Kun-Mao Chao

Oct. 9, 2012

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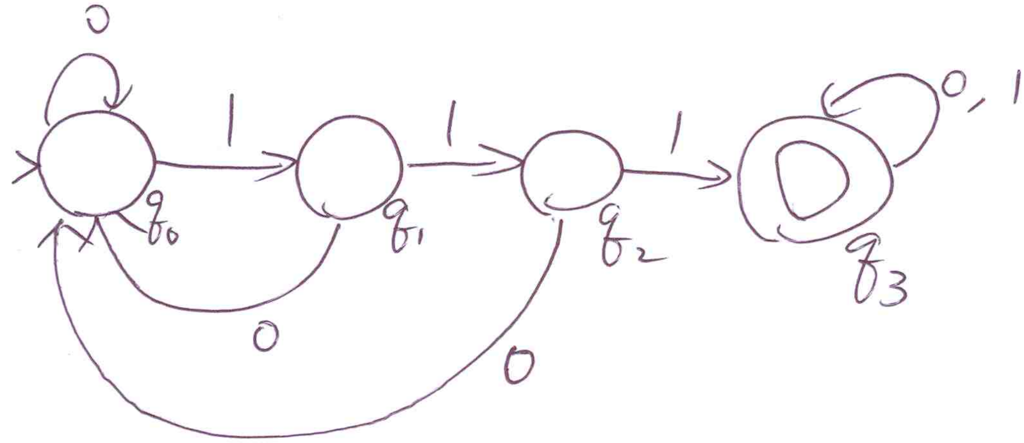
Ex.

$$\alpha = ((0 \cup 1)^* 111 (0 \cup 1)^*)$$

$$\mathcal{L}(\alpha) = \{ w \in \{0,1\}^* : w \text{ has the substring } 111 \}$$

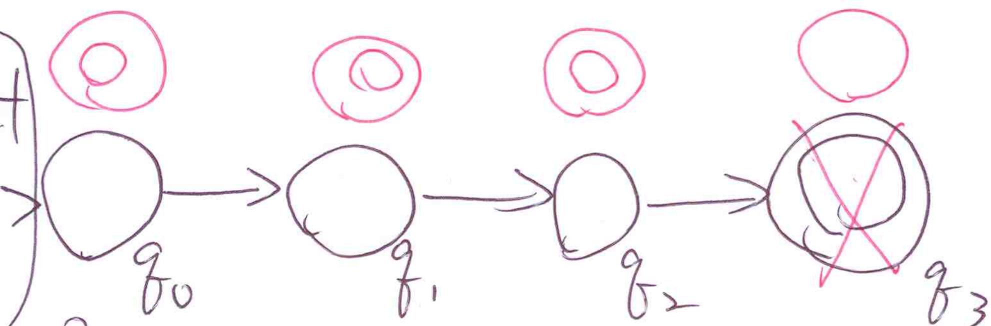
M_α:



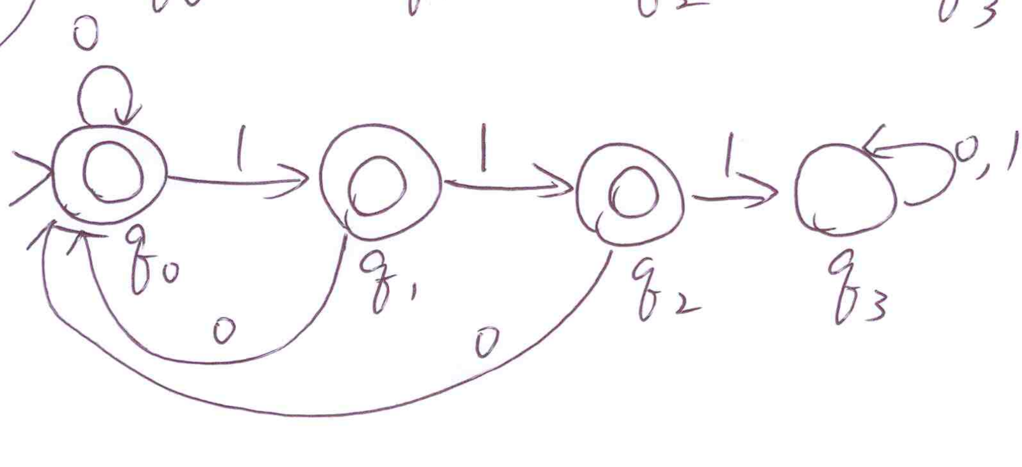
Ex. $$\beta = 0^* \cup 0^* (1 \cup 11)(00^*(1 \cup 11))^* 0^*$$

$$\mathcal{L}(\beta) = \{ w \in \{0,1\}^* : w \text{ does not have the substring } 111 \}$$

$\boxed{\text{A Complement of M for } \alpha.}$



M_β



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Ex. Is 01101110 accepted by M_α?

$$
\begin{aligned}
(q_0, 01101110) &\vdash_{M_\alpha} (q_0, 1101110) \\
&\vdash_{M_\alpha} (q_1, 101110) \\
&\vdash_{M_\alpha} (q_2, 01110) \\
&\vdash_{M_\alpha} (q_0, 1110) \\
&\vdash_{M_\alpha} (q_1, 110) \\
&\vdash_{M_\alpha} (q_2, 10) \\
&\vdash_{M_\alpha} (q_3, 0) \\
&\vdash_{M_\alpha} (q_3, e) \quad \text{accepted.}
\end{aligned}
$$

Input tape		0	1	1	0	1	1	1	0
State	q_0	q_0	q_1	q_2	q_0	q_1	q_2	q_3	q_3 ← accepted.

$$(q_0, 01101110) \vdash^*_{M_\alpha} (q_3, e)$$

[Closure Property, p.37] Def. 1.6.3 Kun-Mao Chao Oct. 16, 2012

Let D be a set, let $n \geq 0$, and let $R \subseteq D^{n+1}$ be a $(n+1)$-ary relation on D. Then a subset B of D is said to be closed under R if $b_{n+1} \in B$ whenever $b_1, \ldots, b_n \in B$ and $(b_1, \ldots, b_n, b_{n+1}) \in R$.

Ex. 1.6.4 The set of a person's ancestors is closed under the relation $\{(a,b)$: a and b are persons, and b is a parent of $a\}$.

[$n=1$]

Ex. 1.6.5 Let A be a fixed set. We say that set S satisfies the inclusion property associated with A if $A \subseteq S$. Any set S satisfying the inclusion property associated with A is closed under the relation $R = \{(a) : a \in A\}$.

[$n=0$; $B=S$]

if $b_{n+1} \in B$ whenever $\underbrace{b_1, \ldots, b_n \in B}_{n=0:\ \text{nothing}}$ and $\underbrace{(b_1, \ldots, b_n, b_{n+1}) \in R}_{n=0 \Rightarrow (b_1)}$

if $b_1 \in S$ whenever $(b_1) \in R$. True.

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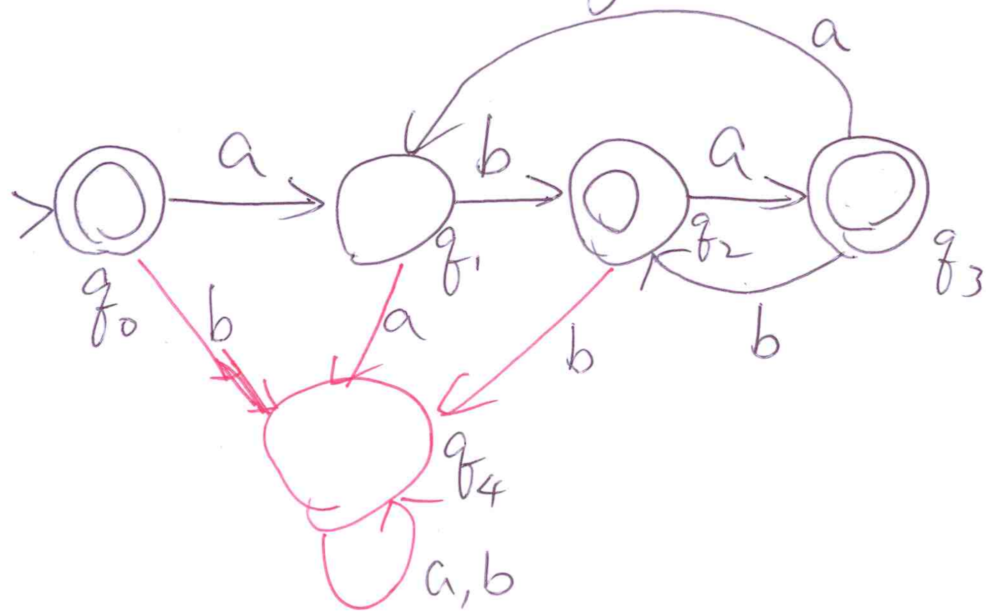
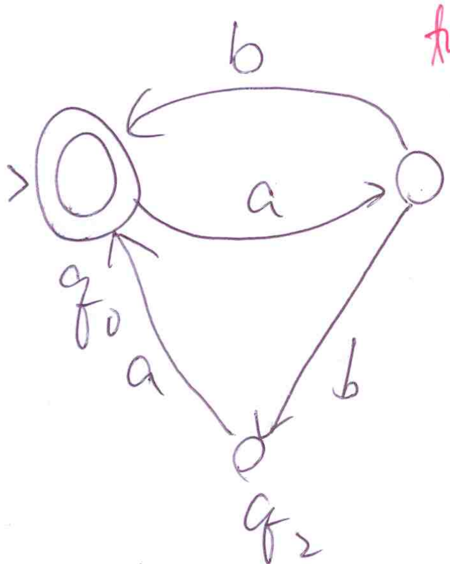
$\alpha = (ab \cup aba)^*$

M_α : A DFA for recognize α

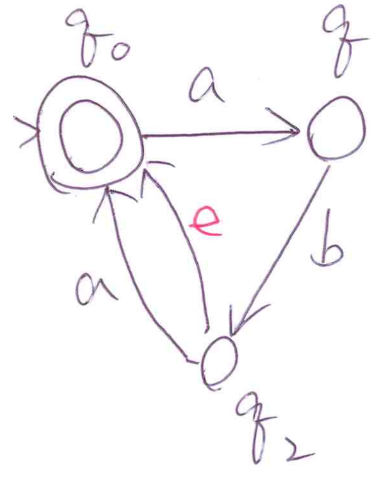


$\hat{M}_\alpha$: An NFA for recognizing α ← Nondeterministic

two arrows out for b's



or



$K = \{q_0, q_1, q_2\}$
$\Sigma = \{a, b\}$
$s = q_0$
$F = \{q_0\}$

Δ:

q	σ	p
q_0	a	q_1
q_1	b	q_0
q_1	b	q_2
q_2	a	q_0

With an "e" move.

Δ:

q	σ	p
q_0	a	q_1
q_1	b	q_2
q_2	a	q_0
q_2	e	q_0

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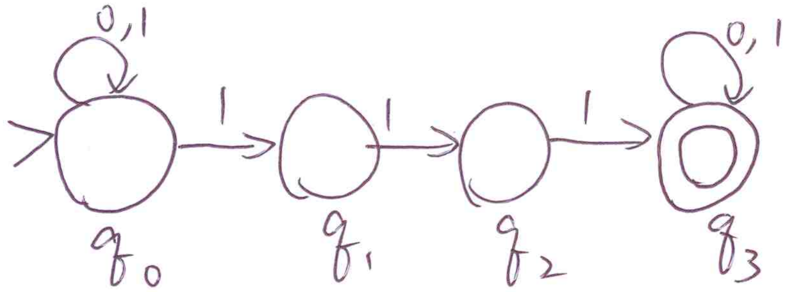
$\alpha = ((0 \cup 1)^* 111 (0 \cup 1)^*)$

$L(\alpha) = \{ w \in \{0,1\}^* : w$ has the substring $111 \}$

NFA M_α :



$$
\begin{aligned}
(q_0, 0110 1111) &\vdash_{M_\alpha} (q_0, 1101111) \\
&\vdash_{M_\alpha} (q_0, 101111) \\
&\vdash_{M_\alpha} (q_0, 01111) \\
&\vdash_{M_\alpha} (q_0, 1111)
\end{aligned}
$$

$\vdash_{M_\alpha} (q_1, 111)$	$\vdash_{M_\alpha} (q_0, 111)$
$\vdash_{M_\alpha} (q_2, 11)$	$\vdash_{M_\alpha} (q_1, 11)$
$\vdash_{M_\alpha} (q_3, 1)$	$\vdash_{M_\alpha} (q_2, 1)$
$\vdash_{M_\alpha} (q_3, e)$	$\vdash_{M_\alpha} (q_3, e)$
accepted	accepted.

A string is accepted by a nondeterministic finite automaton if and only if there is at least one sequence of moves leading to a final state.

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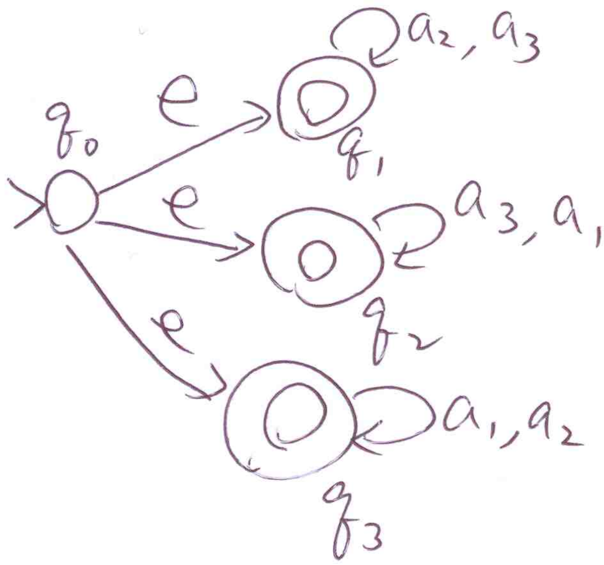
$L = \{ w \in \{a_1, a_2, a_3\} :$ there is a symbol $a_i \in \{a_1, a_2, a_3\}$ not appearing in $w \}$.

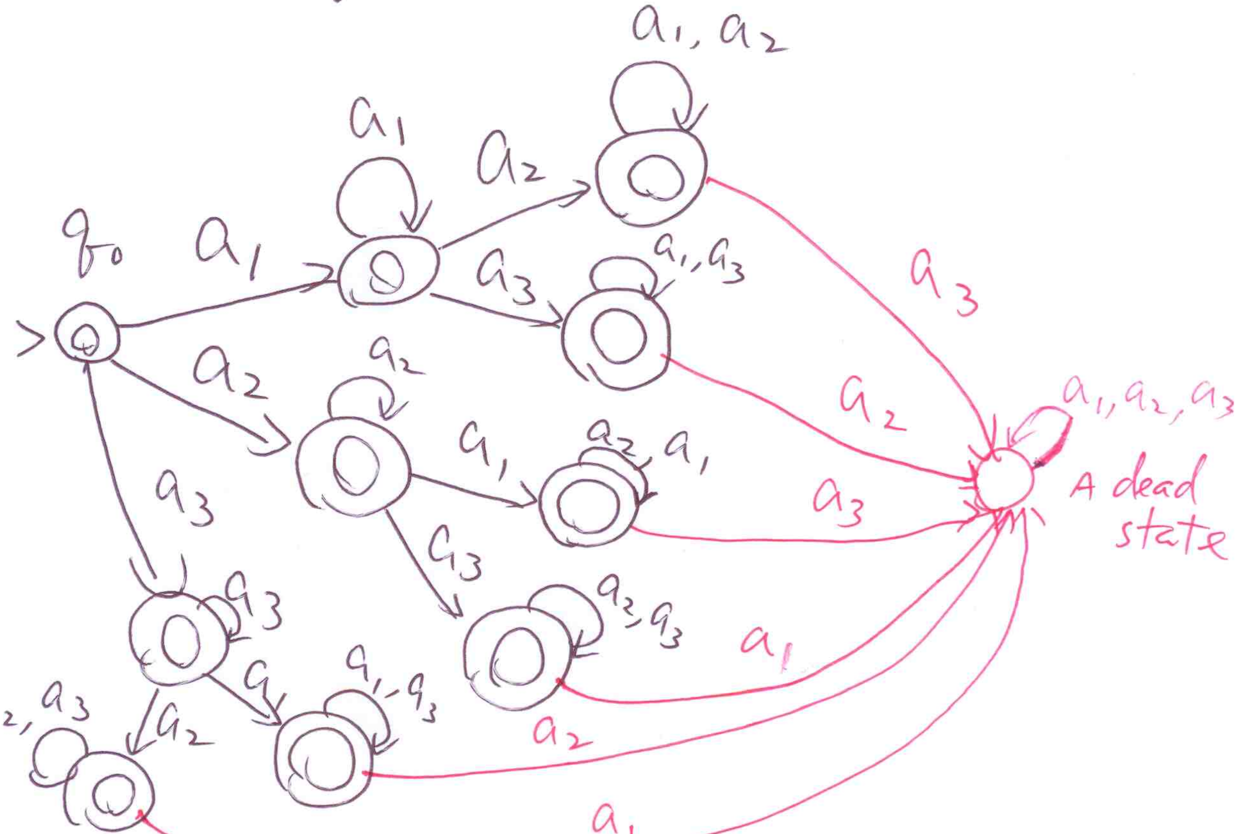
$e,\ a_1,\ a_2,\ a_3,\ a_1a_2,\ a_1a_3,\ a_1a_1,\ a_2a_1, \dots$

$a_1a_2a_3,\ a_1a_1a_3a_2,$

NFA



DFA



NFA → DFA

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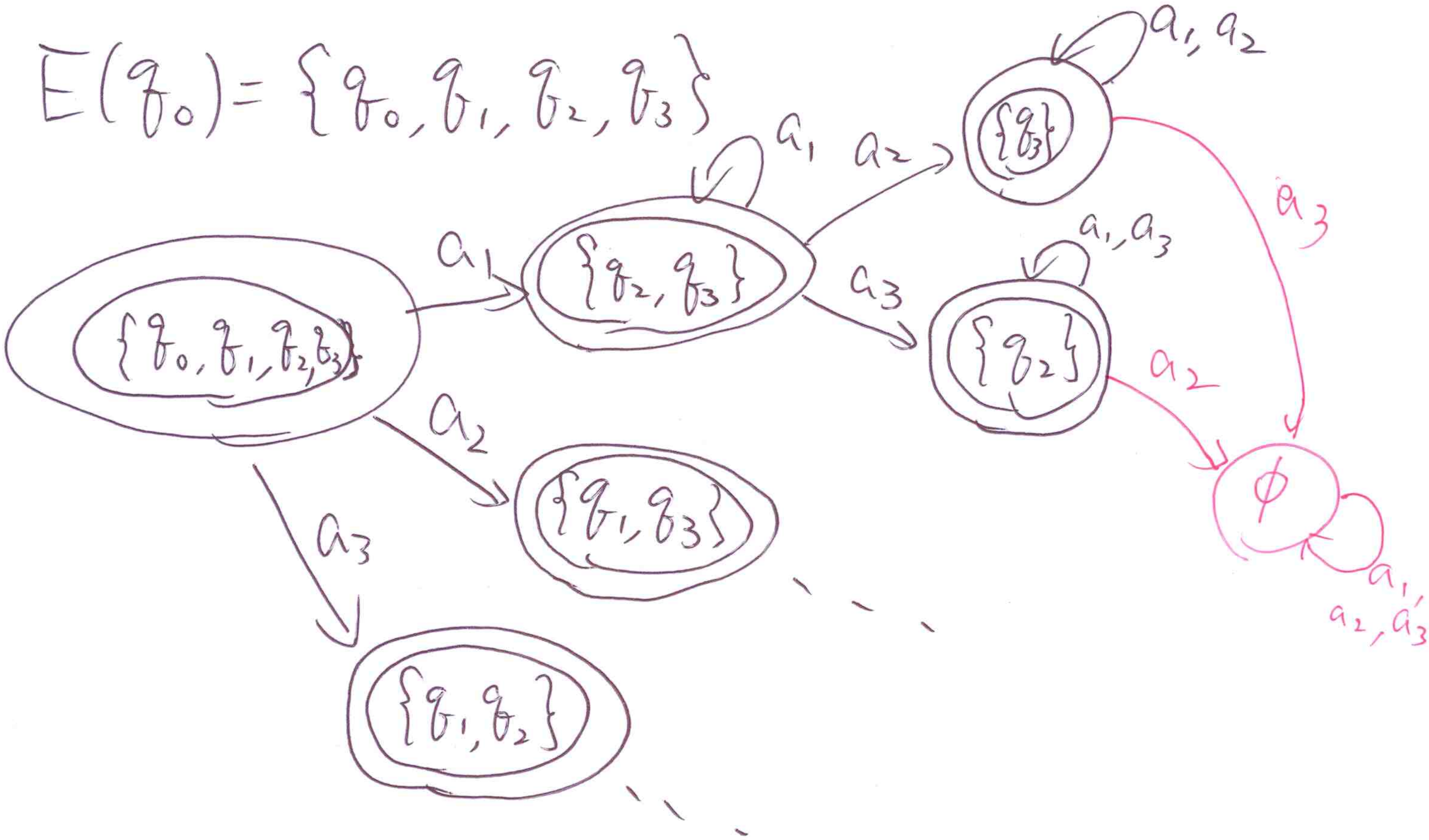
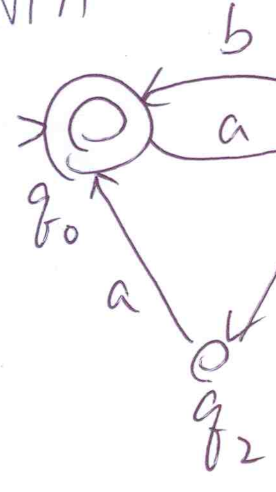
$E(q_0) = \{q_0, q_1, q_2, q_3\}$

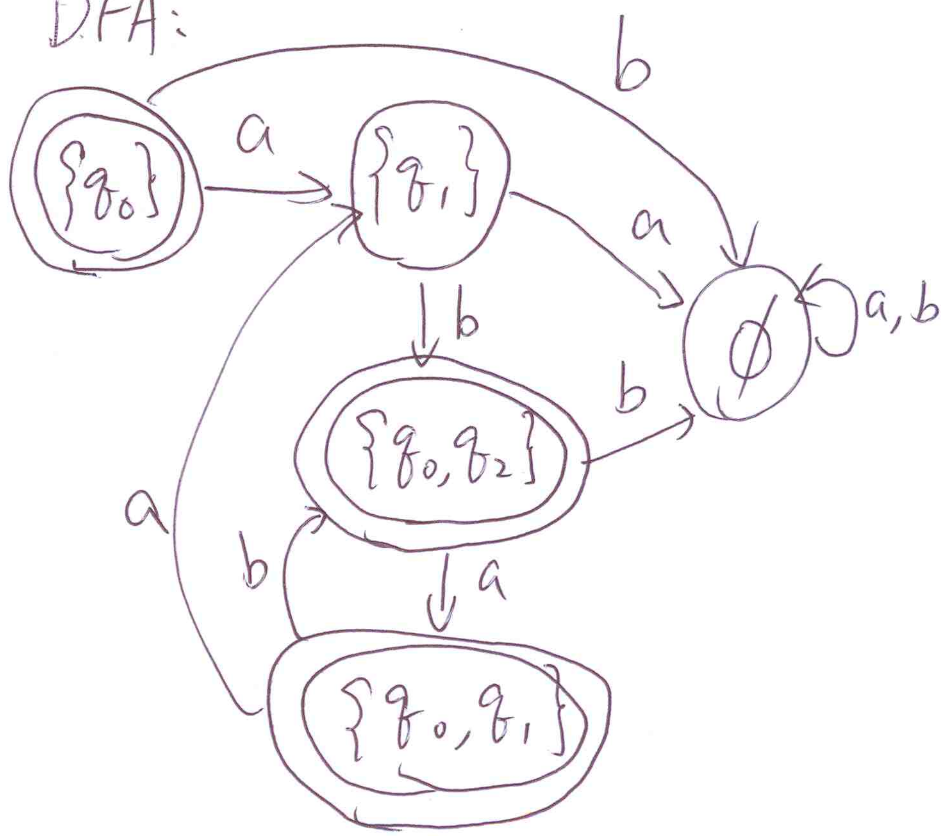


$(ab \cup aba)^*$

NFA:



DFA:



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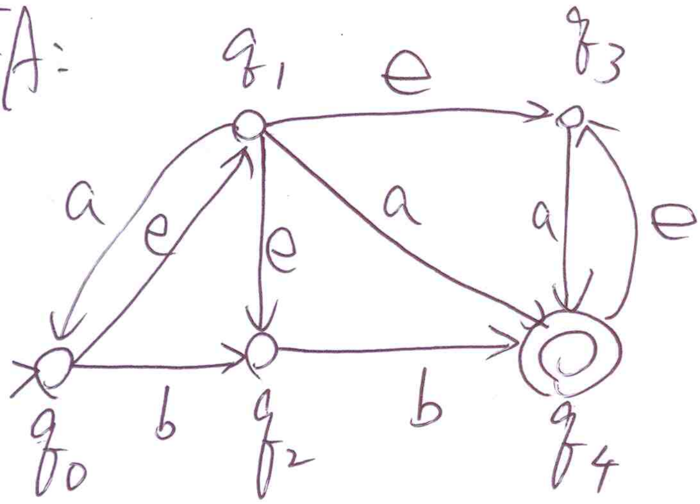
NFA:



$E(q_0) = \{q_0, q_1, q_2, q_3\}$

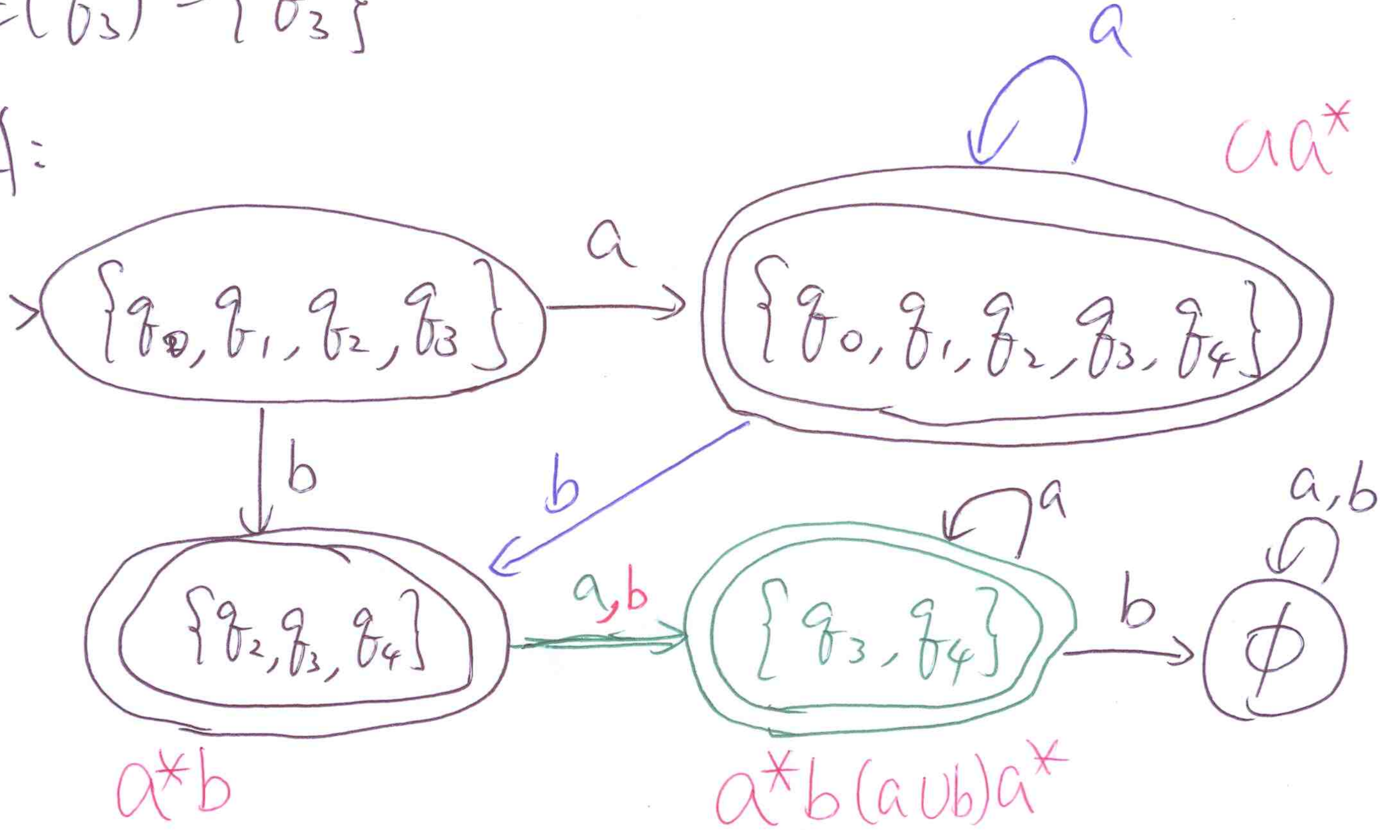
$E(q_1) = \{q_1, q_2, q_3\}$

$E(q_4) = \{q_4, q_3\}$

$E(q_2) = \{q_2\}$

$E(q_3) = \{q_3\}$

DFA:



$aa^* \cup a^*b \cup a^*b(a \cup b)a^*$