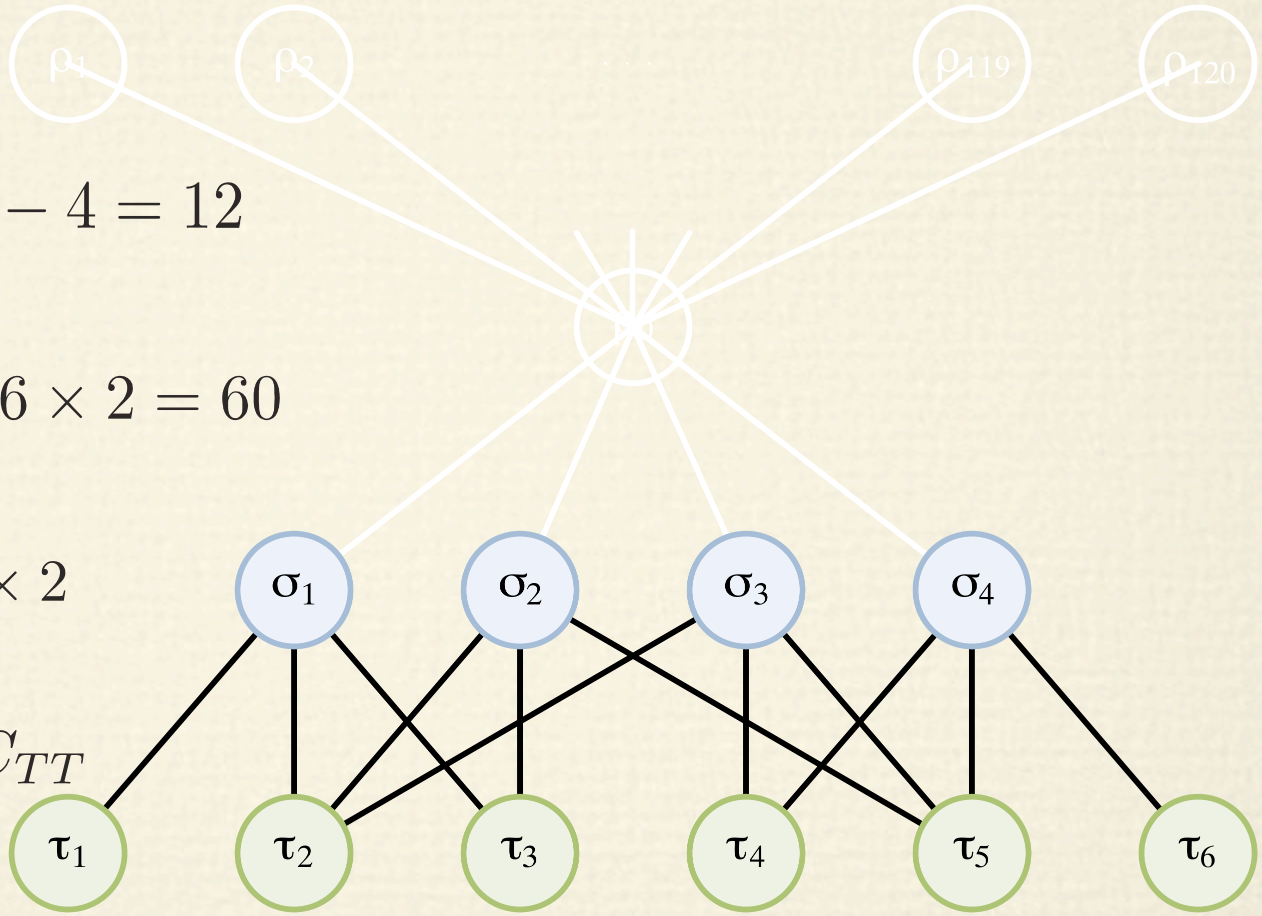

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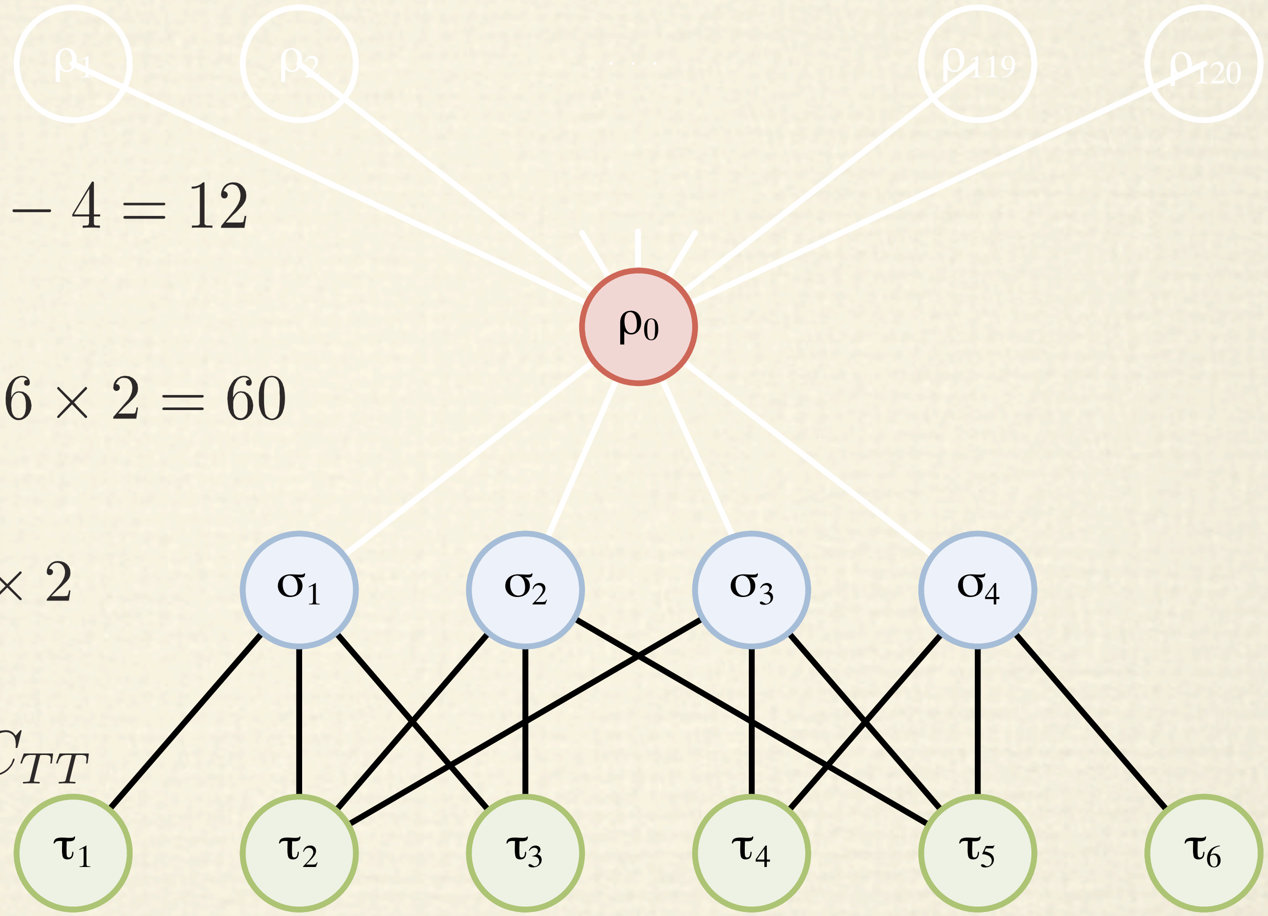

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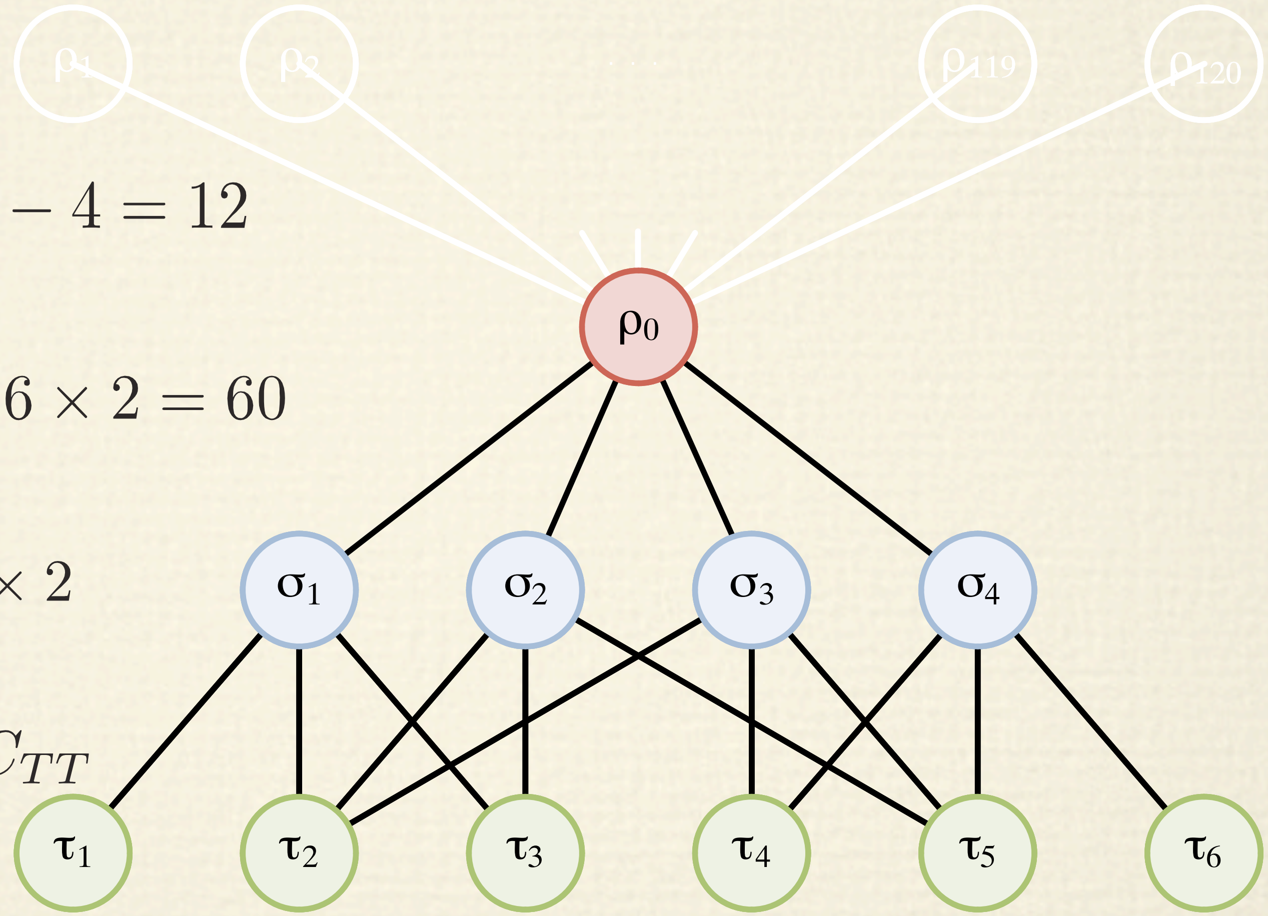

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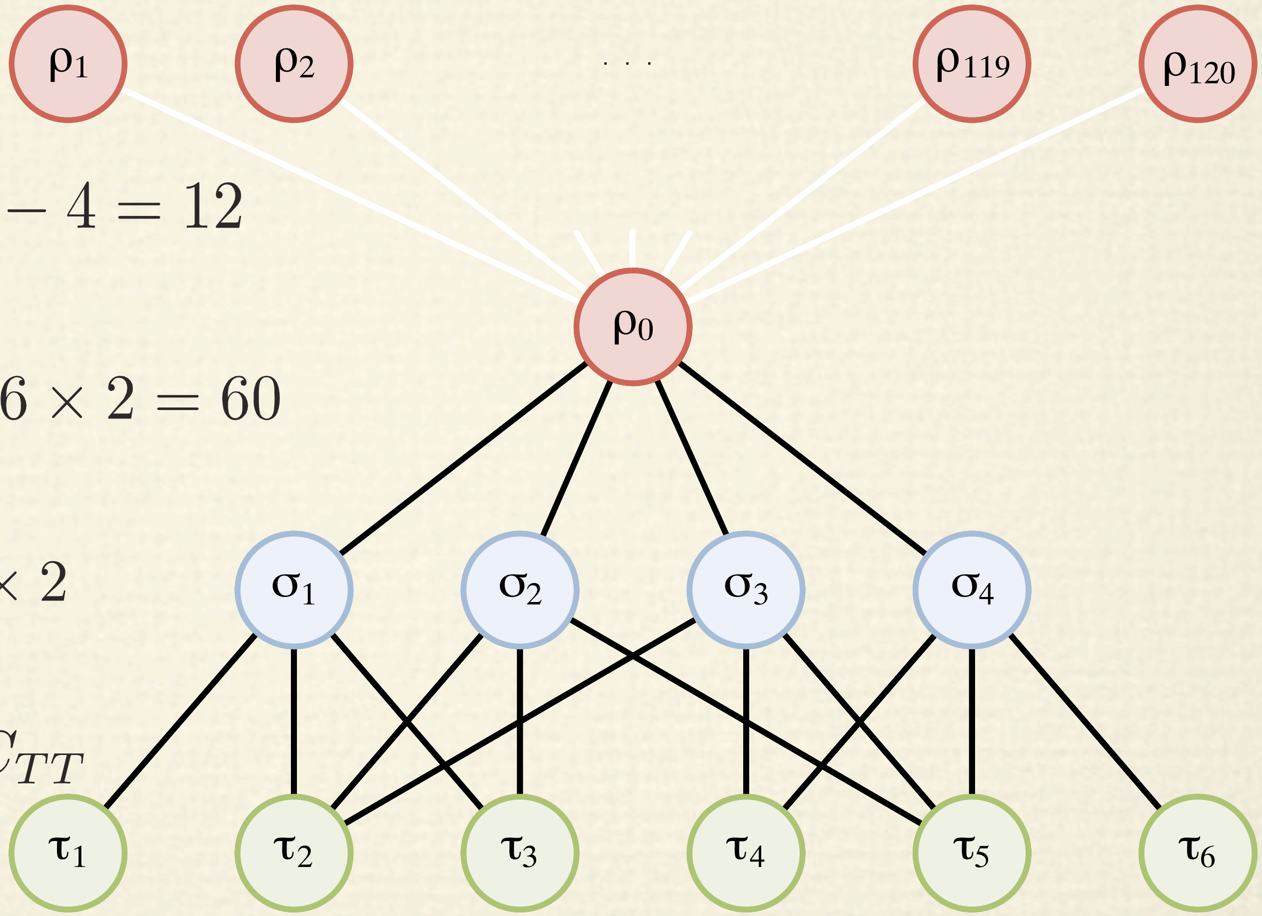

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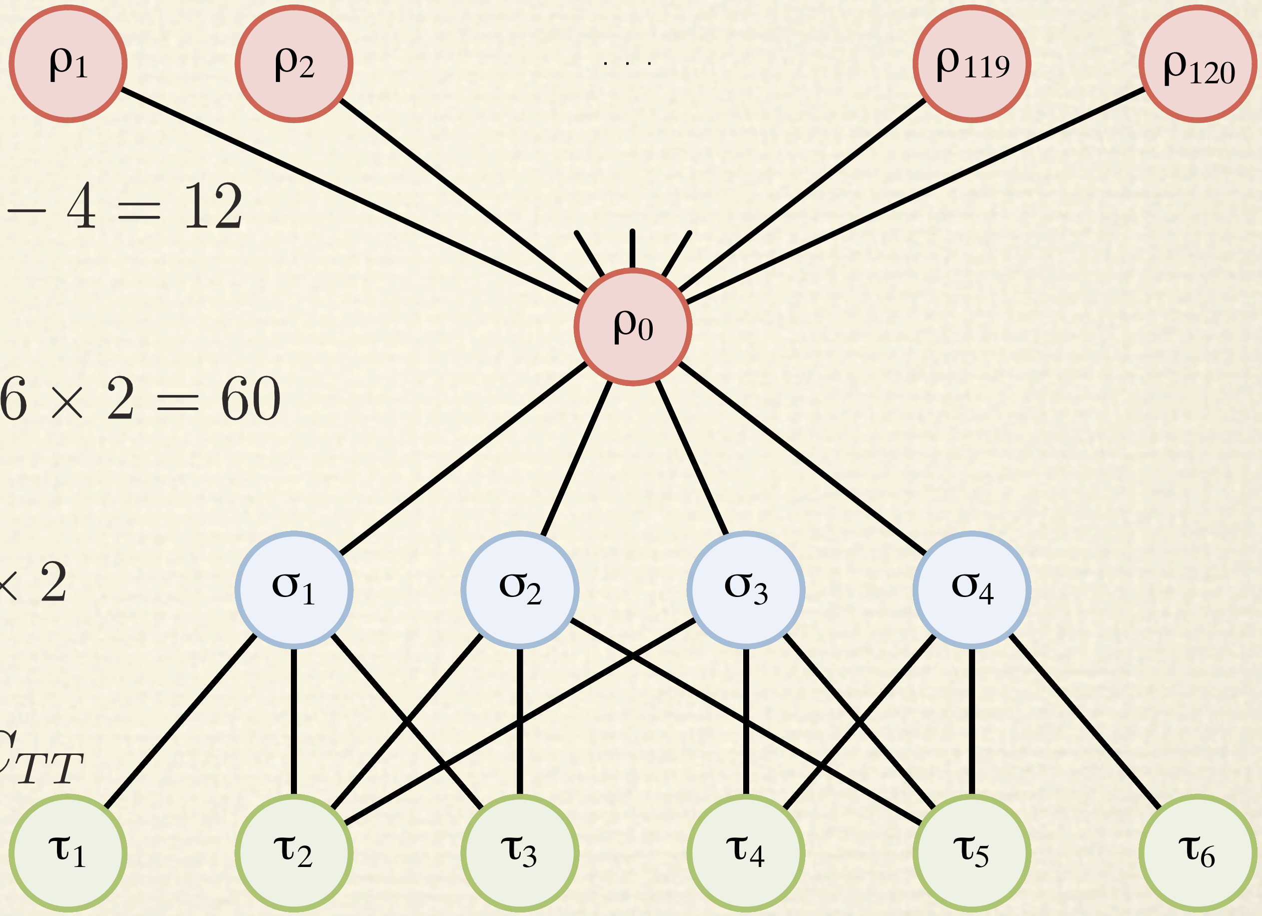

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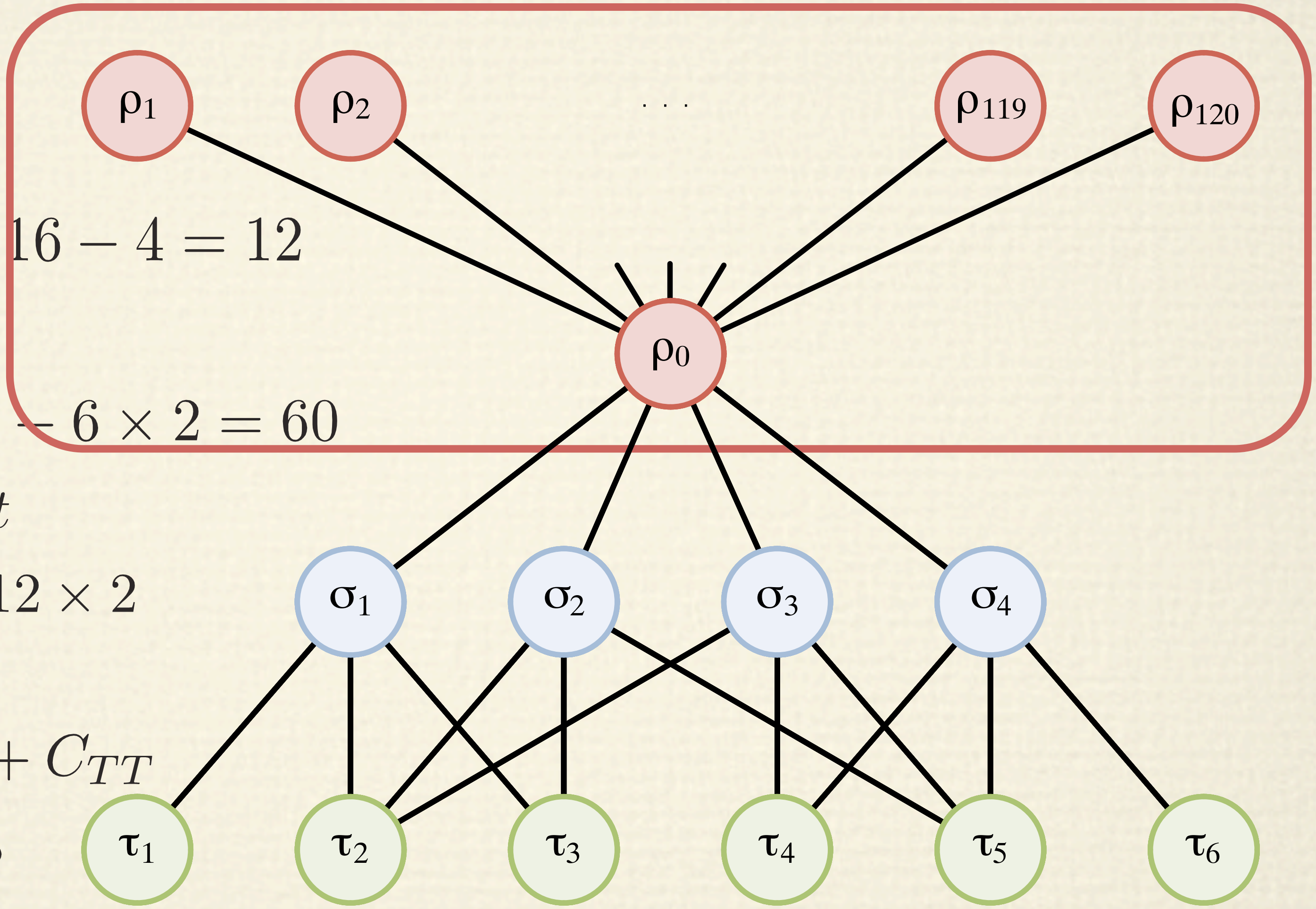

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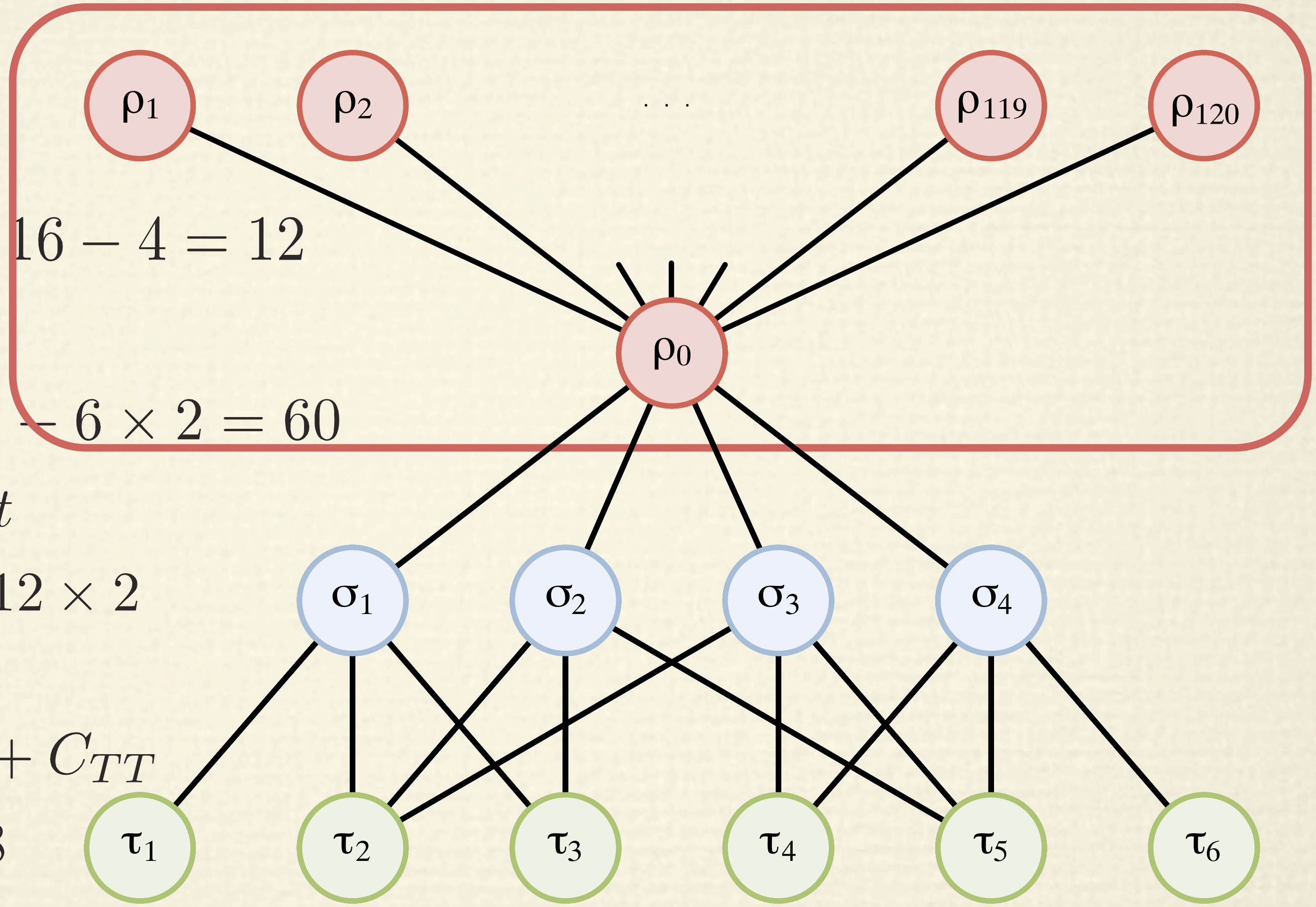

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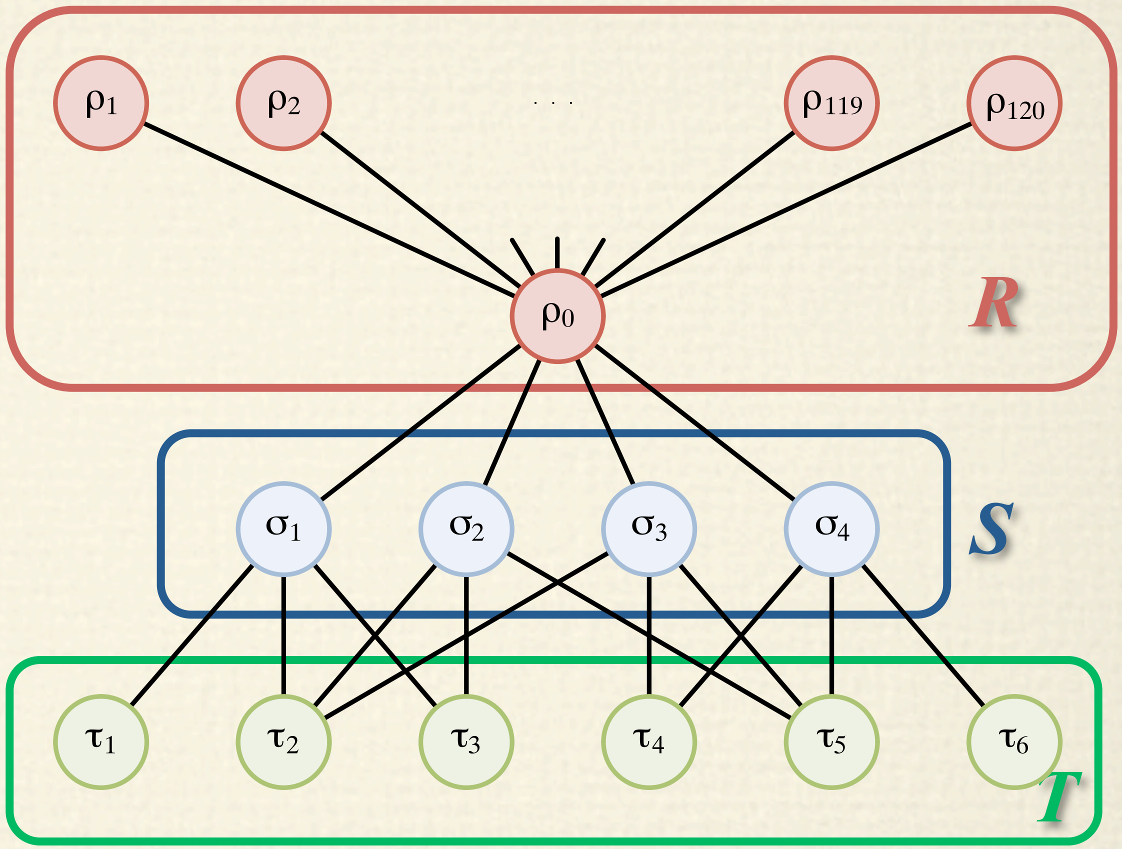


Illustration of Reduction

$$C_{RR} = r^2 = 14400$$

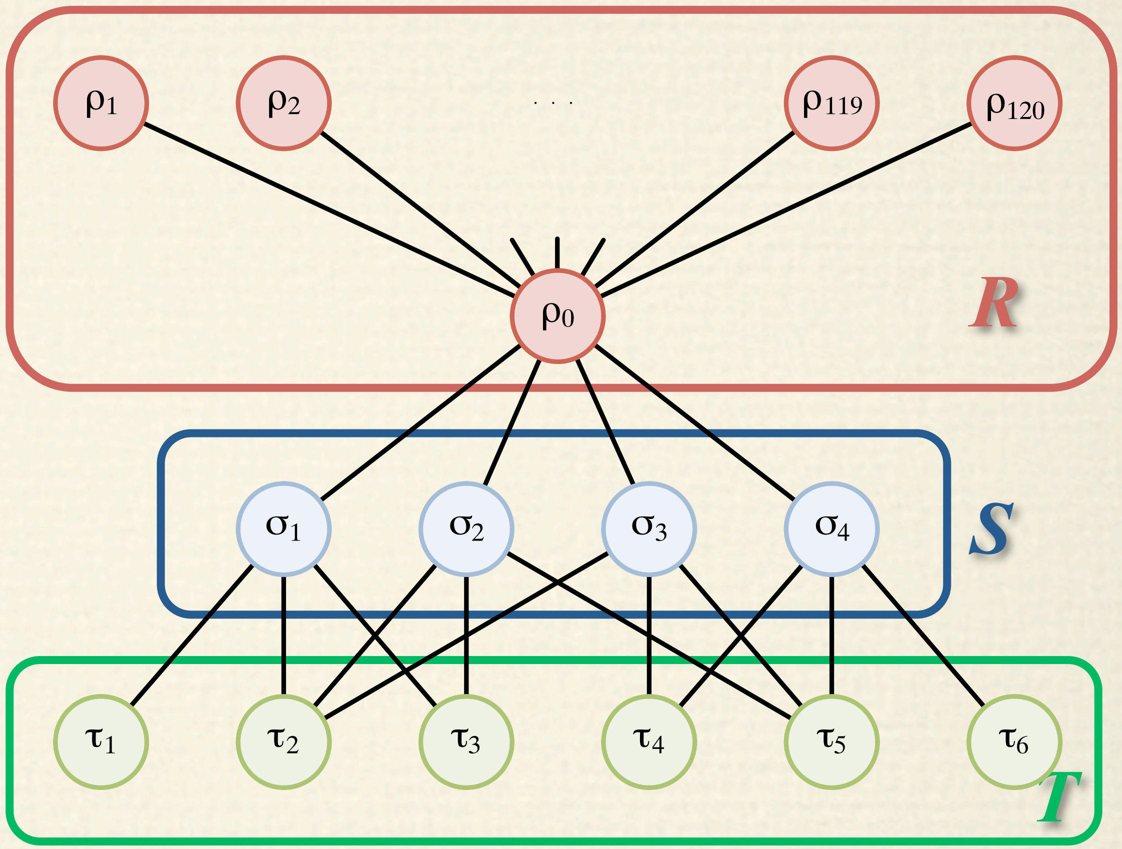
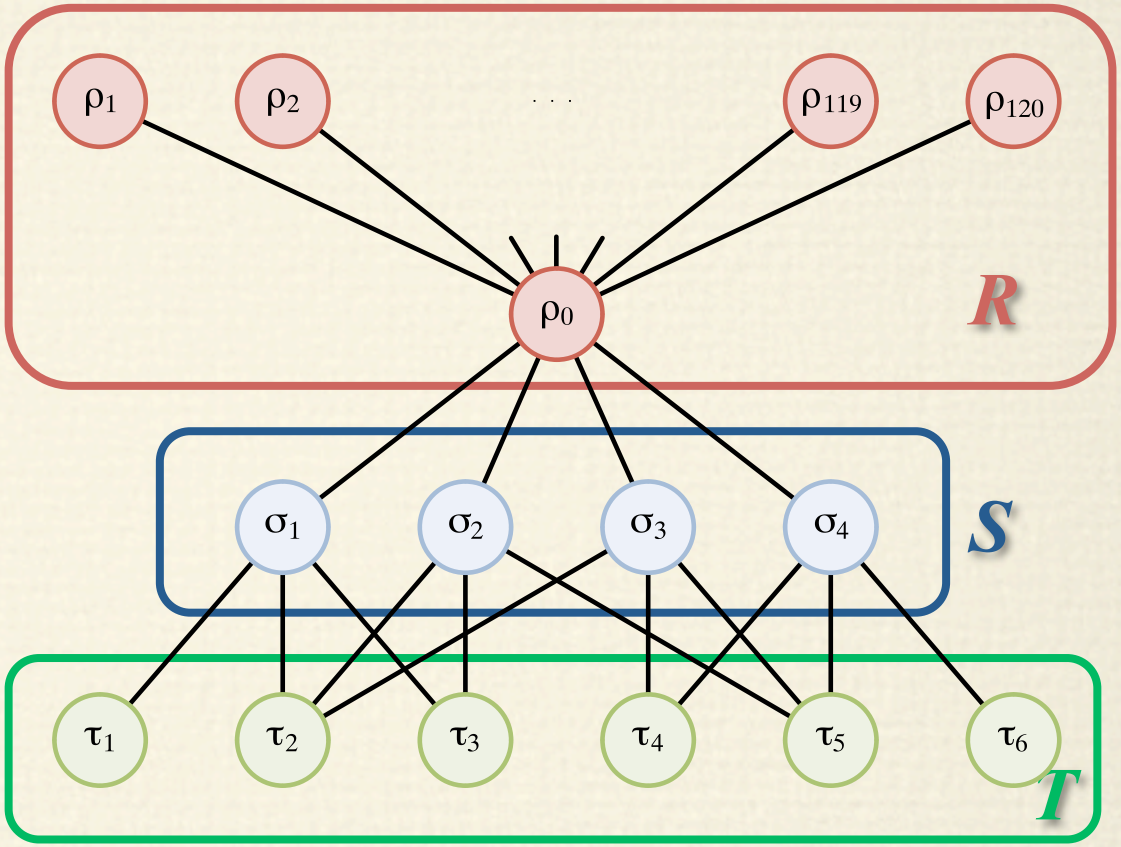


Illustration of Reduction

$$C_{RR} = r^2 = 14400$$

$$\begin{aligned} C_{RS} &= 2rs + s \\ &= 2 \times 120 \times 4 + 4 \\ &= 964 \end{aligned}$$



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$$\begin{aligned} C_{RS} &= 2rs + s \\ &= 2 \times 120 \times 4 + 4 \\ &= 964 \end{aligned}$$

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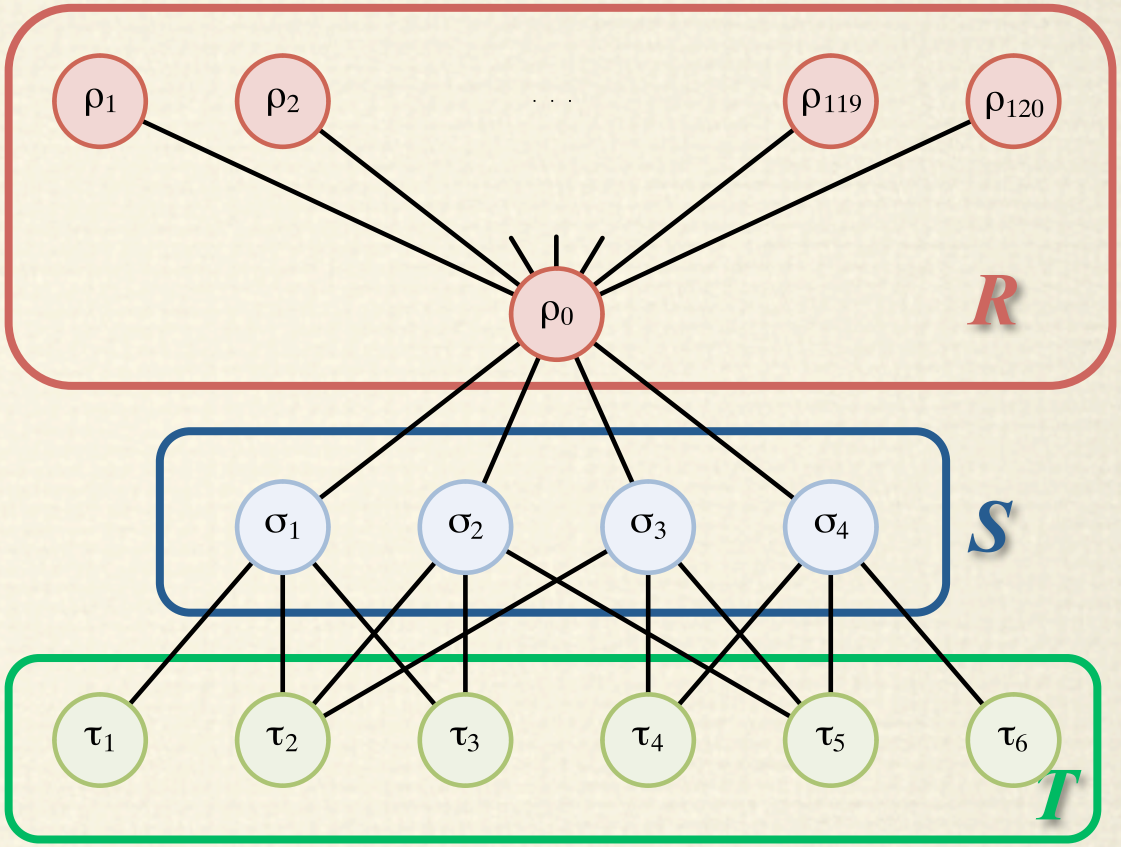

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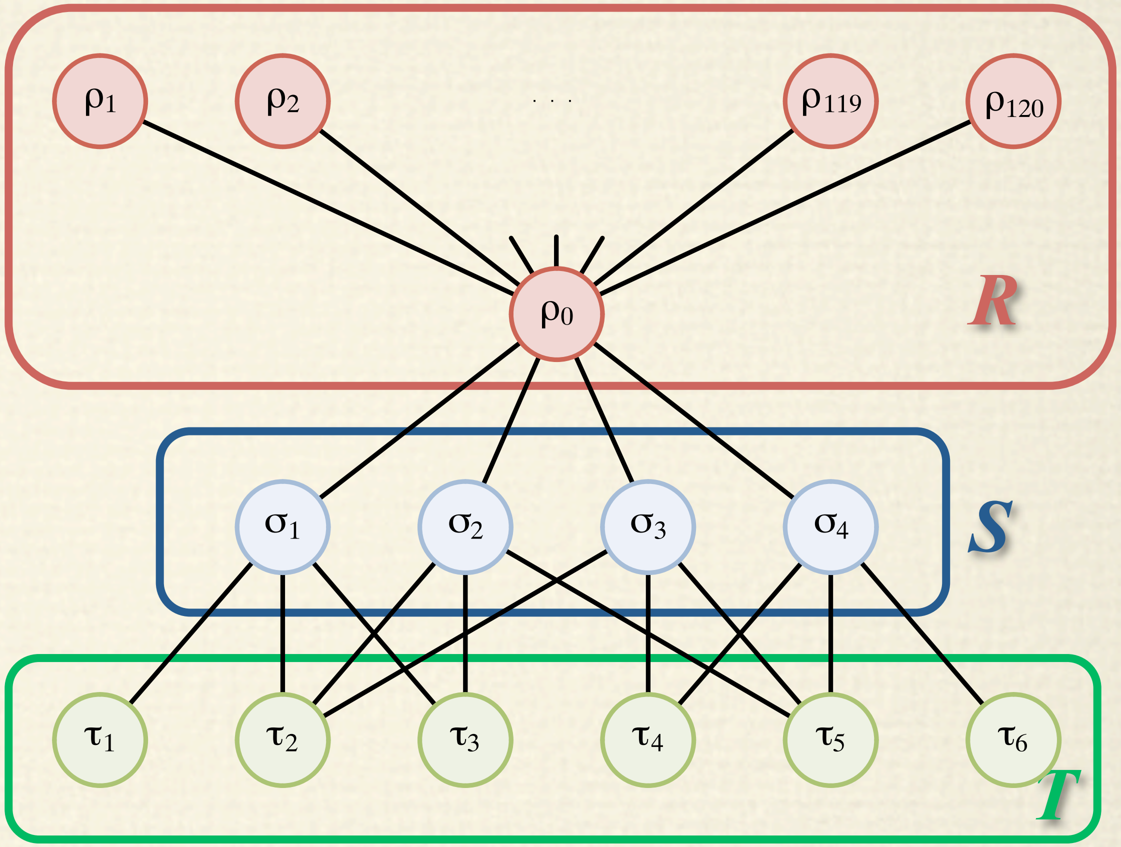
$$\begin{aligned} C &= C_{RR} + C_{RS} + C_{RT} + r \\ &= 14400 + 964 + 2172 + 120 \\ &= 17656 \end{aligned}$$



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Proof of SNDP

R00922102 張庭耀

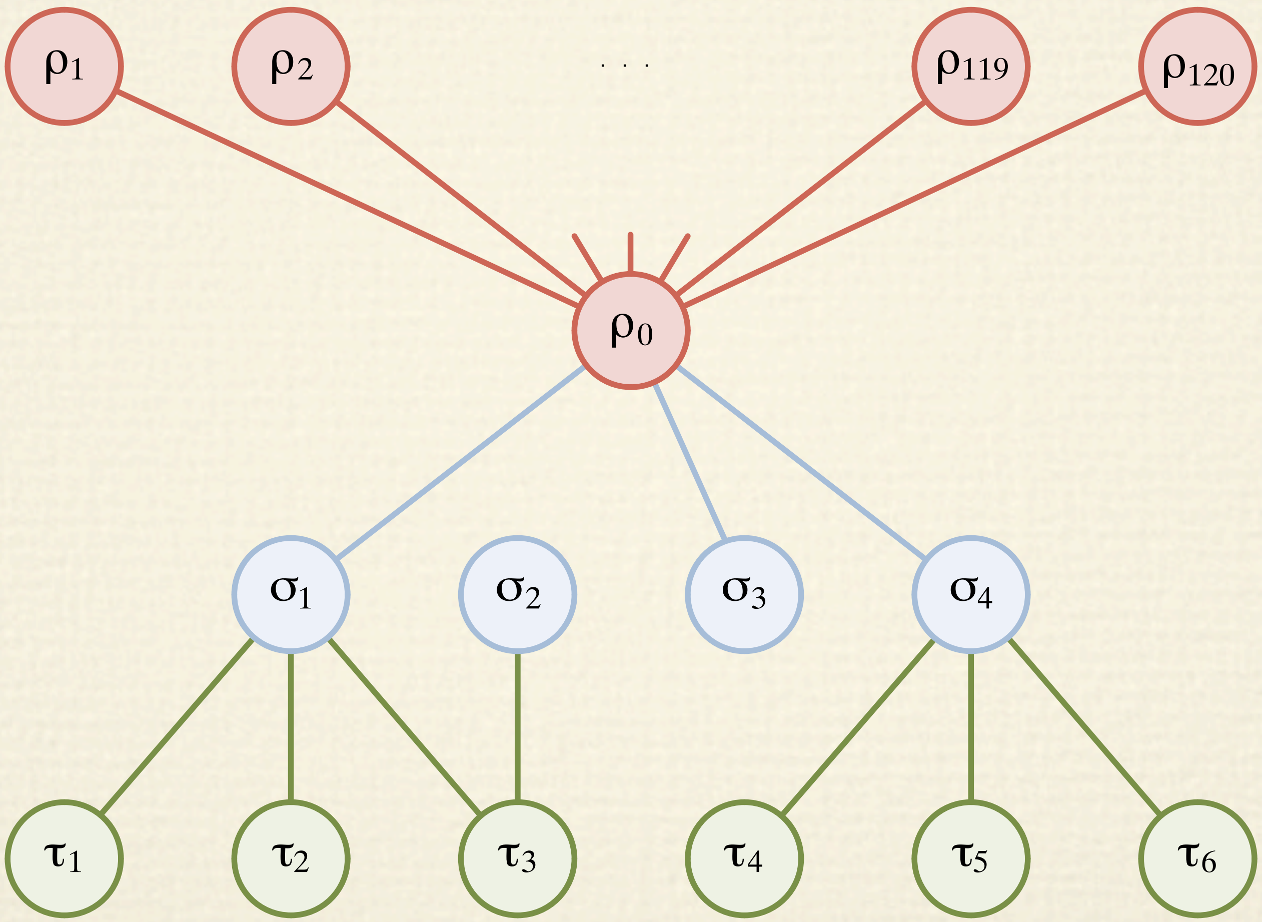
2^{nd} Property

- If $\{\rho_0, \sigma\} \notin E'$ for some $\sigma \in S$, then

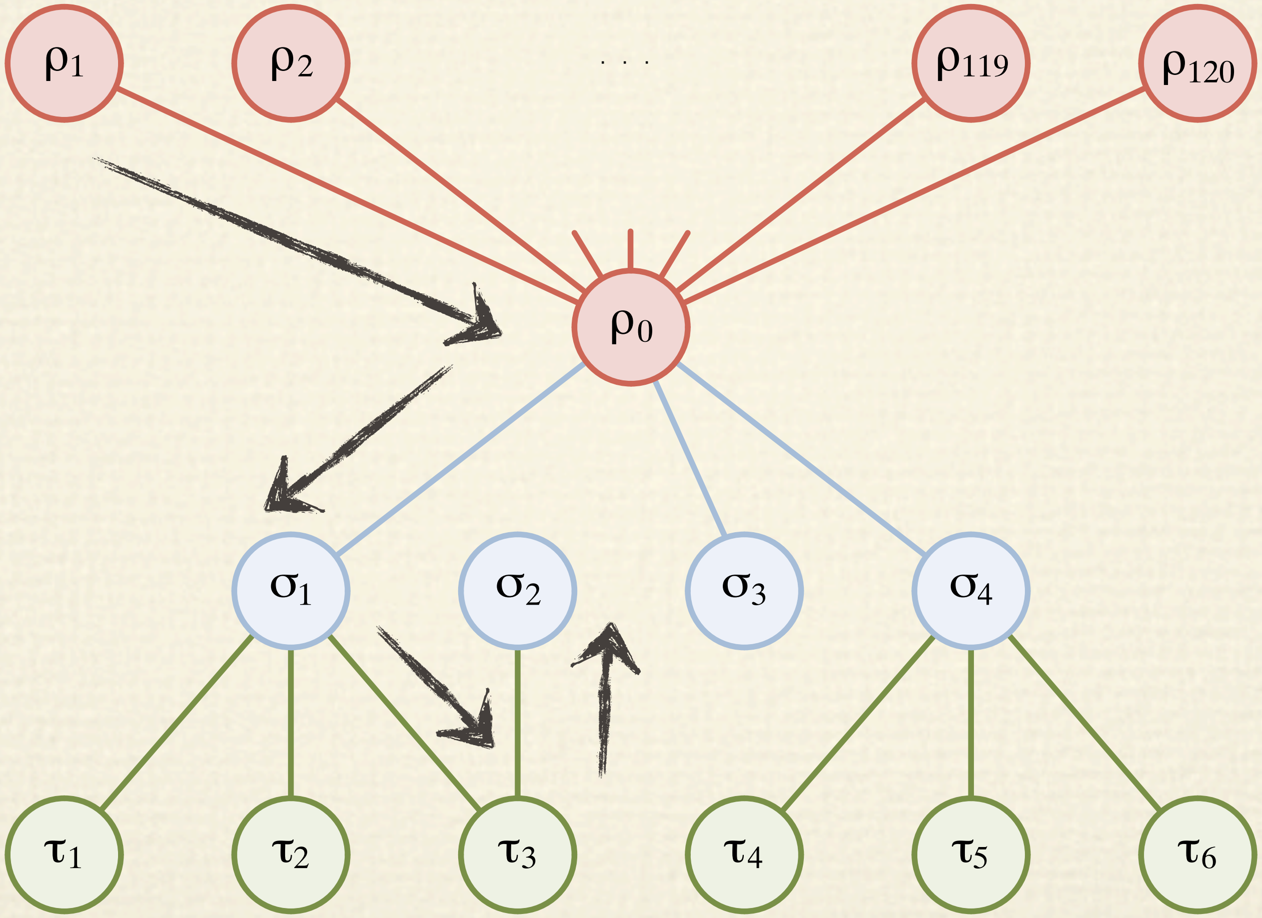
$$
\begin{aligned}
F(G') &= F_{RR}(G')+F_{RS}(G')+F_{RT}(G')+F_{SS}(G')+F_{ST}(G')+F_{TT}(G') \\
&> F_{RR}(G') + F_{RS}(G') + F_{RT}(G') \\
&\geq C_{RR} + C_{RS} + 2(r+1) + C_{RT} \\
&> C_{RR} + C_{RS} + r + C_{RT} \\
&= C_{RR} + C_{RS} + C_{RT} + r \\
&= C_{RR} + C_{RS} + C_{RT} + C_{SS} + C_{ST} + C_{TT} \\
&= C
\end{aligned}
$$

$C_{RR} = r^2$
$C_{RS} = 2rs + s$
$C_{RT} = 9rt + 6t$
$C_{SS} = s^2 - s$
$C_{ST} = 9st - 6t$
$C_{TT} = 18t^2 - 12t$

2^{nd} Property

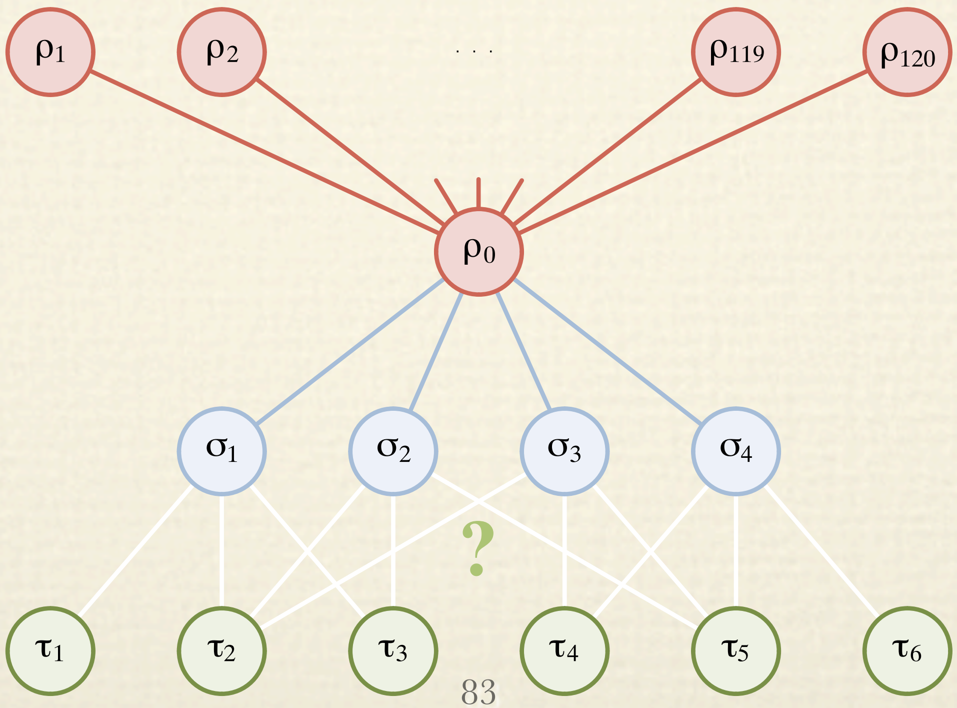


2^{nd} Property



{ϱ$_0$, σ}∈E' for all σ∈S

- Implication: In G', each vertex in T is adjacent to exactly one vertex in S
- $F_{RR}(G')=C_{RR}$, $F_{RS}(G')=C_{RS}$, $F_{SS}(G')=C_{SS}$, $F_{RT}(G')=C_{RT}$, $F_{ST}(G')=C_{ST}$
- **⌈ $F(G')<=C$ ⌋ iff ⌈$F_{TT}(G')=C_{TT}$⌋**



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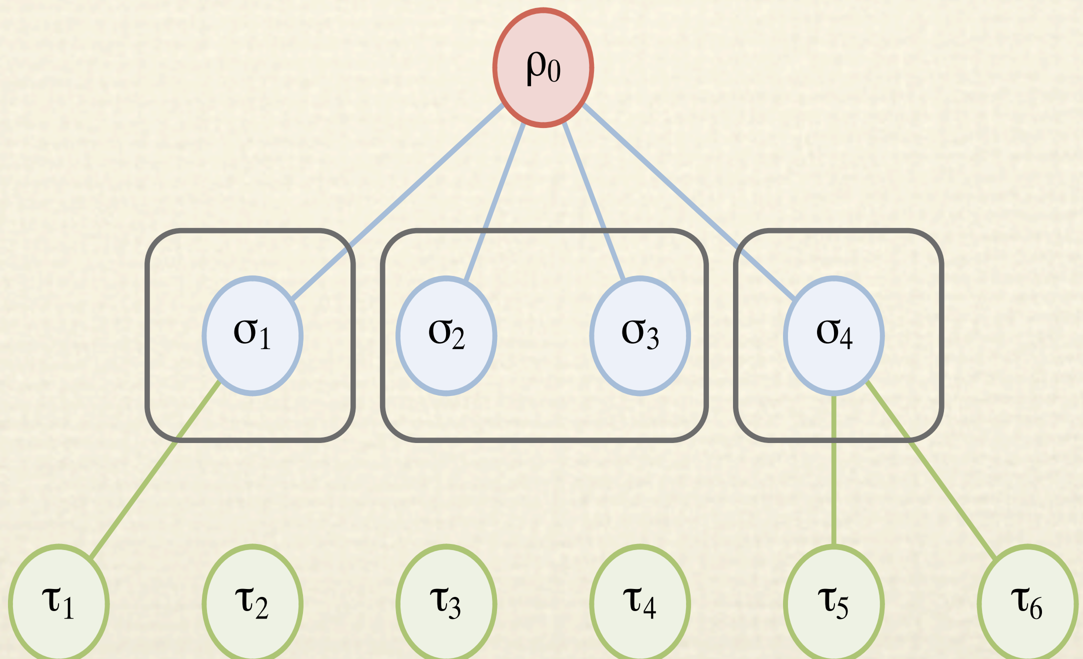
3^{rd} Property

- Denote the number of vertices in S being adjacent in G' to exactly h vertices in T by S_h (h=0,1,2,3)

  Example: $S_0=2$, $S_1=1$, $S_2=1$, $S_3=0$

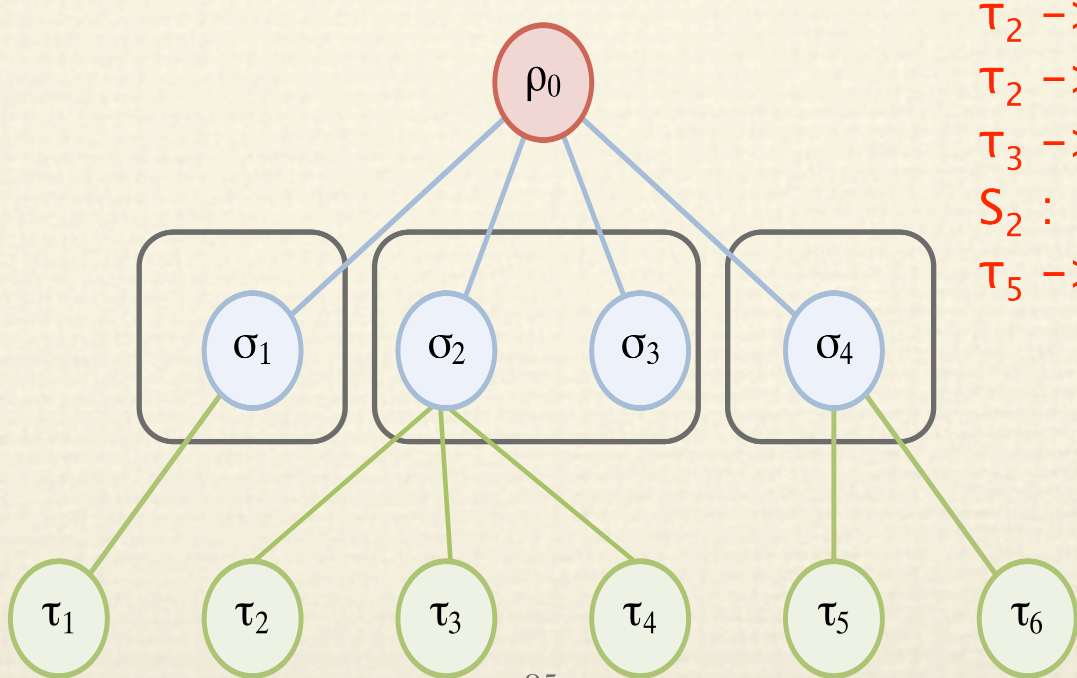


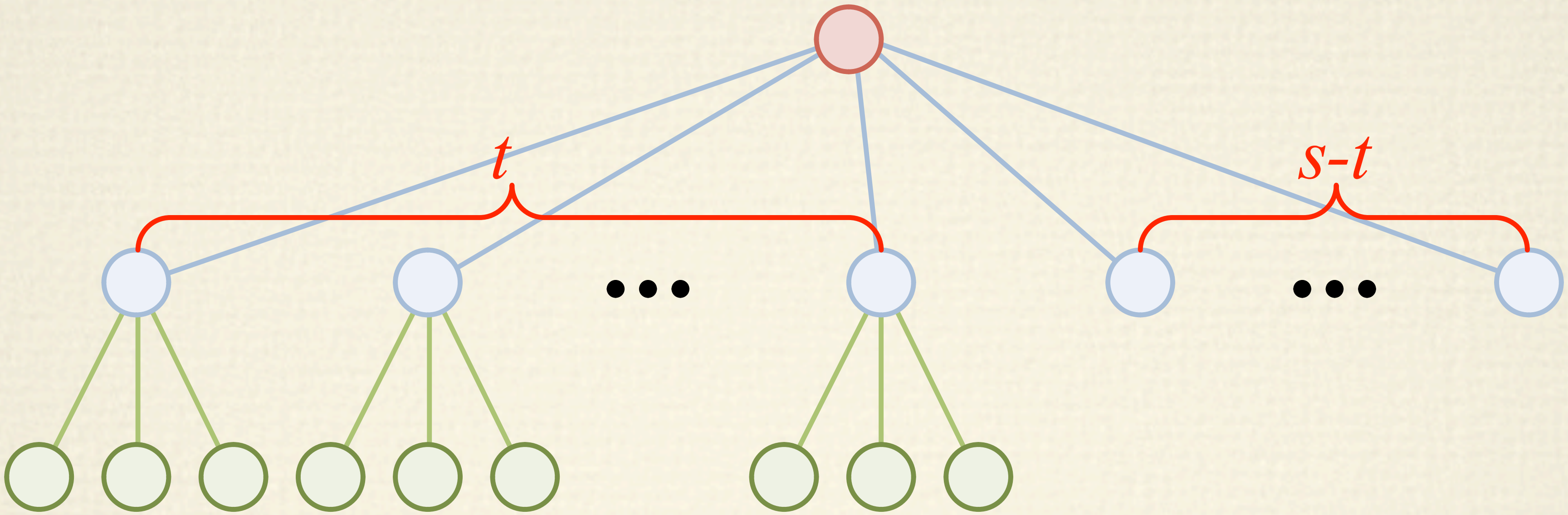
3^{rd} Property

C_2^{3t}

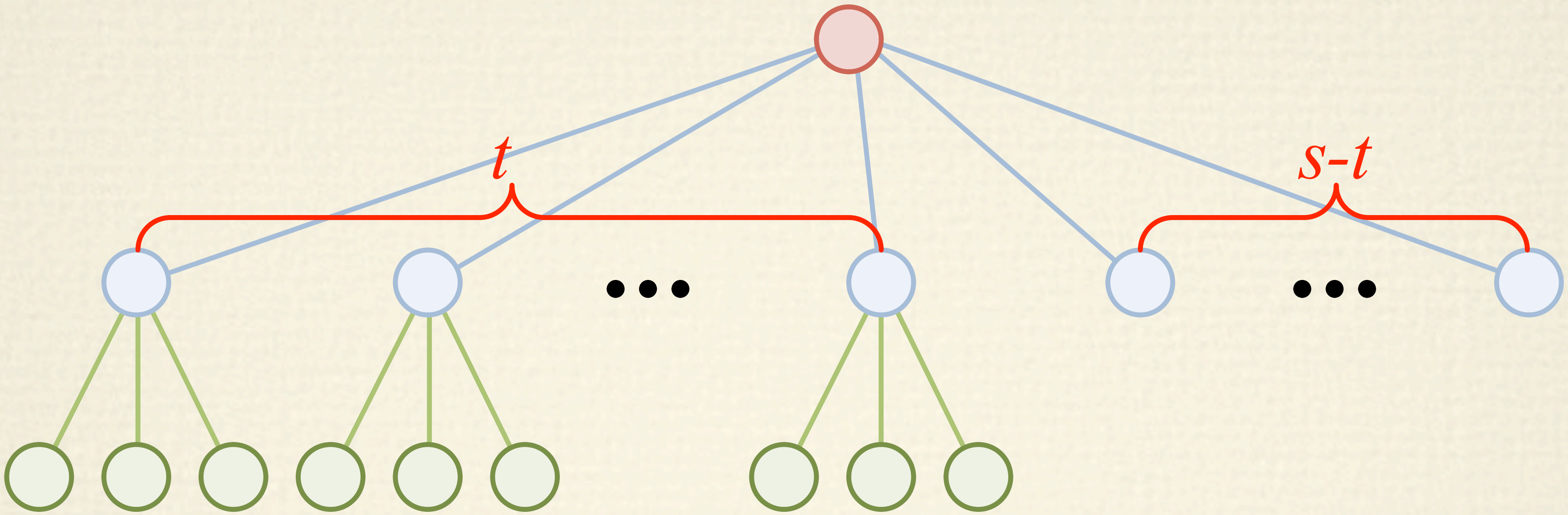
$$
\begin{aligned}
F_{TT}(G') &= 4(3t(3t-1)/2) \\
&\quad - 2|\{\{\tau,\tau'\} : \tau \neq \tau', \{\{\sigma,\tau\},\{\sigma,\tau'\}\} \subset E' \text{ for some } \sigma \in S\}| \\
&= (18t^2 - 6t) - (2s_2 + 6s_3) \\
&= C_{TT} + 6(t - s_3) - 2s_2
\end{aligned}
$$

S_3 :
τ_2 -> τ_3
τ_2 -> τ_4
τ_3 -> τ_4
S_2 :
τ_5 -> τ_6




t
s-t

Clearly, ⌈$F_{TT}(G') = C_{TT} = 18t^2 - 12t$⌋ iff ⌈ $S_3 = t$, $S_0 = s-t$, $S_1 = S_2 = 0$ ⌋



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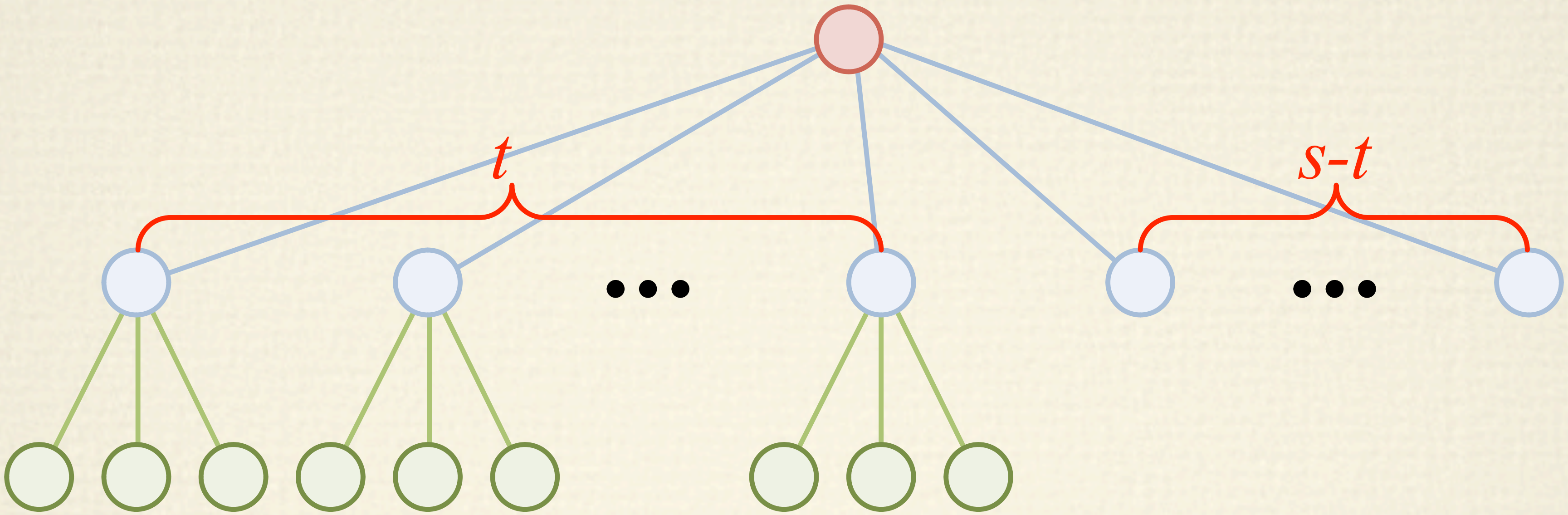
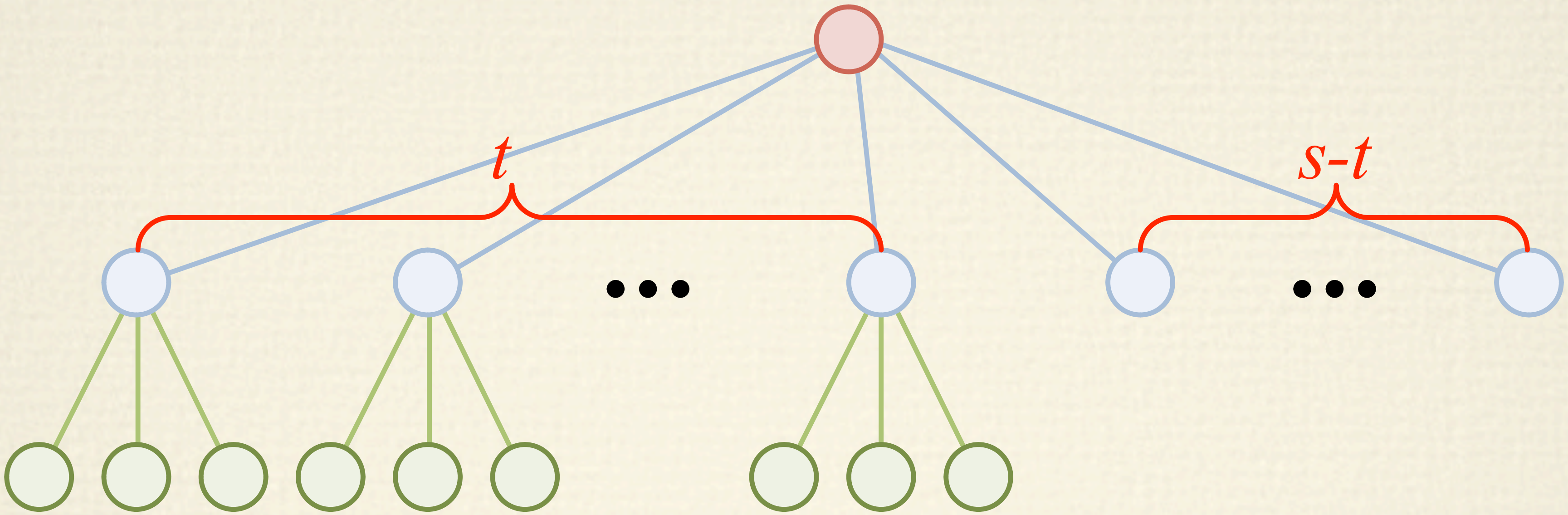


$S' = \{\{\tau_1, \tau_2, \tau_3\}, \{\tau_4, \tau_5, \tau_6\}\}$

Exact 3-cover solution

Clearly, $\lceil F_{TT}(G') = C_{TT} = 18t^2 - 12t \rfloor$ iff $\lceil S_3 = t, S_0 = s-t, S_1 = S_2 = 0 \rfloor$



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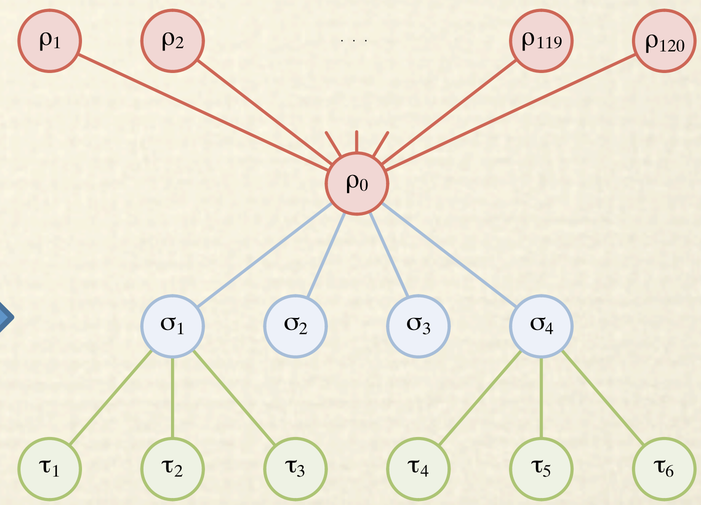

SNDP solution

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Conclusion

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Conclusion

- There's various types of NDP, for example:
- A complete Graph, a Distance and a Requirement for vertex pair. Cost = Distance* Requirement.
  Find SPT to minimize Total Cost.
  distance equal: poly-time, requirement equal: NPC
- A weighted Graph, specific vertex p and integer k, find SPT to minimize Total Weight sbj to: each subtree include p and contain at most k vertices.
  k=2: matching problem, k=3: NPC

Conclusion

- We have shown today:
- how P differs from NP
- what is Network Design Problem
- why NDP and SNDP are NPC
- who involved in this presentation
- and...

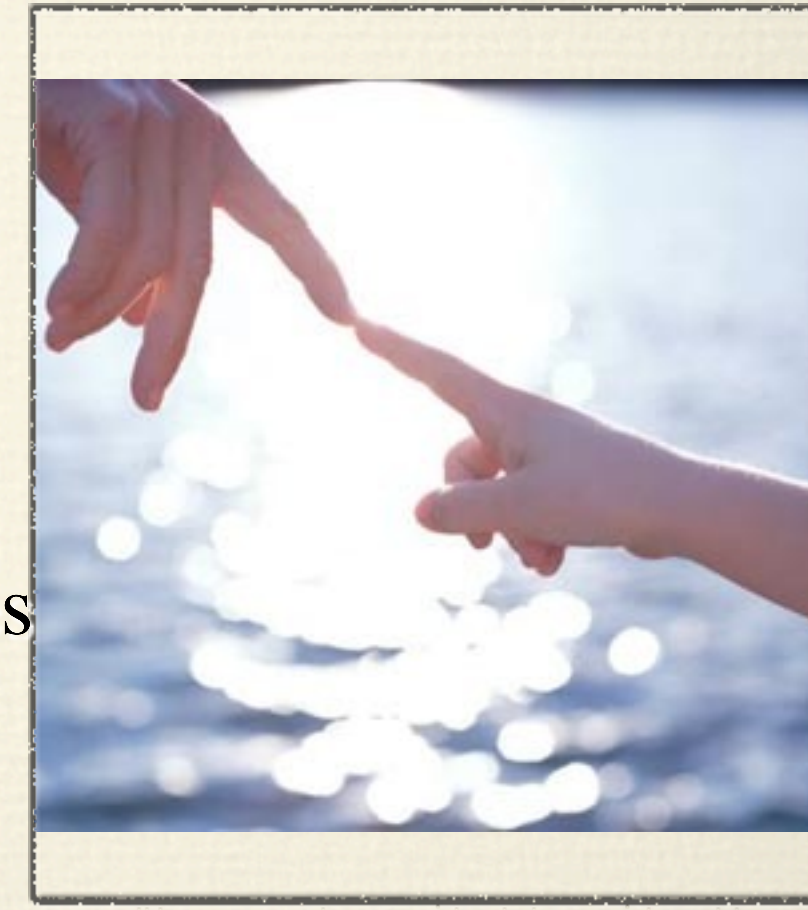
Conclusion

- The most people are interested: when will we finish the demo?

- Let me show applications of NDP
- Bus/MRT Network
- Social Network
- Sensor Network

Conclusion

- A better way to:
- get people connected
- refine transportation
- communicate between machines
- That's why engineers provide people more comfortable life.



Q/A

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Thanks for your attention