From Ordinal Ranking to Binary Classification

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Outline

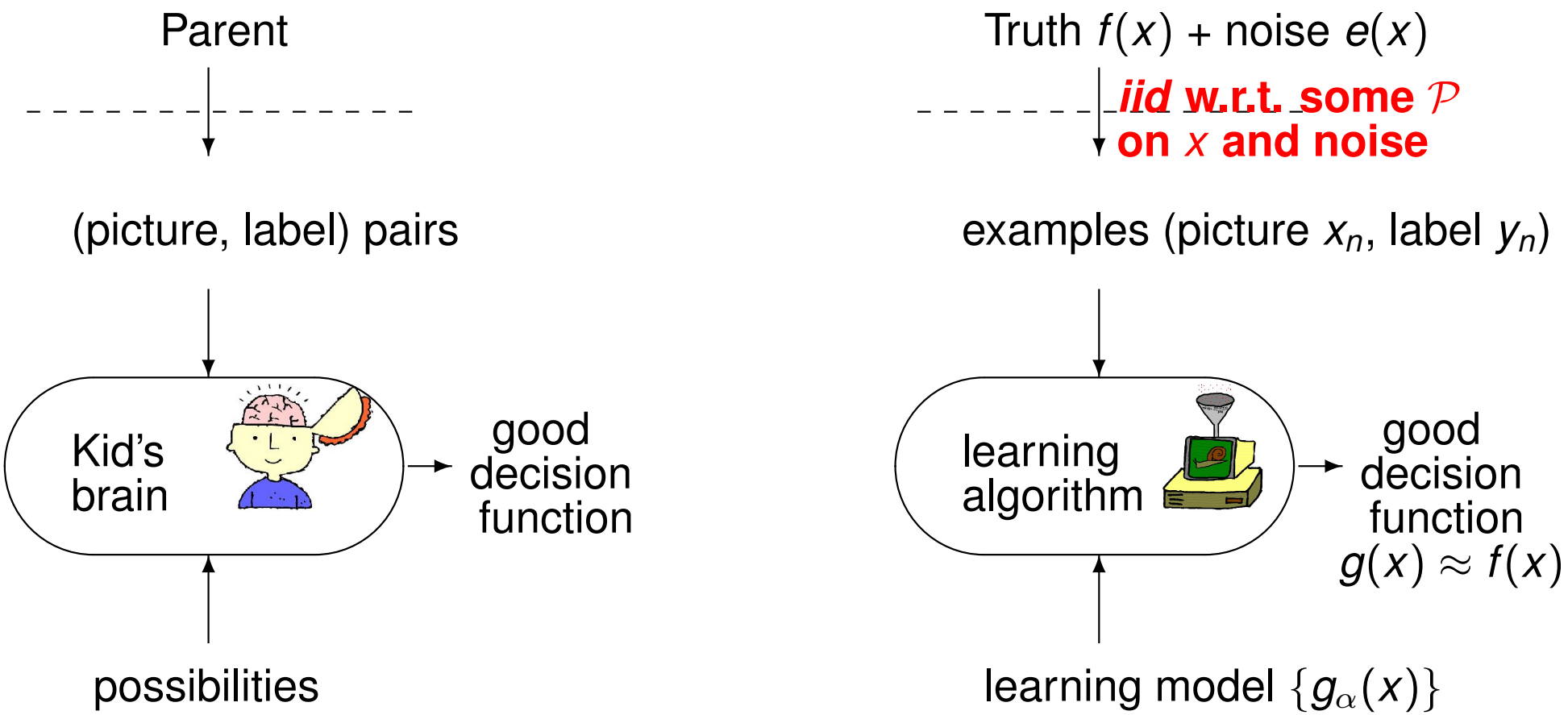
Which Digit Did You Write?

?

one (1) two (2) three (3) four (4)

How can machines learn to classify?

Supervised Statistical Machine Learning



challenge:

see only $\{(x_n, y_n)\}$ without knowing $f(x)$ or $e(x)$

$\stackrel{?}{\Longrightarrow}$ **generalize** to unseen (x, y) w.r.t. $f(x)$

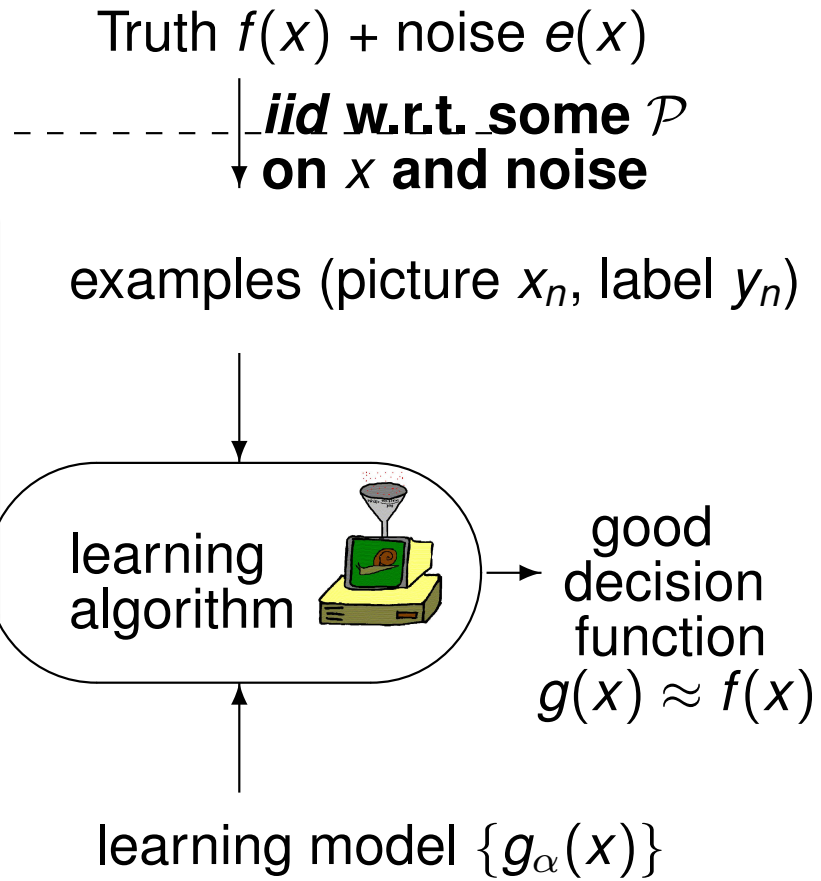
Some Classical Learning Problems

- regression: numerical y_n ($\in \mathbb{R}$)
  - students' scores; stock prices

Translation for Statisticians

- examples == sample from the process
- α == parameters
- decision function == estimate of parameters

- classification: discrete y_n
  - $\{\text{one}, \text{two}, \text{three}, \text{four}\}$
  - $\{\text{apple}, \text{orange}, \text{banana}\}$
  - $\{\text{yes}, \text{no}\}$: **binary classification**

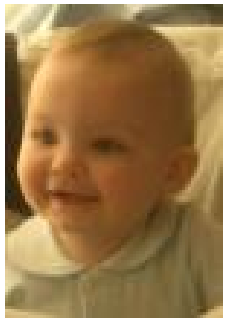


new types of machine learning problems keep coming from new applications

Outline

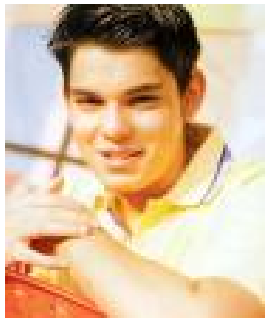
Which Age-Group?

2



infant (1)

child (2)



teen (3)

adult (4)

rank: a finite ordered set of labels $\mathcal{Y} = \{1, 2, \cdots, K\}$

Properties of Ordinal Ranking (1/2)

ranks represent **order** information


infant (1)

<


child (2)

<


teen (3)

<


adult (4)

general classification cannot properly use order information

Hot or Not?

http://www.hotornot.com

Rate People | Meet People | Best Of | Meet Jim and James

HOT or NOT.

Select a rating to see the next picture.

NOT 1 2 3 4 5 6 7 8 9 10 HOT

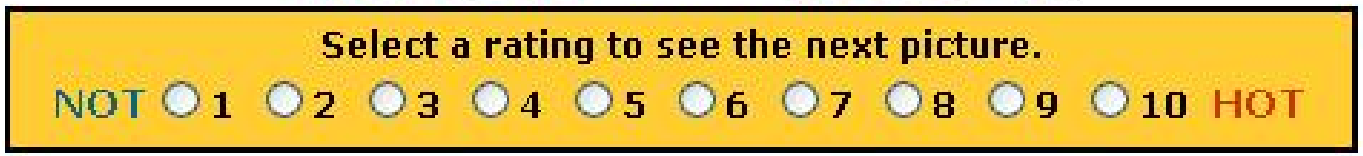
Show me men and women ages 18-25

rank: natural representation of human preferences

Properties of Ordinal Ranking (2/2)

ranks do **not** carry numerical information

- rating 9 not 2.25 times “hotter” than rating 4



- actual metric hidden

infant
(ages 1–3)

child
(ages 4–12)

teen
(ages 13–19)

adult
(ages 20–)

general regression deteriorates without correct numerical information

How Much Did You Like These Movies?

http://www.netflix.com

goal: use “movies you’ve rated” to automatically predict your **preferences (ranks)** on future movies

Ordinal Ranking Setup

Given

N examples (input x_n, rank y_n) $\in \mathcal{X} \times \mathcal{Y}$

- `age-group`: $\mathcal{X}$ = encoding(human pictures), $\mathcal{Y} = \{1, \cdots, 4\}$
- `hotornot`: $\mathcal{X}$ = encoding(human pictures), $\mathcal{Y} = \{1, \cdots, 10\}$
- `netflix`: $\mathcal{X}$ = encoding(movies), $\mathcal{Y} = \{1, \cdots, 5\}$

Goal

an ordinal ranker (decision function) $r(x)$ that "closely predicts" the ranks y associated with some **unseen** inputs x

ordinal ranking: a hot and important research problem

Importance of Ordinal Ranking

- relatively new for machine learning
  (not so new for statisticians)
- connecting classification and regression
- matching human preferences—many applications in social science, information retrieval, psychology and recommendation systems

Formalizing (Non-)Closeness: Cost

- ranks carry no numerical information: how to say "close"?
- artificially quantify the **cost** of being wrong

  e.g. loss of customer loyalty when the system says ★★★★★ but you feel ★★☆☆☆

  Yes! cost == loss for statisticians

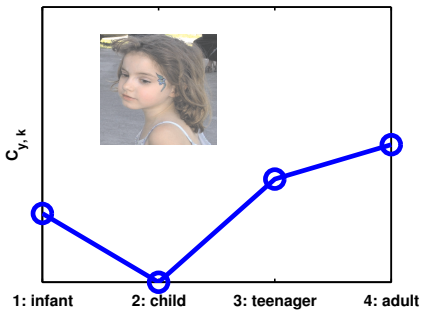
- cost vector $\mathbf{c}$ of example $(x, y, \mathbf{c})$:
  $\mathbf{c}[k] =$ cost when predicting (x, y) as rank k
  e.g. for (`Sweet Home Alabama` , ★★☆☆☆), a proper cost is $\mathbf{c} = (1, 0, 2, 10, 15)$

closely predict: small cost during testing

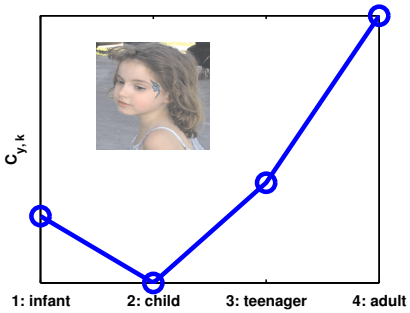
Ordinal Cost Vectors

For an ordinal example $(x, y, \mathbf{c})$, the cost vector $\mathbf{c}$ should

- be consistent with rank y: $\mathbf{c}[y] = \min_k \mathbf{c}[k] \, (= 0)$
- respect order information: V-shaped (**ordinal**) or even convex (**strongly ordinal**)



V-shaped: pay more when predicting further away



convex: pay **increasingly** more when further away

$\mathbf{c}[k] = [\![y \neq k]\!]$	$\mathbf{c}[k] = \lvert y - k \rvert$	$\mathbf{c}[k] = (y - k)^2$
classification: ordinal	absolute: strongly ordinal	squared: strongly ordinal
$(1, 0, 1, 1, 1)$	$(1, 0, 1, 2, 3)$	$(1, 0, 1, 4, 9)$

Our Contributions

a theoretical and algorithmic foundation of ordinal ranking, which reduces ordinal ranking to binary classification, and ...

- provides a methodology for designing new ordinal ranking algorithms with **any** ordinal cost **effortlessly**
- takes many existing ordinal ranking algorithms as **special cases**
- introduces **new theoretical guarantee** on the generalization performance of ordinal rankers
- leads to **superior experimental results**

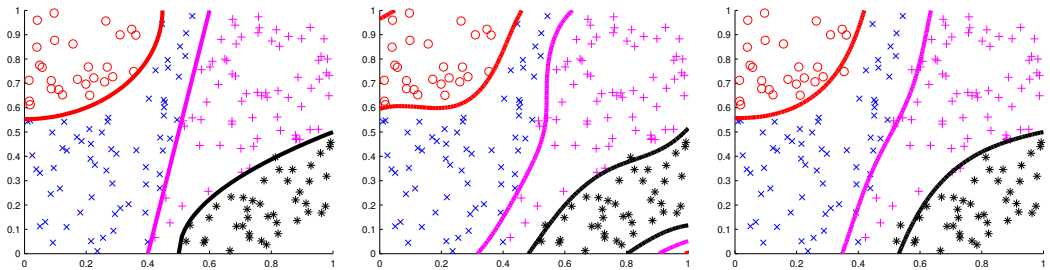


Figure: truth; traditional algorithm; our algorithm

Central Idea: Reduction

(iPod)

(adapter)

(cassette player)

complex ordinal ranking problems

(reduction)

simpler binary classification problems with well-known results on models, algorithms and theories

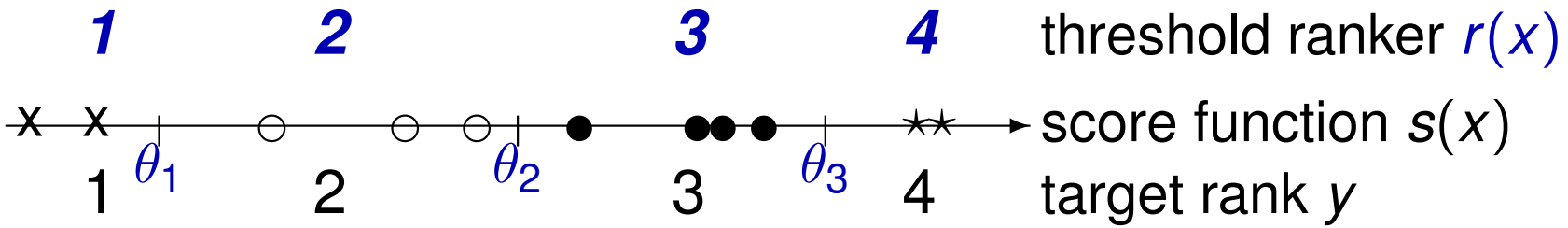
If I have seen further it is by standing on the shoulders of Giants—I. Newton

Outline

Outline

Threshold Ranker

- if getting an ideal score $s(x)$ of a movie x, how to construct the discrete $r(x)$ from an analog $s(x)$?



quantize $s(x)$ by **ordered** (non-uniform) thresholds θ_k

- commonly used in previous work:
  - threshold perceptrons (PRank, Crammer and Singer, 2002)
  - threshold hyperplanes (SVOR, Chu and Keerthi, 2005)
  - threshold ensembles (ORBoost, Lin and Li, 2006)

threshold ranker: $r(x) = \min\{k : s(x) < \theta_k\}$

Key Idea: Associated Binary Queries

getting the rank using a threshold ranker

1. is $s(x) > \theta_1$? Yes
2. is $s(x) > \theta_2$? No
3. is $s(x) > \theta_3$? No
4. is $s(x) > \theta_4$? No

generally, how do we query the rank of a movie x?

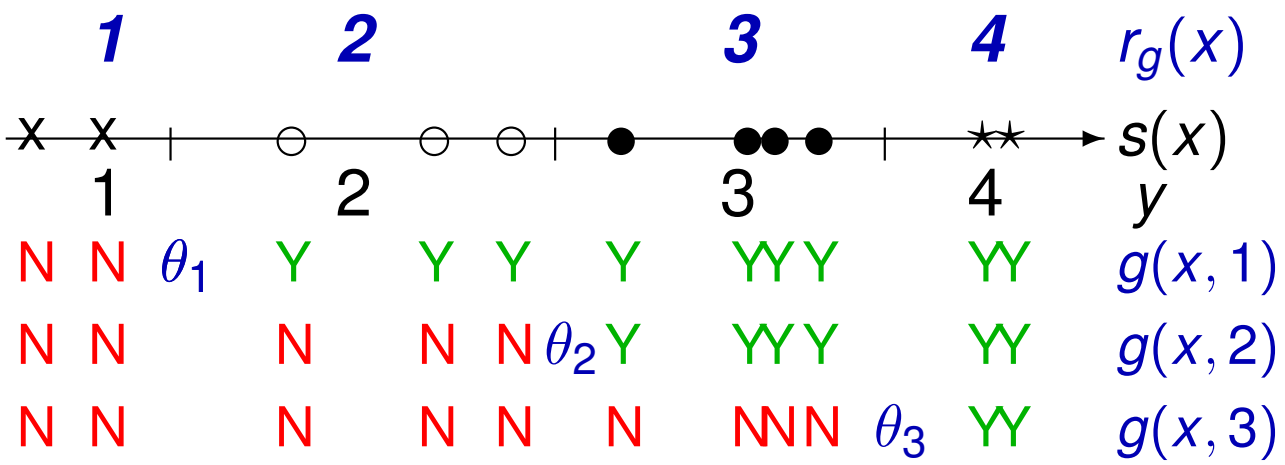
1. is movie x better than rank 1? Yes
2. is movie x better than rank 2? No
3. is movie x better than rank 3? No
4. is movie x better than rank 4? No

associated binary queries:

is movie x better than rank k?

More on Associated Binary Queries

say, the machine uses $g(x, k)$ to answer the query
"*is movie x better than rank k?*"
e.g. for threshold ranker: $g(x, k) = \text{sign}(s(x) - \theta_k)$



associated binary examples:

$$\left(\underbrace{(x, k)}_{k\text{-th associated binary query}}, \underbrace{(z)_k}_{\text{desired answer}} \right)$$

Computing Ranks from Associated Binary Queries

when $g(x,k)$ answers "*is movie x better than rank k?*"

Consider $\big(g(x,1), g(x,2), \cdots, g(x,K-1)\big)$,

- concordant predictions: (Y, Y, N, N, N, N, N)
- extracting the rank from concordant predictions:
  - minimum index searching: $r_g(x) = \min\{k : g(x,k) = \text{N}\}$
  - counting: $r_g(x) = 1 + \sum_k [\![g(x,k) = \text{Y}]\!]$
- two approaches equivalent for concordant predictions
- mistaken/non-concordant predictions? e.g. (Y, N, Y, Y, N, N, Y)

counting: simpler to analyze and robust to mistake

The Counting Approach

Say $y = 5$, i.e., $((z)_1, (z)_2, \cdots, (z)_7) =$ (Y, Y, Y, Y, N, N, N)

- if $g_1(x, k)$ reports concordant predictions (Y, Y, N, N, N, N, N)
  - $g_1(x, k)$ made 2 binary classification errors
  - $r_{g_1}(x) = 3$ by counting: the absolute cost is 2

absolute cost = # of binary classification errors

- if $g_2(x, k)$ reports non-concordant predictions (Y, N, Y, Y, N, N, Y)
  - $g_2(x, k)$ made 2 binary classification errors
  - $r_{g_2}(x) = 5$ by counting: the absolute cost is 0

absolute cost $\leq$ # of binary classification errors

If $(z)_k$ = desired answer & r_g computed by counting,

$$\left|y - r_g(x)\right| \leq \sum_{k=1}^{K-1} [\![(z)_k \neq g(x, k)]\!] .$$

Binary Classification Error v.s. Ordinal Ranking Cost

Say $y = 5$, i.e., $\left((z)_1, (z)_2, \cdots, (z)_7\right) =$ (Y, Y, Y, Y, N, N, N)

- if $g_1(x, k)$ reports concordant predictions (Y, Y, N, N, N, N, N)
  - $g_1(x, k)$ made 2 binary classification errors
  - $r_{g_1}(x) = 3$ by counting: the **squared** cost is 4
- if $g_3(x, k)$ reports concordant predictions (Y, N, N, N, N, N, N)
  - $g_3(x, k)$ made 3 binary classification errors
  - $r_{g_3}(x) = 2$ by counting: the **squared** cost is 9

1 error in binary classification
$\Longrightarrow$ 5 cost in ordinal ranking

Importance of Associated Binary Examples

$(z)_k$	Y	Y	Y	Y	N	N	N	
$g_1(x,k)$	Y	Y	N	N	N	N	N	$\mathbf{c}\left[r_{g_1}(x)\right] = \mathbf{c}[3] = 4$
$g_3(x,k)$	Y	N	N	N	N	N	N	$\mathbf{c}\left[r_{g_3}(x)\right] = \mathbf{c}[2] = 9$
$(w)_k$	7	**5**	3	1	1	3	5	

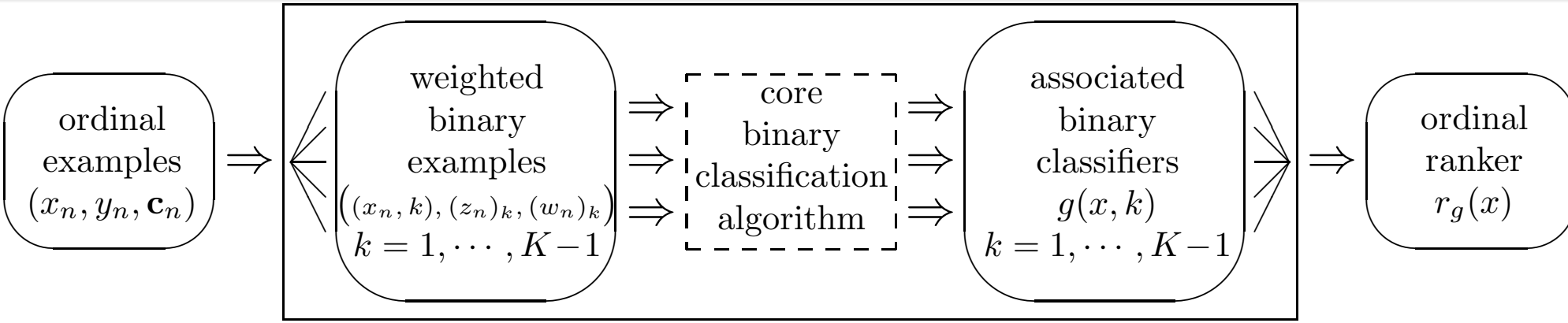
- $(w)_k \equiv \left|\mathbf{c}[k+1] - \mathbf{c}[k]\right|$: the importance of $\left((x,k),(z)_k\right)$

per-example cost bound (Li and Lin, 2007):
for **concordant predictions** or **strongly ordinal costs**

$$\mathbf{c}\left[r_g(x)\right] \leq \sum_{k=1}^{K-1} (w)_k \left[\!\left[(z)_k \neq g(x,k)\right]\!\right]$$

Outline

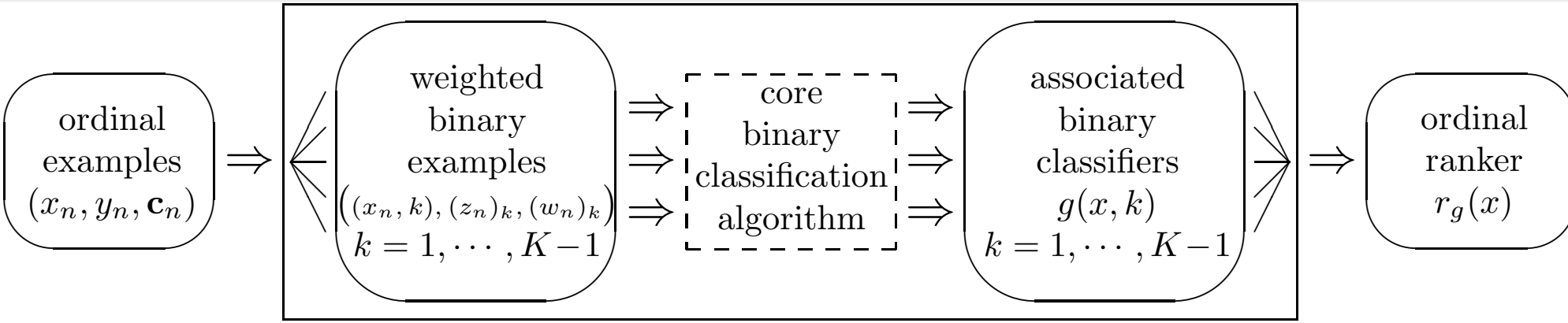
The Reduction Framework (1/2)



1. transform ordinal examples $(x_n, y_n, \mathbf{c}_n)$ to weighted binary examples $((x_n, k), (z_n)_k, (w_n)_k)$
2. use your favorite algorithm on the weighted binary examples and get $K-1$ binary classifiers (i.e., one big joint binary classifier) $g(x, k)$
3. for each new input x, predict its rank using $r_g(x) = 1 + \sum_k [\![g(x, k) = \mathrm{Y}]\!]$

the reduction framework:
systematic & easy to implement

The Reduction Framework (2/2)



- **performance guarantee**:
  accurate binary predictions $\Longrightarrow$ correct ranks
- **wide applicability**:
  works with any ordinal **c** & any binary classification algorithm
- **simplicity**:
  mild computation overheads with $O(NK)$ binary examples
- **state-of-the-art**:
  allows new improvements in binary classification to be immediately inherited by ordinal ranking

Theoretical Guarantees of Reduction (1/3)

1. absolutely good binary classifier
   $\Longrightarrow$ absolutely good ranker? **YES!**

error transformation theorem (Li and Lin, 2007)

For **concordant predictions** or **strongly ordinal costs**, if g makes test error Δ in the induced binary problem, then r_g pays test cost at most Δ in ordinal ranking.

- a one-step extension of the per-example cost bound
- conditions: general and minor
- performance guarantee in the absolute sense

what if no "absolutely good" binary classifier?

Theoretical Guarantees of Reduction (2/3)

1. absolutely good binary classifier
   $\Longrightarrow$ absolutely good ranker? **YES!**
2. relatively good binary classifier
   $\Longrightarrow$ relatively good ranker? **YES!**

regret transformation theorem (Lin, 2008)

For **concordant predictions** or **strongly ordinal costs**,
if g is ϵ-close to the optimal binary classifier g_*,
then r_g is ϵ-close to the optimal ranker r_*.

"reduction to binary" sufficient for algorithm design,
but necessary?

Theoretical Guarantees of Reduction (3/3)

1. absolutely good binary classifier
   $\Longrightarrow$ absolutely good ranker? **YES!**
2. relatively good binary classifier
   $\Longrightarrow$ relatively good ranker? **YES!**
3. algorithm producing relatively good binary classifier
   $\Longleftrightarrow$ algorithm producing relatively good ranker? **YES!**

equivalence theorem (Lin, 2008)

For a general family of **ordinal costs**,
a good ordinal ranking algorithm exists
if & only if a good binary classification algorithm exists
for the corresponding learning model.

ordinal ranking is **equivalent to** binary classification

Outline

Proving New Generalization Theorems

Ordinal Ranking (Li and Lin, 2007)

For linear rankers, with pr. $> 1 - \delta$,

expected test cost of r

$$\leq \underbrace{\frac{\beta}{N} \sum_{n=1}^{N} \sum_{k=1}^{K-1} [\![\bar{\rho}(r(x_n), y_n, k) \leq \Phi]\!]}_{\text{goodness of fit}} + \underbrace{O\left(\text{poly}\left(K, \frac{\log N}{\sqrt{N}}, \frac{1}{\Phi}, \sqrt{\log \frac{1}{\delta}}\right)\right)}_{\text{confidence interval}}$$

Bi. Cl. (Bartlett and Shawe-Taylor, 1998)

For linear classifiers, with pr. $> 1 - \delta$,

expected test err. of g

$$\leq \underbrace{\frac{1}{N} \sum_{n=1}^{N} [\![\bar{\rho}(g(x_n), y_n) \leq \Phi]\!]}_{\text{goodness of fit}} + \underbrace{O\left(\text{poly}\left(\frac{\log N}{\sqrt{N}}, \frac{1}{\Phi}, \sqrt{\log \frac{1}{\delta}}\right)\right)}_{\text{confidence interval}}$$

new ordinal ranking theorem
= reduction + any cost + bin. thm. + math derivation

Outline

Unifying Existing Algorithms

ordinal ranking = reduction + cost + binary classification

ordinal ranking	cost	binary classification algorithm
PRank (Crammer and Singer, 2002)	absolute	modified perceptron rule
kernel ranking (Rajaram et al., 2003)	classification	modified hard-margin SVM
SVOR-EXP	classification	modified soft-margin SVM
SVOR-IMC (Chu and Keerthi, 2005)	absolute	modified soft-margin SVM
ORBoost-LR	classification	modified AdaBoost
ORBoost-All (Lin and Li, 2006)	absolute	modified AdaBoost

- development and implementation time could have been saved
- algorithmic structure revealed (SVOR, ORBoost)

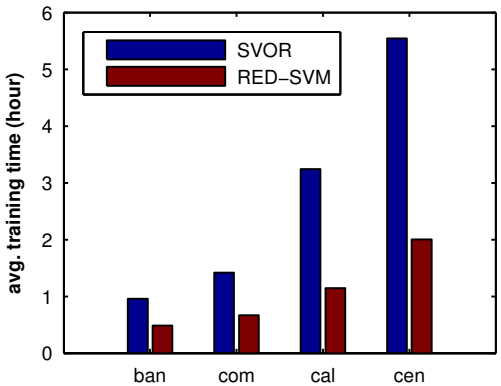
variants of existing algorithms can be designed quickly by tweaking reduction

Designing New Algorithms Effortlessly

ordinal ranking = reduction + cost + binary classification

ordinal ranking	cost	binary classification algorithm
RED-SVM	absolute	standard soft-margin SVM
RED-C4.5 (Li and Lin, 2007)	absolute	standard C4.5 decision tree

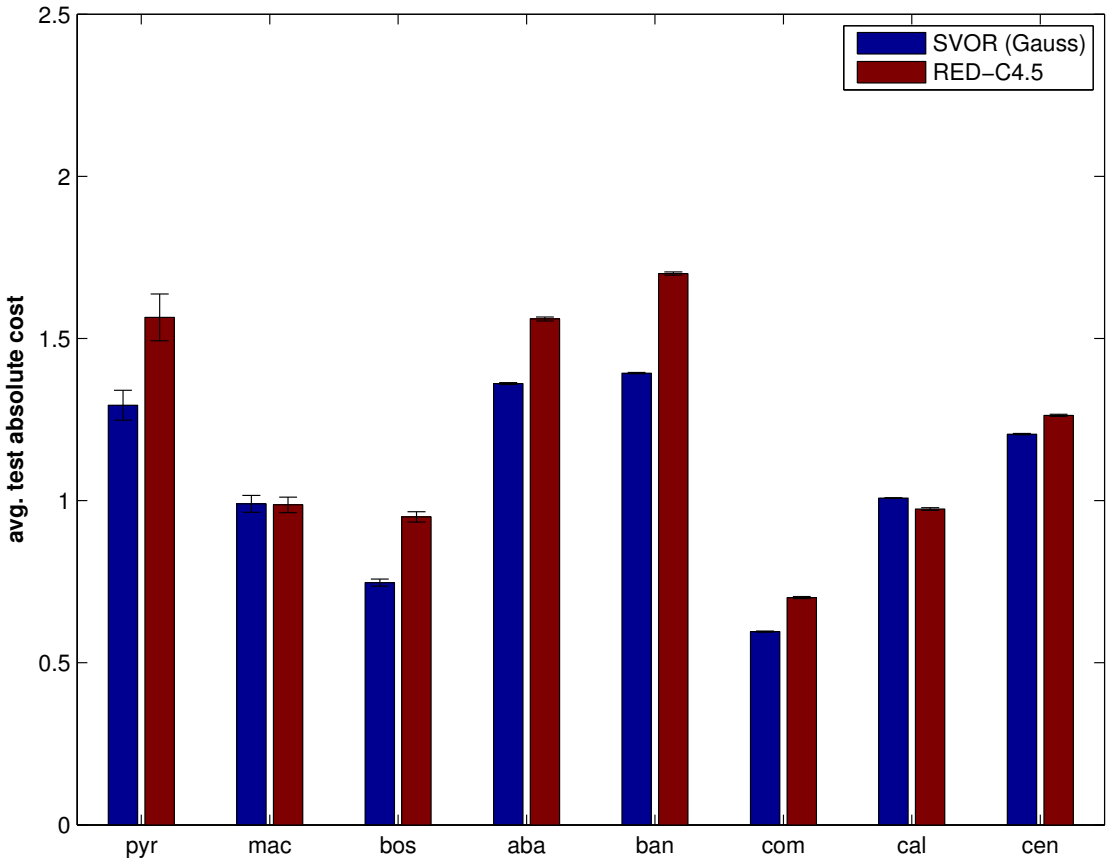
SVOR (modified SVM) v.s. RED-SVM (standard SVM):



advantages of core binary classification algorithm inherited in the new ordinal ranking one

Outline

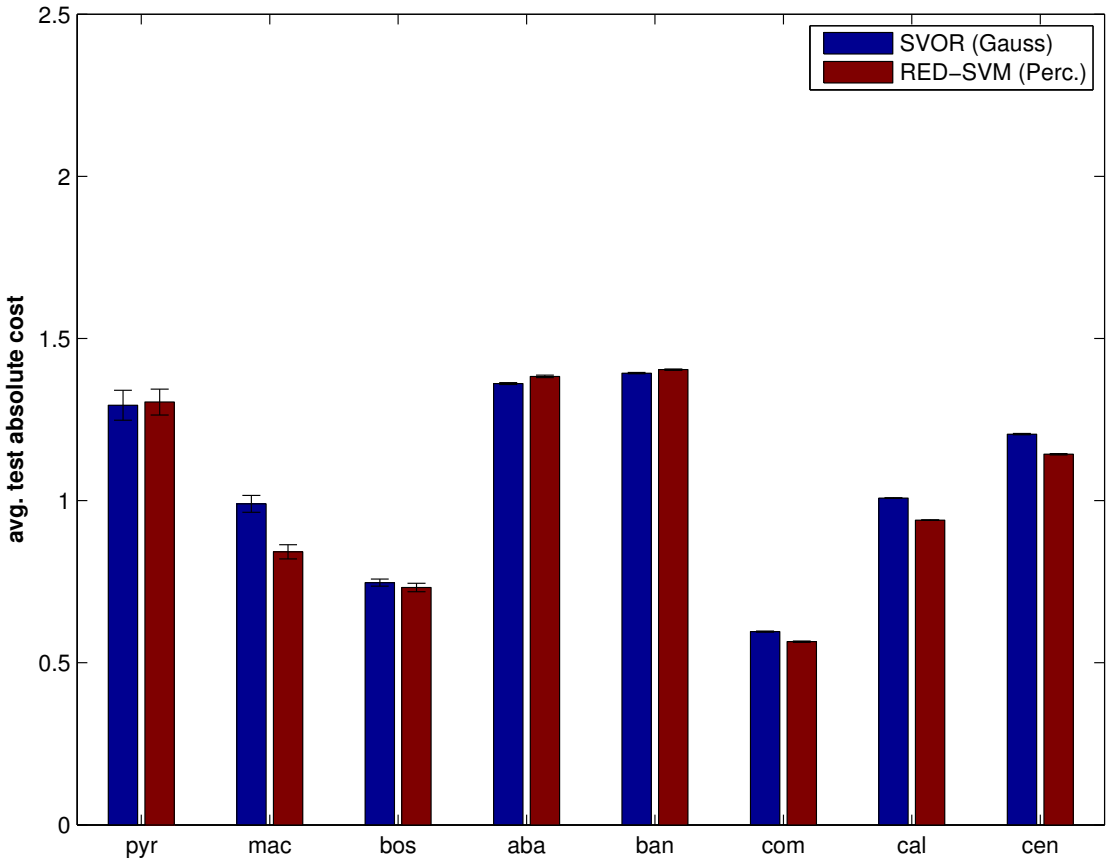
Reduction-C4.5 v.s. SVOR



- C4.5: a (too) simple binary classifier —decision trees
- SVOR: state-of-the-art ordinal ranking algorithm

even simple Reduction-C4.5 sometimes beats SVOR

Reduction-SVM v.s. SVOR



- SVM: one of the most powerful binary classification algorithm
- SVOR: state-of-the-art ordinal ranking algorithm extended from modified SVM

Reduction-SVM without modification often better than SVOR and faster

Outline

Conclusion

- reduction framework: simple but useful
  - **establish** equivalence to binary classification
  - **unify** existing algorithms
  - **simplify** design of new algorithms
  - **facilitate** derivation of new theoretical guarantees
- **superior** experimental results:
  better performance and faster training time

reduction keeps ordinal ranking
up-to-date with binary classification