DATE . . NO.

※ 以下千篇，探討 marriage problem (bilateral preference lists), Roommates problem (不一定具 bipartite 性質) 中, popular matching 的相關議題.

[補充]

Stable Marriage Problem (SM)

Complete版 ▦ 所有 instances 都有 stable matching 解，可在 $O(n^2)$ time 找出

Incomplete版 ▩ = 同size版 = , = $O(m)$ =

Incomplete with tie 版 ▩ 所有 instances 都有 stable matching 解，但 size 不定，欲求出 max/min size 的 stable matching 解是 NP-hard

Stable Roommates Problem (SR)

Complete版 ▦ 未必所有 instance 皆有 stable matching，若有，可在 $O(n^2)$ time 找出。

with tie版 ▥ 欲辨別某個 instance 是否具 stable matching，屬 NP-complete.

DATE . . NO.

15. [CIAC 2010] Popular matchings in the Marriage and Roommates problems by Biro, Irving, and Manlove

本文探討 popularity 和 Stability 的關係，並证明「分辨一個 with tie 的 Roommate / Marriage instance 是否具有 popular matching，屬於 NP-hard」

① without tie 時

对 RP instance I 而言

strongly popular ⇒ Stable ⇒ popular ⇒ maximal

(⇐× ⇐× ⇐×)

▲本文 gives 一个 O(m) algorithm 来「determine if I 具 strongly popular matching」

▲ = $O(\sqrt{n \times (n,m)}\, m \log^{\frac{3}{2}} n)$ = popular =

▲ Given I，「求出一个任意 size 的 popular matching，if exists」仍 open

对 MP instance I 而言

因 MP 为 RP 的 special case，故亦具同性质。

▲本文 gives 一个 $O(\sqrt{n}\, m)$ time algo.「determine if I 具 popular 解」

▲ Given I，「求出 max-cardinality popular matching 解」仍 open

② with ties 時

透过 reduction to Exact-Maximal-Matching problem，证明 MPT 及 RRT 均为 NP-complete problem.

DATE . . NO.

16. [ESA 2011] Near-popular matchings in the Roommates problem

在上文中已证明，欲判别一个 RP instance 存在 popular matching 与否是 NP-hard，故本文退求其次，找一個工中必定存在的 near-popular matching = least unpopularity factor matching.

定義 $\Delta(M_0, M_1) =$ 支持 M_1 人数 / 支持 M_0 人数

unpopularity factor of M: $\mu(M) = \max_{M'} \Delta(M, M')$

即 $\mu(M)$ 象徵 M 最憐烈的一仗

而所有 matchings 中有最小 $\mu(M)$ 者，即不當憐敗的，即為所求(必存在)

⚠ 可惜的是，雖然定義題示 least unpopularity factor matching 必存在但要找到它亦十分困難，連想求一个常倍逼近的解都被证明是 NP-hard。

⚠ 所幸，本文证明，任何 instance 皆有 unpopularity factor 在 $O(\log n)$ 以內的 matching，且提出 linear time method 找出它

DATE . . NO.

17. [ICALP 2011] Popular Matchings in the Stable Marriage Problem by Huang, Telikepalli

在第1篇論文中提到

① Given MP instance, 求 max-cardinality popular matching 解決open

➡ 本文提出一個 $O(mn_0)$ time方法, where $n_0 = \min(|A|, |B|)$

② 一個在 $O(\sqrt{n}m)$ time分辨上是否有popular matching的方法

➡ 本文提出一个 linear time 解法

18. [SODA 2012] Popularity vs Maximum Cardinality in the stable marriage setting by Telikepalli

➡ 再改進第17篇①的 time complexity 到 $O(m)$

⚠ 另外提出一個 local-popular 的問題, 希望在 max-cardinality matchings 所成集合 $\mathcal{M}$ 中, 找到一個 popular than $\mathcal{M}$ 中其他配對的 solution M^*。(其實 M^* 未必 popular within all matchings in G)

⇨ 提出一个 $O(mn_0)$ time方法

(or equal to)

課題 Popular matching 相關 open problems

DATE . . NO.

[SODA 2005] Popular Matchings

Assume I 具 popular matching 的 instance,
Given a not popular matching M_0,
try to give an efficient algorithm for computing a shortest-length path $\langle M_0, M_1, M_2, \cdots, M_k \rangle$ such that 「M_i is more popular than M_{i+1}, and M_k is a popular matching」

[ICALP 2006] Weighted Popular Matchings

原 popular 的定義:

「M_1 is more popular than M_2」↔「prefer M_1 的人比 prefer M_2 者多」

新定義

「M_1 is more popular than M_2」↔「M_1 中所有 applicant 配到對自己的偏好順序總和較 M_2 為大」

try to find an efficient algorithm.

[ESA 2006] Popular Matchings in the capacitated House Allocation Problem

Capacitated House Allocation problem (CHA) 及 CHA with ties (CHAT) problem 的 complexity 為何?

已具 $O(nm)$ / $O((|C|+n_1)m)$ 的解 so far

DATE . . NO.

[Journal of Discrete Algorithm 2010] Popular Matchings in the Weighted Capacitated House Allocation Problem

Weighted Capacitated House Allocation problem (with ties)
即 WCHAT，是否存在 polynomial time algorithm 可 give a popular matching if exists, or report that none exists.

[CIAC 2010] Popular Matchings in the Marriage and Roommates Problem

① Given 一个 Marriage problem instance without tie,
what 是 the complexity of finding a max-cardinality popular matching
➡ [ICALP 2011] 解掉了

② "finding a popular matching or reporting that none exists for an instance of Roommates Problem" 是否可在 polynomial time 完成

③ 同②，但限定 max-cardinality popular matching

[ESA 2011] Near-popular matchings in the Roommates Problem

design an algorithm with approximation better than $O(\log|V|)$ to compute a least unpopularity factor matching.